Update on the RM123 method

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Map

RM123: The Story So Far

The method

The observable $\mathcal{O}[U, A, \chi]$ on the full path integral[a]:

$$\langle \mathcal{O} \rangle = \frac{\int dU d\chi dA \; e^{-S_{\mathrm{Iso}}[U,\chi]} e^{-S_{\mathrm{IB}}[U,A,\chi]} e^{-S_{\gamma}[A]} \mathcal{O}[U, A, \chi]}{\int dU d\chi dA \; e^{-S_{\mathrm{Iso}}[U,\chi]} e^{-S_{\mathrm{IB}}[A,\chi,U]} e^{-S_{\gamma}[A]}}$$

$$= \frac{\langle \int dA \; e^{-S_{\mathrm{IB}}[U,A,\chi]} e^{-S_{\gamma}[A]} \mathcal{O}[U, A, \chi] \rangle_{\mathrm{Iso}}}{\langle \int dA \; e^{-S_{\mathrm{IB}}[U,A,\chi]} e^{-S_{\gamma}[A]} \rangle_{\mathrm{Iso}}}$$

$$= \langle \overset{\parallel}{\mathcal{O}} \rangle_{\mathrm{Iso}} \underbrace{- \langle S_{\mathrm{IB}} \mathcal{O} \rangle_{\mathrm{Iso}\gamma,c} + \frac{1}{2} \langle S_{\mathrm{IB}} S_{\mathrm{IB}} \mathcal{O} \rangle_{\mathrm{Iso}\gamma,c}}_{valence-valence} + \underbrace{\langle \overset{\parallel}{S}_{\mathrm{IB}} S_{\mathrm{IB}} \mathcal{O} \rangle_{\mathrm{Iso}\gamma,c}}_{sea-valence}$$

$$\underbrace{- \langle \overset{\parallel}{S}_{\mathrm{IB}} \overset{\parallel}{\mathcal{O}} \rangle_{\mathrm{Iso}\gamma,c} + \frac{1}{2} \langle \overset{\parallel}{S}_{\mathrm{IB}} \overset{\parallel}{S}_{\mathrm{IB}} \overset{\parallel}{\mathcal{O}} \rangle_{\mathrm{Iso}\gamma,c} + \frac{1}{2} \langle S_{\mathrm{IB}} S_{\mathrm{IB}} \overset{\parallel}{\mathcal{O}} \rangle_{\mathrm{Iso}\gamma,c}}_{sea-sea}$$

$$+ o\left(\delta m_{ud}^2, e^4, e^2 \delta m_{ud}\right).$$

[a] Divitiis et al., "Leading isospin breaking effects on the lattice".

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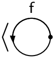
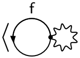
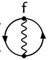
Sea-Sea Diagrams

Considering a pure gluonic observable $\mathcal{O}$ the disconnected terms are:

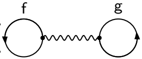
$\langle\mathcal{O}\rangle = \langle\mathcal{O}\rangle_{\text{Iso}} - \delta\beta\langle S_{gauge}\mathcal{O}\rangle_{\text{Iso},c} + \sum_f \delta m_f \langle$ 

$\mathcal{O}\rangle_{\text{Iso},c}$

$+e^2 \Big[\sum_f \hat{q}_f^2 \Big(\langle$ 

$\mathcal{O}\rangle_{\text{Iso},c} + \langle$ 

$\mathcal{O}\rangle_{\text{Iso},c} \Big)$

$+ \sum_{fg} \hat{q}_f \hat{q}_g \langle$ 

$\mathcal{O}\rangle_{\text{Iso},c} \Big] .$

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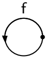
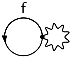
Sea-Sea Diagrams

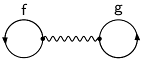
$$\frac{1}{2}\sum_{x}\text{Re tr}\left[D_f^{-1}(x,x)\right] =$$



