Isospin breaking corrections to the muon's $g-2$: an update

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RC* collaboration meeting

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Outline

Isospin breaking in lattice calculations

- lattice calculations usually done in the isosymmetric limit
- sources of isospin breaking effects (IBE)
  - strong IBE $\sim \mathcal{O}((m_d - m_u)/\Lambda_{QCD})$
  - QED effects $\sim \mathcal{O}(\alpha_{EM})$

$\Longrightarrow$ IBE effects are important for calculations with precision $\leq 1\ \%$

RC* program: focus on the IB corrections (*masses of mesons, HVP, etc.*):

- non-isosymmetric configurations at several unphysical values of α_{EM} and $m_u - m_d$ + extrapolation to the physical point
- isosymmetric configurations + RM123 method

Outline

Definition of QCD+QED

- theory: QCD+QED with four quarks
- bare parameters: $\beta, \alpha, m_{f=u,d,s,c}$
- six conditions define the **renormalization scheme**:
  - needed to ensure a well-defined continuum limit
  - six observables that can be evaluated precisely on the lattice
  - six inputs (either theoretical estimates or exp. quantities)

Note: the choice of the scheme is arbitrary $\implies$ no effects on the observable quantities at the continuum limit

Definition of QCD+QED

Following the hadronic scheme in 1608.08900 , 2108.11989

Observables		Targets
$(8t_0/a^2)^{1/2}\cdot a$	$\overset{!}{=}$	$(8t_0)^{1/2,\text{phys}} = 0.415$ fm [Bruno et al., 1608.08900]
α_R	$\overset{!}{=}$	$\alpha^{\text{phys}} = 0.007297$
$\phi_0 = 8t_0(m^2_{K^\pm} - m^2_{\pi^\pm})$	$\overset{!}{=}$	$\phi_0^{\text{phys}} = 0.992$
$\phi_1 = 8t_0(m^2_{K^\pm} + m^2_{\pi^\pm} + m^2_{K^0})$	$\overset{!}{=}$	$\phi_1^{\text{phys}} = 2.26$
$\phi_2 = 8t_0(m^2_{K^0} - m^2_{K^\pm})/\alpha_R$	$\overset{!}{=}$	$\phi_2^{\text{phys}} = 2.36$
$\phi_3 = \sqrt{8t_0}(m_{D_S^\pm} + m_{D^\pm} + m_{D^0})$	$\overset{!}{=}$	$\phi_3^{\text{phys}} = 12.0$

PDG values (α^{phys}, ϕ_0^{phys} – ϕ_3^{phys})

- t_0: sets the SU(3) bare coupling
- α_R: sets the U(1) bare coupling
- ϕ_0: sets $m_s - m_d$
- ϕ_1: sets $m_s + m_d + m_u$
- ϕ_2: sets $\delta m_{ud}/\delta_{EM}$
- ϕ_3: sets m_c

Definition of isoQCD

- isosymmetric QCD has four parameters: $\beta, m_{f=l,s,c}$
- same scheme as QCD+QED along $\phi_2 =$const, $\alpha_R \to 0$

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$\phi_3 = \sqrt{8t_0}(m_{D^\pm_S} + m_{D^\pm} + m_{D^0})$	$\stackrel{!}{=}$	$\phi_3^{\text{phys}} = 12.0$	PDG values

Note: the separation of isosymmetric and IB contributions to the observarbles is scheme-dependent!

Our line of constant physics

- we use the hadronic scheme for tuning: $(8t_0)^{1/2}$, $\alpha_R(t_0)$, ϕ_0, ϕ_1, ϕ_2, ϕ_3
- unphysical choice of the targets as starting point

  $(8t_0/a^2)^{1/2} \cdot a \overset{!}{=} 0.415$ fm [Bruno et al., 1608.08900]

  $\alpha_R \in [0, 0.04]$

  $\phi_0 = 8t_0(m^2_{K^\pm} - m^2_{\pi^\pm}) \overset{!}{=} 0 \quad [\phi_0^{\text{phys}} = 0.992]$

  $\phi_1 = 8t_0(m^2_{K^\pm} + m^2_{\pi^\pm} + m^2_{K^0}) \overset{!}{=} 2.11 \quad [\phi_1^{\text{phys}} = 2.26]$

