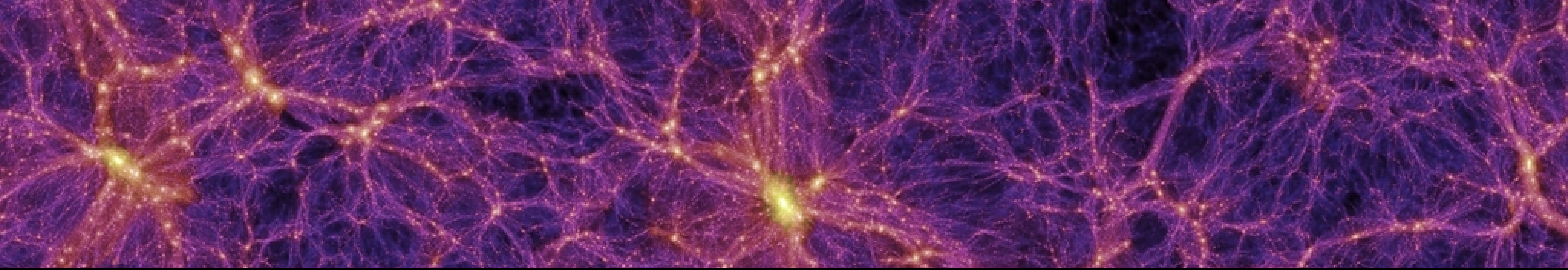


Field-level inference for galaxy clustering

Beatriz Tucci

On behalf of the LEFTfield group:
Ivana Babić, Safak Celik, Andrija Kostić, Nhat-Minh Nguyen, Ivana Nikolac, Charalampos Nikolis, Martin Reinecke, Henrique Rubira, Fabian Schmidt, Moritz Singhartinger, Julia Stadler

LEFTfield

MAX-PLANCK-INSTITUT FÜR ASTROPHYSIK

Is Einstein's General Relativity the final theory of Gravity?

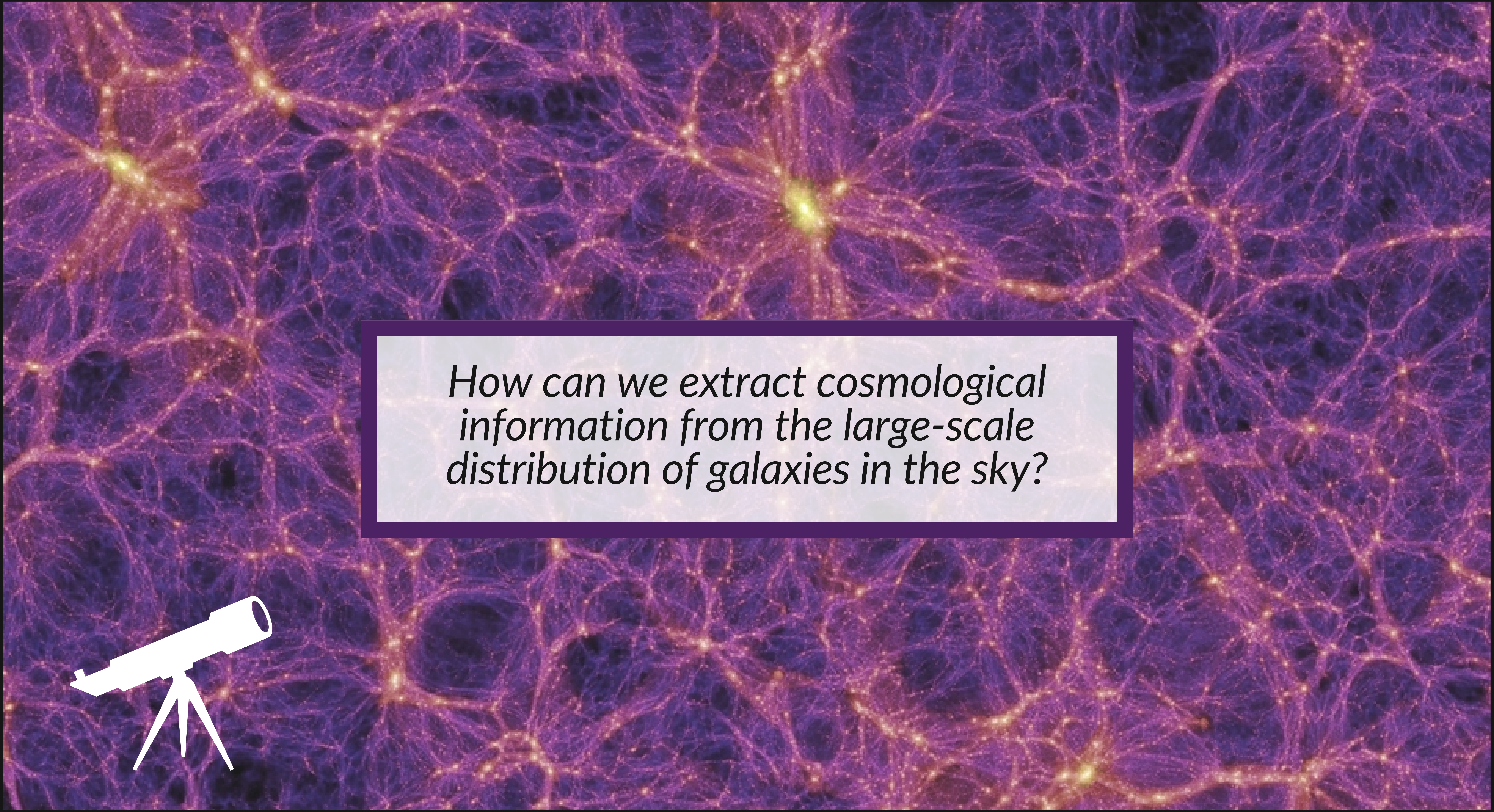
What were the initial conditions of the Universe?