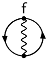
$$\frac{1}{4}\sum_{xy}\langle\text{Re tr}\left[D_f^{-1}(x,y)T(y,x)\right]\rangle_\gamma =$$



$$\left.\begin{array}{l}
\frac{1}{8}\sum_{xyzw,\mu}\langle\text{Im tr}\left[J(x,y)D_f^{-1}(y,x)\right]\text{Im tr}\left[J(z,w)D_g^{-1}(w,z)\right]\rangle_\gamma \\
\frac{c_f c_s}{128}\sum_{xy,\mu\nu\rho\sigma}\langle\text{Re tr}\left[\sigma_{\mu\nu}\hat{A}_{\mu\nu}(x)D_f^{-1}(x,x)\right]\text{Re tr}\left[\sigma_{\rho\sigma}\hat{A}_{\rho\sigma}(y)D_g^{-1}(y,y)\right]\rangle_\gamma \\
\frac{c_f}{16}\sum_{xyz,\mu\nu}\langle\text{Re tr}\left[\sigma_{\mu\nu}\hat{A}_{\mu\nu}(x)D_f^{-1}(x,x)\right]\text{Im tr}\left[J(y,z)D_g^{-1}(z,y)\right]\rangle_\gamma
\end{array}\right\} =$$



$$\left.\begin{array}{l}
\frac{1}{4}\sum_{xyzw,\mu}\langle\text{Re tr}\left[J(x,y)D_f^{-1}(y,z)J(z,w)D_f^{-1}(w,x)\right]\rangle_\gamma \\
-\frac{c_f^2}{64}\sum_{xy,\mu\nu\rho\sigma}\langle\text{Re tr}\left[\sigma_{\mu\nu}\hat{A}_{\mu\nu}(x)D_f^{-1}(x,y)\sigma_{\rho\sigma}\hat{A}_{\rho\sigma}(y)D_f^{-1}(y,x)\right]\rangle_\gamma \\
-\frac{c_f}{8}\sum_{xyz,\mu\nu}\langle\text{Im tr}\left[\sigma_{\mu\nu}\hat{A}_{\mu\nu}(x)D_f^{-1}(x,y)J(y,z)D_f^{-1}(z,x)\right]\rangle_\gamma
\end{array}\right\} =$$



RM123: The Story So Far

Sea-Sea Diagrams: Results

lattice	a [fm]	m_π [MeV]	m_D [MeV]	no. cnfg	*nsrc* per lv per cnfg
64×32^3	0.05393(24)	398.5(4.7)	1912.7(5.7)	50	400
80×48^3	0.05400(14)	401.9(1.4)	1908.5(4.5)	50	100

Table: Configurations used[b].

Parameters used:

$$\delta\beta = \delta c_{\text{SW}}^{\text{SU}(3)f} = 0$$

$$c_{\text{SW}}^{\text{U}(1)f} = 1$$

$$e = e_{\text{phys}}$$

δm_f from QCD+QED simulation.

From QCD $\text{N}_f = 3 + 1$ to QCD+QED $\text{N}_f = 1 + 2 + 1$.

[b] Bushnaq et al., "First results on QCD+QED with C* boundary conditions".

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Sea-Sea Diagrams: t_0

The scale is set using the auxiliary Wilson-flow observable t_0:

$$t_0^2 \langle E(t) \rangle \Big|_{t=t_0} = 0.3$$

Observable	Value [lattice units]
t_0^{QCD}	7.36 ± 0.04
δt_0^{Mass}	0.28 ± 0.21
δt_0^{Tad}	-0.49 ± 0.27
$\delta t_0^{\text{Bubbles}}$	-0.01 ± 0.06
δt_0^{Self}	0.12 ± 0.13
δt_0^{Tot}	-0.11 ± 0.13
$t_0^{\text{QCD+QED}_{\text{RM123}}}$	7.26 ± 0.14
$t_0^{\text{QCD+QED}}$	7.54 ± 0.05

Table: IBE effects on t_0 for A400a00b324.