  $\phi_2 = 8t_0(m^2_{K^0} - m^2_{K^\pm})/\alpha_R \overset{!}{=} 2.36 \quad [\phi_2^{\text{phys}} = 2.36]$

  $\phi_3 = \sqrt{8t_0}(m_{D_S^\pm} + m_{D^\pm} + m_{D^0}) \overset{!}{=} 12.1 \quad [\phi_3^{\text{phys}} = 12.0]$

- same inputs for QCD+QED and (3+1) isoQCD simulations ($\phi_2 = 0$)

Outline

RM123 method

- idea: perturbative expansion in $\alpha_{em} = e^2/4\pi$ and $\delta m_{ud} = m_u - m_d$

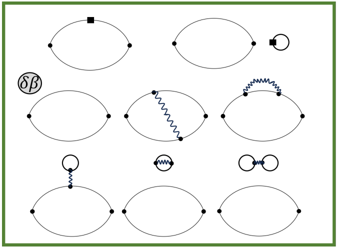
  [De Divitiis et al, 1303.4896]

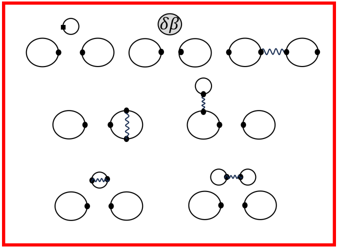
$$\langle \mathcal{O} \rangle (\vec{\varepsilon}) = \langle \mathcal{O} \rangle (m_l, m_s, m_c, \beta, e^2 = 0, \delta m_{ud} = 0) + \delta m_{ud} \partial_{m_l} \langle \mathcal{O} \rangle |_{\delta m_{ud}=0} + e^2 \partial_{e^2} \langle \mathcal{O} \rangle |_{e^2=0}$$

- expansion around the isosymmetric point

$$\langle \mathcal{O} \rangle (\vec{\varepsilon}) = \langle \mathcal{O} \rangle_{(0)} (m_l', m_s', m_c', \beta') + \sum_f \delta m_f \partial_{m_f} \langle \mathcal{O} \rangle |_{(0)} + e^2 \partial_{e^2} \langle \mathcal{O} \rangle |_{(0)} + \delta\beta \partial_\beta \langle \mathcal{O} \rangle |_{(0)}$$

- need to find six IB parameters $\delta m_i \equiv (m_i - m_i'), \delta\beta \equiv (\beta - \beta'), e^2$





Definition of IBE

Each renormalization condition is expanded in $\delta\vec{\varepsilon} \equiv (a\delta m_i, \delta\beta, e^2)$, e.g:

$$\phi_0^{QCD+QED}(am_i, \beta, e^2) = \phi_0^{isoQCD}(am_i', \beta')$$

$$\Big\downarrow \text{RM123}$$

$$\phi_0^{isoQCD}(am_i', \beta') + \sum_i \delta\varepsilon_i \frac{\partial}{\partial\varepsilon_i}\phi_0|_{am_i',\beta'} = \phi_0^{isoQCD}(am_i', \beta')$$

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1) $\sum_i \delta\varepsilon_i \frac{\partial}{\partial\varepsilon_i} t_0 = 0$

2) $\alpha_R(t_0) = \alpha_{em}$

3) $\sum_i \delta\varepsilon_i \frac{\partial}{\partial\varepsilon_i} \phi_0 = 0$

4) $\sum_i \delta\varepsilon_i \frac{\partial}{\partial\varepsilon_i} \phi_1 = 0$

5) $\sum_i \delta\varepsilon_i \frac{\partial}{\partial\varepsilon_i} \phi_2 = \phi_2^{phys}$

6) $\sum_i \delta\varepsilon_i \frac{\partial}{\partial\varepsilon_i} \phi_3 = 0$

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- electro-quenched approximation

$\Longrightarrow$ no corrections from sea quarks

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$\downarrow$ RM123

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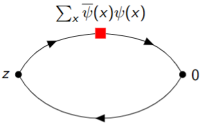
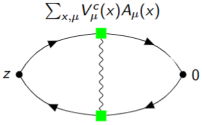
Strategy

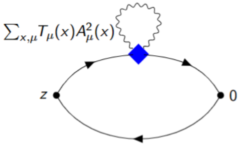
Steps:

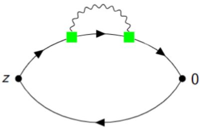
1. Computation of derivatives of pseudoscalar correlator
2. Solution of the renormalization conditions system to derive the quark mass shifts (u and d/s)
3. Computation of derivatives of the vector-vector correlator
4. Analysis of δa_μ^{HVP}

Computation of derivatives of pseudoscalar correlator on 200/250 configurations, 10 point sources per conf.