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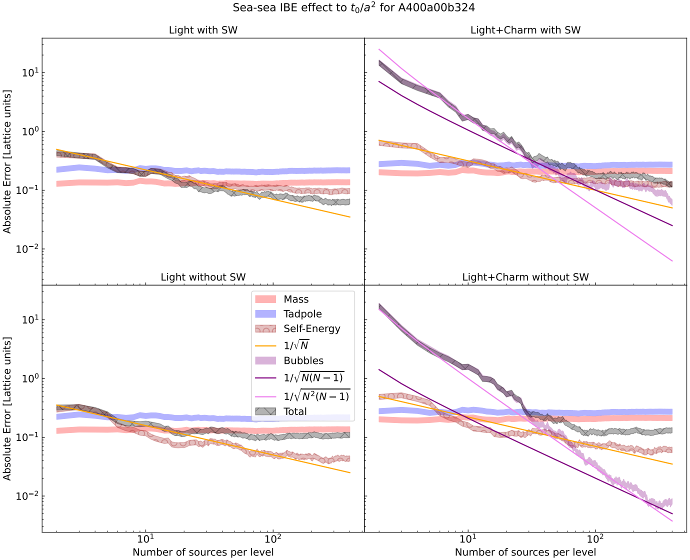
Sea-Sea Diagrams: t_0 error on A400a00b324



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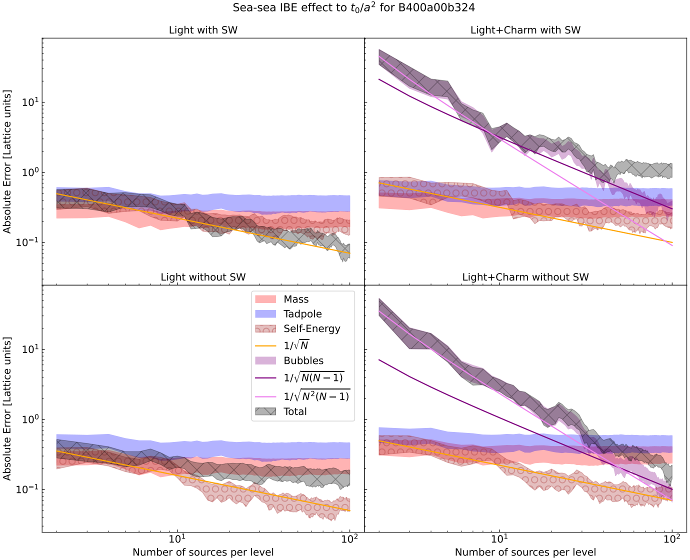
Sea-Sea Diagrams: t_0 error on B400a00b324



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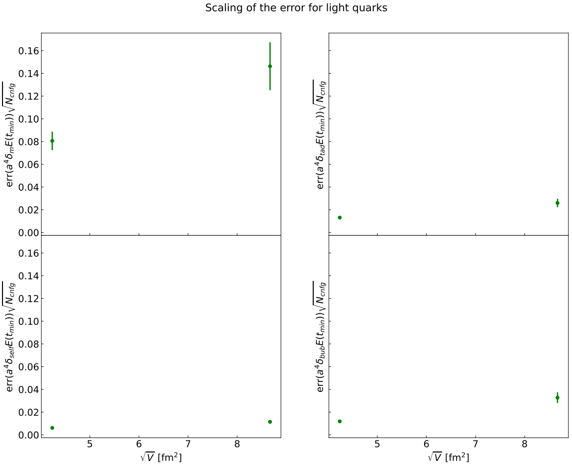
Sea-Sea Diagrams: Volume scaling of the variance

For a gluonic quantity like E(t) the scaling of the error is $a^{-1}\sqrt{V}$ for strong isospin-breaking effects and $a^{/2}\sqrt{V}$ for electro-magnetic isospin-breaking effects.



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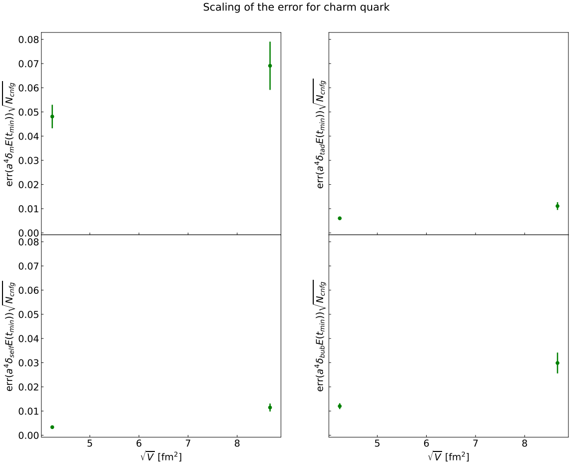
Sea-Sea Diagrams: Volume scaling of the variance



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Sea-Sea Diagrams: $m_{\pi^\pm}$

Sea-sea contribution to the pion mass.

Observable	Value [MeV]
$m_{\pi^\pm}^{\text{QCD}}$	408 ± 7
$\delta m_{\pi^\pm}^{\text{Mass}}$	-63 ± 41
$\delta m_{\pi^\pm}^{\text{Tad}}$	50 ± 60
$\delta m_{\pi^\pm}^{\text{Bubbles}}$	7 ± 9
$\delta m_{\pi^\pm}^{\text{Self}}$	-27 ± 25
$\delta m_{\pi^\pm}^{\text{Tot}}$	-32 ± 20
$m_{\pi^\pm}^{\text{QCD+QED}_{\text{RM123}}}$	375 ± 21
$m_{\pi^\pm}^{\text{QCD+QED}}$	401 ± 7

Table: Sea-Sea IBE effects on $m_{\pi^\pm}$ on A400a00b324.

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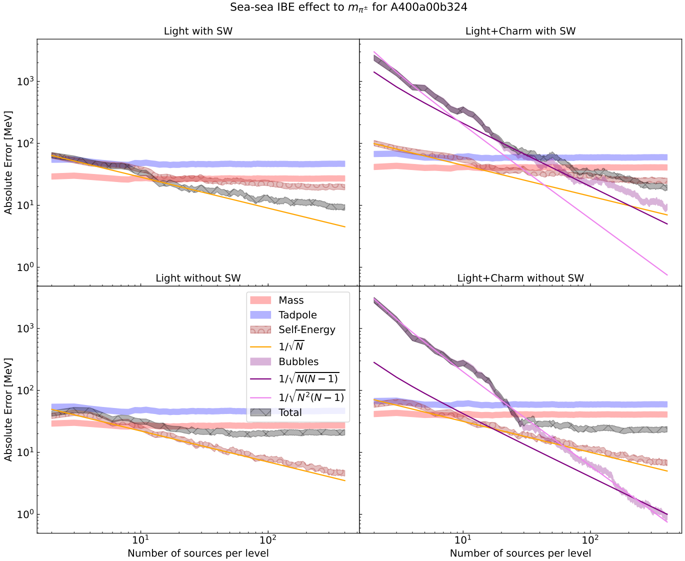
Sea-Sea Diagrams: $m_{\pi^\pm}$ error on A400a00b324



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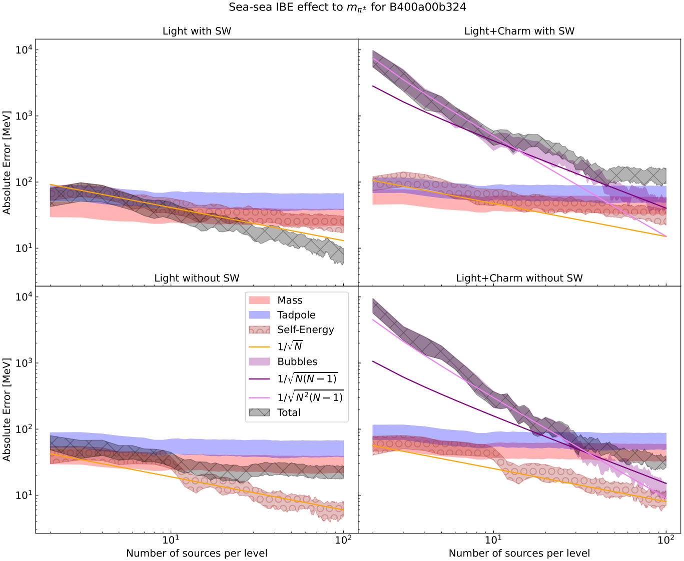
Sea-Sea Diagrams: $m_{\pi^\pm}$ error on B400a00b324



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Sea-Sea Diagrams: Achievements and Future Plan

Achievements:

- ✓ The gauge noise can be achieved for the sea-sea diagrams;
- ✓ The $O(a)$ improvement term reach faster the gauge noise;
- ✓ The precision in the RM123 method is worse than the full simulations for the same number of gauge configurations;

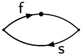
Future plan:

- To compare stochastic and exact photon;
- To study the scale of the error in $a \to 0$, $V \to \infty$ and $m_\pi \to m_{\pi^{\text{phys.}}}$;