- Sequential propagators: further inversions with a modified source
- Photon field (Feynman gauge) estimated stochastically (1 source per point source)

$$\hat{A}_\mu(x) = \tfrac{1}{\sqrt{N}} \sum_k \frac{e^{-ikx}}{\sqrt{\hat{k}^2}} \tilde{B}_\mu(k), \qquad P(B) \propto \exp\bigl(-B^2_\mu(k)\bigr)$$

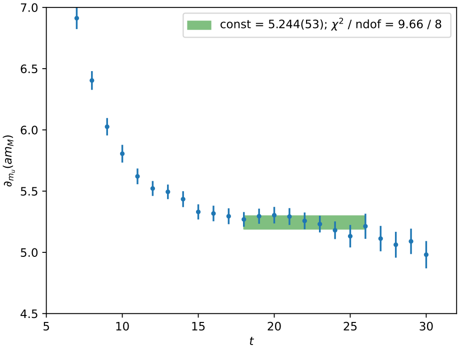
$$\Lambda_{\mu\nu}(x-y) = \tfrac{\delta_{\mu\nu}}{N} \sum_k \frac{e^{ik(x-y)}}{\hat{k}^2} \simeq \tfrac{1}{n_{src}} \sum_{i=1}^{n_{src}} \hat{A}^i_\mu(x) \hat{A}^i_\nu(y)$$

Corrections to mesons' masses

- Each diagram gives a contribution to $a\delta m_M = \sum_i \frac{\partial a m_M}{\partial \varepsilon_i}\delta\varepsilon_i$

$$\partial_{\varepsilon_i}(am_M)(t) = \left[\frac{\partial_{\varepsilon_i}G(t)}{G^{(0)}(t)} - \frac{\partial_{\varepsilon_i}G(t+1)}{G^{(0)}(t+1)}\right]\times$$

$$\times\frac{1}{(T/2-t)\tanh\Big(am_M^{(0)}(T/2-t)\Big) - (T/2-(t+1))\tanh\Big(am_M^{(0)}(T/2-(t+1))\Big)}$$



Corrections to mesons' masses

Ensemble	V	n.cnfg	ϕ_1(meas)	a[fm]	m_π[MeV]
A400	64×32^3	200	2.110(32)	0.05394(27)	398.9(3.7)
B400	80×48^3	250	2.172(20)	0.05404(14)	404.5(1.9)

Derivatives:

Quantity	Mass der.	Tad.	Ph. ex	Ph. self	Tot QED
$am_{\pi^\pm}$	5.21(8)	0.638(10)	0.00219(5)	-0.03003(46)	0.611(10)
$am_{K^\pm}$	5.21(8)	0.638(10)	0.00219(5)	-0.03003(46)	0.611(10)
am_{K^0}	5.21(8)	0.255(4)	-0.00109(2)	-0.01201(18)	0.242(4)

Quantity	Mass der.	Tad.	Ph. ex	Ph. self	Tot QED
$am_{\pi^\pm}$	5.19(5)	0.639(6)	0.00274(5)	-0.02944(31)	0.612(6)
$am_{K^\pm}$	5.19(5)	0.639(6)	0.00274(5)	-0.02944(31)	0.612(6)
am_{K^0}	5.19(5)	0.255(2)	-0.00137(3)	-0.01178(13)	0.243(2)

IB parameters

Solution of the system:

$$3)\ \sum_i \delta\varepsilon_i \frac{\partial}{\partial\varepsilon_i}\phi_0 = 0$$
$$4)\ \sum_i \delta\varepsilon_i \frac{\partial}{\partial\varepsilon_i}\phi_1 = 0$$
$$5)\ \sum_i \delta\varepsilon_i \frac{\partial}{\partial\varepsilon_i}\phi_2 = \phi_2^{phys}$$

A400a00b324

$$\delta\beta = 0$$
$$e^2 = 0.091701237$$
$$a\delta m_u = -0.008781(14)$$
$$a\delta m_d = -0.002046(5)$$
$$a\delta m_s = -0.002046(5)$$