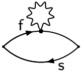
RM123 The Story So Far

Valence-Valence for mesons

$$\sum_{z} \text{Re tr}\left[\gamma_5 D_s^{-1}(x,y)\gamma_5 D_f^{-1}(y,z)D_f^{-1}(z,x)\right] =$$



$$\frac{1}{2}\sum_{zw}\langle \text{Re tr}\left[\gamma_5 D_s^{-1}(x,y)\gamma_5 D_f^{-1}(y,z)T(z,w)D_f^{-1}(w,x)\right]\rangle_\gamma =$$



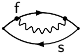
$$-\sum_{zwlm}\langle \text{Re tr}\left[\gamma_5 D_s^{-1}(x,y)\gamma_5 D_f^{-1}(y,z)J(z,w)D_f^{-1}(w,l)J(l,m)D_f^{-1}(m,x)\right]\rangle_\gamma$$

$$\frac{c_f c_s}{16}\sum_{\substack{zw\\ \mu\nu\rho\sigma}}\langle \text{Re tr}\left[\gamma_5 D_s^{-1}(x,y)\gamma_5 D_f^{-1}(y,z)\sigma_{\mu\nu}\hat{A}_{\mu\nu}(z)D_f^{-1}(z,w)\sigma_{\rho\sigma}\hat{A}_{\rho\sigma}(w)D_f^{-1}(w,x)\right]\rangle_\gamma$$

$$\frac{c_f}{2}\sum_{\substack{zwk\\ \mu\nu}}\langle \text{Im tr}\left[\gamma_5 D_s^{-1}(x,y)\gamma_5 D_f^{-1}(y,z)\sigma_{\mu\nu}\hat{A}_{\mu\nu}(z)D_f^{-1}(z,w)J(w,k)D_f^{-1}(k,x)\right]\rangle_\gamma$$

$$\frac{c_f}{2}\sum_{\substack{zwk\\ \mu\nu}}\langle \text{Im tr}\left[\gamma_5 D_s^{-1}(x,y)\gamma_5 D_f^{-1}(y,w)J(w,z)D_f^{-1}(z,k)\sigma_{\mu\nu}\hat{A}_{\mu\nu}(k)D_f^{-1}(k,x)\right]\rangle_\gamma$$

$=$

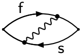


$$-\sum_{zwlm}\langle \text{Retr}\left[\gamma_5 D_s^{-1}(x,l)J(l,m)D_s^{-1}(m,y)\gamma_5 D_f^{-1}(y,w)J(w,z)D_f^{-1}(z,x)\right]\rangle_\gamma$$

$$\frac{c_f c_s}{8}\sum_{\substack{zw\\ \mu\nu\rho\sigma}}\langle \text{Re tr}\left[\gamma_5 D_s^{-1}(x,z)\sigma_{\mu\nu}\hat{A}_{\mu\nu}(z)D_s^{-1}(z,y)\gamma_5 D_f^{-1}(y,w)\sigma_{\rho\sigma}\hat{A}_{\rho\sigma}(w)D_f^{-1}(w,x)\right]\rangle_\gamma$$

$$\frac{c_f}{2}\sum_{\substack{zwk\\ \mu\nu}}\langle \text{Im tr}\left[\gamma_5 D_s^{-1}(x,z)J(z,w)D_s^{-1}(w,y)\gamma_5 D_f^{-1}(y,k)\sigma_{\mu\nu}\hat{A}_{\mu\nu}(k)D_f^{-1}(k,x)\right]\rangle_\gamma$$

$=$



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Valence-Valence: Mass and Tadpole on A400a00b324

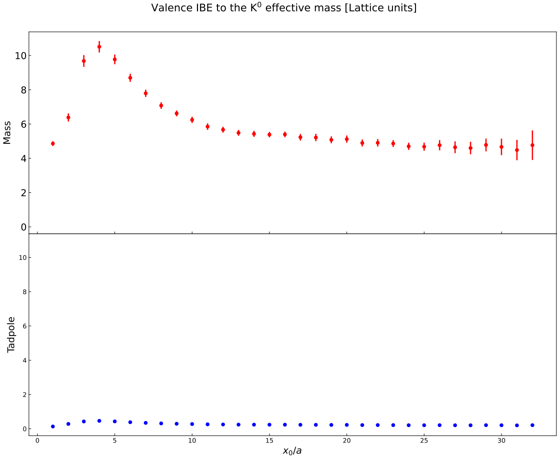


Figure: Valence-Valence IBE correction to the effective mass m_{K^0} (only from the mass and tadpole) with 30 random sources [Preliminary].

RM123 The Story So Far

Valence-Valence on A400a00b324

Looking at what Paola already computed for m_{K^0} we have:

$$\delta am_{K^0} = (\delta am_d + \delta am_s)4.7(2) + e^2 0.22(1)$$

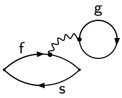
Compatible inside of 2σ.

RM123 The Story So Far

Sea-Valence for mesons

The Sea-Valence effects to compute are:

$$
\underbrace{
\begin{aligned}
&- \frac{1}{4} \sum_{zwkl} \langle \text{Im tr} \left[\gamma_5 D_s^{-1}(x,y) \gamma_5 D_f^{-1}(y,z) J(z,w) D_f^{-1}(w,x) \right] \text{Im tr} \left[J(k,l) D_g^{-1}(l,k) \right] \rangle_\gamma \\
&- \frac{c_g c_f}{64} \sum_{\substack{zw \\ \mu\nu\rho\sigma}} \langle \text{Re tr} \left[\gamma_5 D_s^{-1}(x,y) \gamma_5 D_f^{-1}(y,z) \sigma_{\mu\nu} \hat{A}_{\mu\nu}(z) D_f^{-1}(z,x) \right] \text{Re tr} \left[\sigma_{\rho\sigma} \hat{A}_{\rho\sigma}(w) D_g^{-1}(w,w) \right] \rangle_\gamma \\
&- \frac{c_g}{16} \sum_{\substack{zwk \\ \mu\nu}} \langle \text{Im tr} \left[\gamma_5 D_s^{-1}(x,y) \gamma_5 D_f^{-1}(y,z) J(z,w) D_f^{-1}(w,x) \right] \text{Re tr} \left[\sigma_{\mu\nu} \hat{A}_{\mu\nu}(k) D_g^{-1}(k,k) \right] \rangle_\gamma \\
&- \frac{c_f}{16} \sum_{\substack{zwk \\ \mu\nu}} \langle \text{Re tr} \left[\gamma_5 D_s^{-1}(x,y) \gamma_5 D_f^{-1}(y,z) \sigma_{\mu\nu} \hat{A}_{\mu\nu}(z) D_f^{-1}(z,x) \right] \text{Im tr} \left[J(w,k) D_g^{-1}(k,w) \right] \rangle_\gamma
\end{aligned}
}
$$



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Sea-Valence: Noise problem

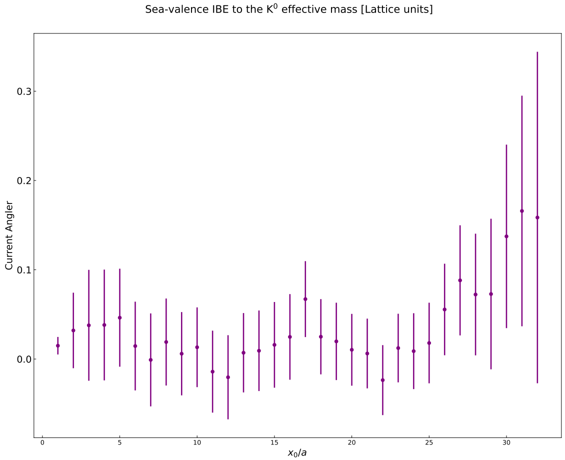


Figure: Sea-Valence IBE correction to the effective mass m_{K^0} (only from the current) with 30 random sources on A400a00b324 [Preliminary].