B400a00b324

$$\delta\beta = 0$$
$$e^2 = 0.091701237$$
$$a\delta m_u = -0.008837(3)$$
$$a\delta m_d = -0.002055(1)$$
$$a\delta m_s = -0.002055(1)$$

IB parameters

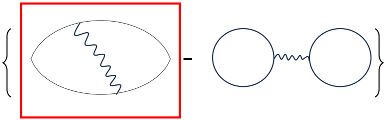
Expansion of the meson masses :

- A400: $am_{\pi^\pm} = 0.109(1) + 5.22(8)(a\delta m_u + a\delta m_d) + 0.612(9)e^2 = 0.1086(10)$ [397.3(3.5) Mev]
- A400: $am_{K^0} = 0.109(1) + 5.22(8)(a\delta m_d + a\delta m_s) + 0.243(4)e^2 = 0.1099(9)$ [402.2(3.5) Mev]
- B400: $am_{\pi^\pm} = 0.1108(7) + 5.19(5)(a\delta m_u + a\delta m_d) + 0.612(6)e^2 = 0.1103(7)$ [403(2) MeV]
- B400: $am_{K^0} = 0.1108(7) + 5.19(5)(a\delta m_d + a\delta m_s) + 0.242(2)e^2 = 0.1117(7)$ [407(2) MeV]
- A400/B400: $am_{K^\pm} = am_{\pi^\pm}$

(Half-)Prediction:

- pion-splitting

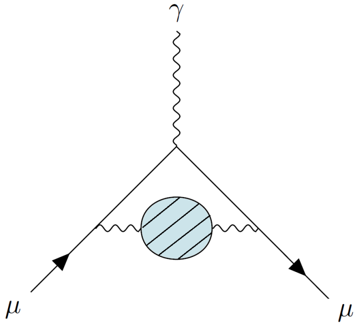
$$am_{\pi^\pm} - am_{\pi^0} \propto e^2 \frac{(q_u - q_d)^2}{2}$$



A400: 0.00045(1) [1.65(4) MeV]
B400: 0.00056(1) [2.06(3) MeV]

Outline

HVP calculation



- time-momentum representation

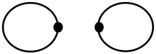
$$G(t) = -\frac{1}{3} \sum_{k=1,2,3} \sum_{\vec{x}} \langle V_k^{em}(x) V_k^{em}(0) \rangle$$

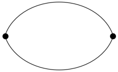
$$a_\mu^{HVP} = \left(\frac{\alpha}{\pi}\right)^2 \sum_t G(t) \tilde{K}(t; m_\mu)$$

- two discretizations of the correlator (local-local,conserved-local)

- two types of contributions

$$\langle V_k^l(x) V_k^l(0) \rangle = \underbrace{\textstyle\sum_{f,f'} q_f q_{f'} \, \mathrm{tr}\left[\gamma_k D_f^{-1}(x|x)\right] \cdot \mathrm{tr}\left[\gamma_k D_{f'}^{-1}(0|0)\right]}_{\text{disconnected } (\sim 2\% \text{ of the total})} - \underbrace{\textstyle\sum_f q_f^2 \, \mathrm{tr}\left[\gamma_k D_f^{-1}(x|0) \gamma_k D_f^{-1}(0|x)\right]}_{\text{connected}}$$





IB corrections to the HVP

For instance, we consider the local local discretization

$$a_\mu^{HVP} = \left(\frac{\alpha}{\pi}\right)^2 \sum_t Z_V^2 G^{ll}(t) \tilde{K}(t; m_\mu)$$

1) corrections to the correlator

$$\delta a_{\mu,(1)}^{HVP} = \left(\frac{\alpha}{\pi}\right)^2 \sum_t (Z_V^{(0)})^2 \delta G^{ll}(t) \tilde{K}(t; m_\mu)$$

$$G^{ll}(t) = G^{ll}(t)^{(0)} + \delta G^{ll}(t) = G^{ll}(t)^{(0)} + \sum_f \delta m_f \frac{\partial G^{ll}(t)}{\partial m_f}\bigg|_{(0)} + \frac{1}{2} e^2 \frac{\partial^2 G^{ll}(t)}{\partial e^2}\bigg|_{(0)}$$