Plans:

- Strict test with numerical derivatives;
- Use the analytic photon for the sea-valence to recycle the sea-sea bubble;
- Extend to the complete ensembles A400a00b324 and B400a00b324;

$N_f = 3$ Project

New Ensembles

We plan to generate the following ensembles:

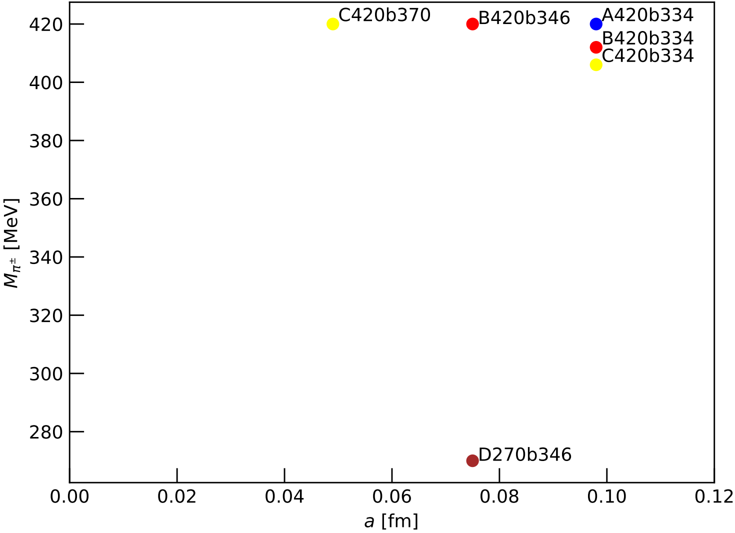
ensemble	lattice	β	c_{SW}	n.cnfg.	$\kappa_{u,d,s}$	M_π [MeV]	a [fm]	$M_\pi L$
C420a00b370	64×32^3	3.70	1.70477	2000	0.137	420	0.049	3.3
B420a00b346	48×24^3	3.46	1.915595	2000	0.13689	420	0.075	3.8
D270a00b346	96×48^3	3.46	1.915595	500	0.136994	270	0.075	4.9
A420a00b334	32×16^3	3.34	2.066858	2000	0.1365716	420	0.098	3.3
B420a00b334	48×24^3	3.34	2.066858	2000	0.1365716	420	0.098	5.0
C420a00b334	64×32^3	3.34	2.066858	2000	0.1365716	420	0.098	6.7

Table: $N_f = 3$ ensembles that we plan to generate in the project, for all the ensembles we will use CLS tuned parameters[c].

[c] Bali et al., "Scale setting and the light baryon spectrum in $N_f = 2 + 1$ QCD with Wilson fermions".

$N_f = 3$ Project

New Ensembles



$N_f = 3$ Project

Objective

The steps of the project are:

- To generate of $N_f = 3$ with different a, V, m_π;
- To measure sea-sea, sea-valence and valence valence diagrams.

The main objective is to study the scaling of the sea-sea, sea-valence and valence-valence contributions in the limits $a \to 0$, $V \to \infty$ and $m_\pi \to m_\pi^{\text{phys.}}$.

What is next?

- Can these configurations be useful for HVP, Baryons and other observables?
- What is the effect of these sea-sea diagrams on other observables?

Thank you for your attention!

Acknowledgments

The work was supported by the North-German Supercomputing Alliance (HLRN) with the project bep00102 and bep00116.

References I

[1] Gunnar S. Bali et al. "Scale setting and the light baryon spectrum in $N_f = 2 + 1$ QCD with Wilson fermions". In: *JHEP* 05 (2023), p. 035. DOI: 10.1007/JHEP05(2023)035. arXiv: 2211.03744 [hep-lat].

[2] Lucius Bushnaq et al. "First results on QCD+QED with C^* boundary conditions". In: *JHEP* 03 (2023), p. 012. DOI: 10.1007/JHEP03(2023)012. arXiv: 2209.13183 [hep-lat].

[3] G. M. de Divitiis et al. "Leading isospin breaking effects on the lattice". In: *Phys. Rev. D* 87 (11 2013), p. 114505. DOI: 10.1103/PhysRevD.87.114505. URL: https://link.aps.org/doi/10.1103/PhysRevD.87.114505.