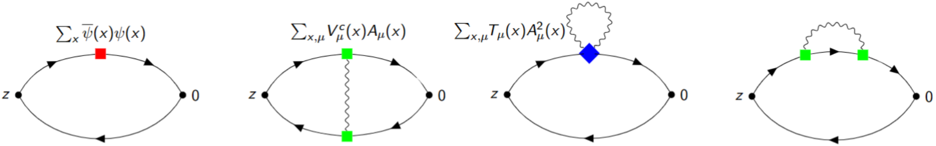
2) corrections to the renormalization constant

$$\delta a_{\mu,(2)}^{HVP} = \left(\frac{\alpha}{\pi}\right)^2 \sum_t 2 Z_V^{(0)} \delta Z_V G^{ll}(t)^{(0)} \tilde{K}(t; m_\mu)$$

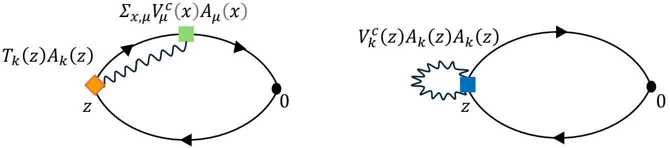
$$Z_V = Z_V^{(0)} + \delta Z_V = Z_V^{(0)} + \sum_f \delta m_f \frac{\partial Z_V}{\partial m_f}\bigg|_{(0)} + \frac{1}{2} e^2 \frac{\partial^2 Z_V}{\partial e^2}\bigg|_{(0)}$$

Corrections to the correlator

- leading IB effects in the electro-quenched approximation

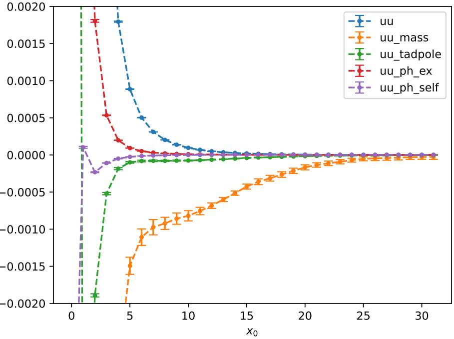


- if conserved current at the sink (no additional propagators needed)



Corrections to the correlator

- reconstruction of the vector correlator derivatives

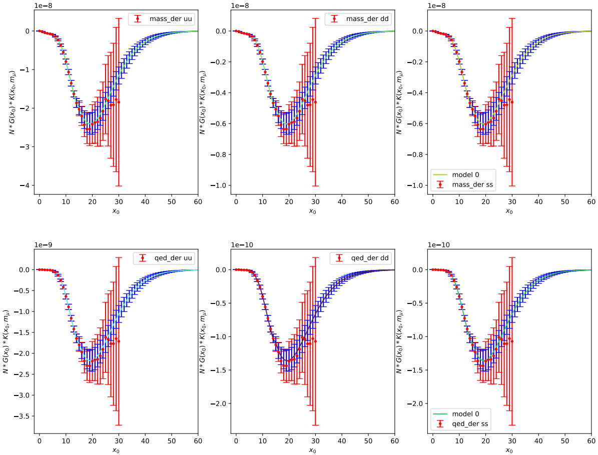


$$G(t) \simeq (A^{(0)} + \delta A)e^{-(m^{(0)}+\delta m)t}$$

$$G^{(1)}(t) \simeq A^{(0)}e^{-m^{(0)}t}(1 + \delta A/A^{(0)} - \delta m t)$$

$$\frac{G^{(1)}(t)-G^{(0)}(t)}{G^{(0)}(t)} \simeq \delta A/A^{(0)} - \delta m t$$

Preliminary results (Ensemble A400a00b324)



- After the x_0-integration:

$$\delta a^{\mathrm{HVP}}_{\mu,(1)} = -4.8(7)\times 10^{-7} a\delta m_u +$$

$$-1.20(17)\times 10^{-7}(a\delta m_d \times 2)+$$

$$-(4.3(6)+0.24(7)\times 2)\times 10^{-8}e^2$$

- Inserting the quark mass shifts:

$$\delta a^{\mathrm{HVP}}_{\mu,(1)} = 2.88(24)\times 10^{-10}$$

$$a^{\mathrm{HVP,LO}}_{\mu,(u+d+s)} = 284.5(7.8)\times 10^{-10}$$

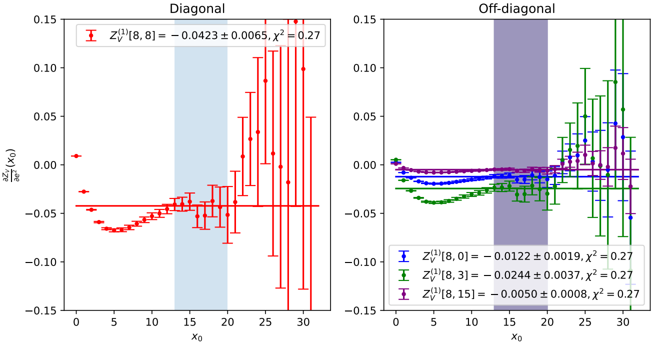
Corrections to Z_V

- Renormalization condition

$$Z_{V_R V_l} = \lim_{x_0\to\infty} G^{cl}(x_0)(G^{ll}(x_0))^{-1} \to \begin{pmatrix} 0.6578(9) & 0.0(0) & 0.0(0) & 0.0220(6) \\ 0.0(0) & 0.6766(12) & 0.0(0) & 0.0(0) \\ 0.0(0) & 0.0(0) & 0.6766(12) & 0.0(0) \\ 0.0439(12) & 0.0(0) & 0.0(0) & 0.6224(11) \end{pmatrix}$$

- Taking derivatives

$$\frac{\partial Z_{V_R V_l}}{\partial \varepsilon_i} = \lim_{x_0\to\infty}\left[\frac{\partial G^{cl}}{\partial \varepsilon_i}(x_0) - G^{cl}(x_0)\,(G^{ll}(x_0))^{-1}\frac{\partial G^{ll}}{\partial \varepsilon_i}(x_0)\right]\cdot (G^{ll}(x_0))^{-1}$$



Preliminary results (Ensemble A400a00b324)

$$(\delta Z_{V_R V_l})_{m_u} = \begin{pmatrix} -0.076(33) & -0.152(65) & -0.088(38) & -0.031(13) \\ -0.076(33) & -0.152(65) & -0.088(38) & -0.031(13) \\ -0.044(19) & -0.088(38) & -0.051(22) & -0.018(8) \\ -0.062(27) & -0.124(53) & -0.072(31) & -0.025(11) \end{pmatrix} \cdot a\delta m_u$$

$$(\delta Z_{V_R V_l})_{m_d} = \begin{pmatrix} -0.076(33) & 0.152(65) & -0.088(38) & -0.031(13) \\ -0.076(33) & -0.152(65) & 0.088(38) & 0.031(13) \\ -0.044(19) & 0.088(38) & -0.051(22) & -0.018(8) \\ -0.062(27) & 0.124(53) & -0.072(31) & -0.025(11) \end{pmatrix} \cdot a\delta m_d$$

$$(\delta Z_{V_R V_l})_{m_s} = \begin{pmatrix} -0.076(33) & 0.0(0) & 0.175(74) & -0.031(13) \\ 0.0(0) & 0.0(0) & 0.0(0) & 0.0(0) \\ 0.088(38) & 0.0(0) & -0.202(87) & 0.039(15) \\ -0.062(27) & 0.0(0) & 0.143(62) & -0.025(11) \end{pmatrix} \cdot a\delta m_s$$

$$(\delta Z_{V_R V_l})_{e^2} = \begin{pmatrix} -0.0423(65) & -0.0423(65) & -0.0244(37) & -0.0173(26) \\ -0.0212(32) & -0.071(11) & -0.0244(37) & -0.0086(13) \\ -0.0122(19) & -0.0244(37) & -0.0423(65) & -0.0050(8) \\ -0.0346(53) & -0.0346(53) & -0.0199(30) & -0.0141(22) \end{pmatrix} \cdot e^2$$

Conclusions

To summarize:

- Possible strategy for defining the isospin-breaking effects to the HVP
- Computation of the derivatives (valence contributions) of the light quark pseudoscalar correlator and corrections to meson masses on A400 and B400
- Corrections to the vector correlator and the renormalization constant (in progress)

Next steps:

- Finalize the analysis (AIC for combining models)
- Estimate finite-volume corrections

Conclusions

- Write-up: paper on the isospin-breaking corrections to the HVP

PREPARED FOR SUBMISSION TO JHEP

Isospin breaking corrections to the hadronic vacuum polarisation from lattice simulations with C⋆ boundary conditions

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ABSTRACT: Abstract...

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