Backup

Mass Term Estimator

The mass term has been estimated using the following method:

$$\mathrm{Tr}\left[D_{m_u}^{-1}\right] \sim (m_c - m_u)\frac{1}{N_s}\sum_i^{N_s}\left(D_{m_c}^{\dagger}\eta_i\right)^{\dagger} D_{m_u}\eta_i$$

$$+\sum_i^{N_{pr}} s_i^{\dagger} SW^{-1}\sum_{n=0}^{3}\left(-HSW^{-1}\right)^n s_i$$

$$+\frac{1}{N_s}\sum_i^{N_s}\xi_i^{\dagger}\left(-SW^{-1}H\right)^4 D_{m_c}^{-1}\xi_i$$

Backup

Tadpole Term Estimator

The tadpole term has been estimated using the following method:

$$\begin{aligned}\mathrm{Tr}\left[TD_{m_u}^{-1}\right] \sim (m_c - m_u)\,\frac{1}{N_s}\sum_{i}^{N_s}\left(D_{m_c}^{\dagger}\eta_i\right)^{\dagger} TD_{m_u}\eta_i \\ + \sum_{i}^{N_{pr}} s_i^{\dagger}\,TSW^{-1}\sum_{n=0}^{2}\left(-HSW^{-1}\right)^{n} s_i \\ + \frac{1}{N_s}\sum_{i}^{N_s}\xi_i^{\dagger}\,T\left(-SW^{-1}H\right)^{3} D_{m_c}^{-1}\xi_i\end{aligned}$$

Backup

Bubbles Estimator

Due to SU(3) flavour symmetry, the only bubbles contributing are the charm bubbles:

$$\left[JD_{m_c}^{-1}\right]_i^a = \sum_j^{N_{pr}} s_j^\dagger J^a SW^{-1} \sum_{n=0}^{2} \left(-HSW^{-1}\right)^n s_j + \xi_i^\dagger J^a \left(-SW^{-1}H\right)^3 D_{m_c}^{-1}\xi_i$$

$$\left[\sigma_{\mu\nu}\hat{A}_{\mu\nu}D_{m_c}^{-1}\right]_i^a = \sum_j^{N_{pr}} s_j^\dagger \sigma_{\mu\nu}\hat{A}_{\mu\nu}^a SW^{-1} \sum_{n=0}^{3} \left(-HSW^{-1}\right)^n s_j + \xi_i^\dagger \sigma_{\mu\nu}\hat{A}_{\mu\nu}^a \left(-SW^{-1}H\right)^4 D_{m_c}^{-1}\xi_i$$

The stochastic estimator will be:

$$\mathrm{Tr}\left[JD_{m_c}^{-1}\right]\mathrm{Tr}\left[JD_{m_c}^{-1}\right] \sim \frac{2}{N_s\left(N_s-1\right)} \sum_{i\neq j}^{N_s} \frac{1}{N_A} \sum_a^{N_A} \left[JD_{m_c}^{-1}\right]_i^a \left[JD_{m_c}^{-1}\right]_j^a$$

Backup

Self-Energy Estimator

The tadpole term has been estimated using the following method:

$$\mathrm{Tr}\left[JD_{m_u}^{-1}JD_{m_u}^{-1}\right] \sim (m_c - m_u)\,\frac{1}{N_s}\sum_{i}^{N_s}\left(D_{m_c}^{\dagger}\eta_i\right)^{\dagger} JD_{m_u}^{-1}JD_{m_u}\eta_i$$

$$+\frac{1}{N_s}\sum_{i}^{N_s}\xi_i^{\dagger}JD_{m_u}^{-1}JD_{m_c}^{-1}\xi_i$$

RM123: The Story So Far

Valence-Valence Diagrams: m_{K^0}

Valence-Valence contribution to the kaon mass.

Observable	Value [MeV]
$m_{K^0}^{\mathrm{QCD}}$	408 ± 7
$\delta m_{K^0}^{\mathrm{Mass}}$	-26 ± 1
$\delta m_{K^0}^{\mathrm{Tad}}$	69 ± 5
$\delta m_{K^0}^{\mathrm{Self}}$	-18 ± 1
$\delta m_{K^0}^{\mathrm{Exch.}}$	0.54 ± 0.06
$\delta m_{K^0}^{\mathrm{Angl.}}$	37 ± 19
$\delta m_{K^0}^{\mathrm{Tot}}$	62 ± 20
$m_{K^0}^{\mathrm{QCD+QED_{RM123}}}$	470 ± 22
$m_{K^0}^{\mathrm{QCD+QED}}$	405 ± 8

Table: Valence-Valence IBE effects on m_{K^0} on A400a00b324.