What is the nature of the dark sector?

Which are the Neutrinos masses?

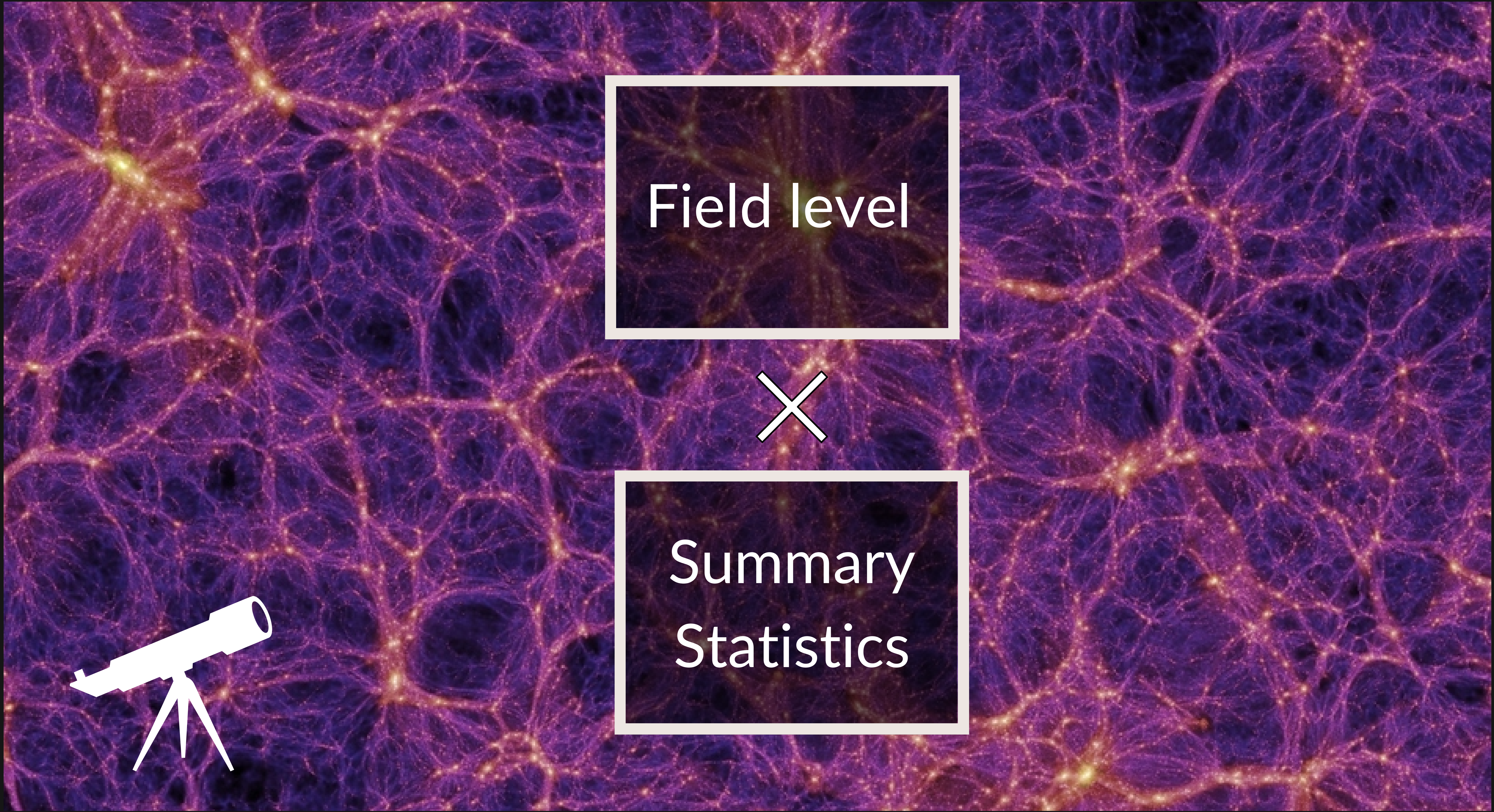
....

The Millennium Simulation Project

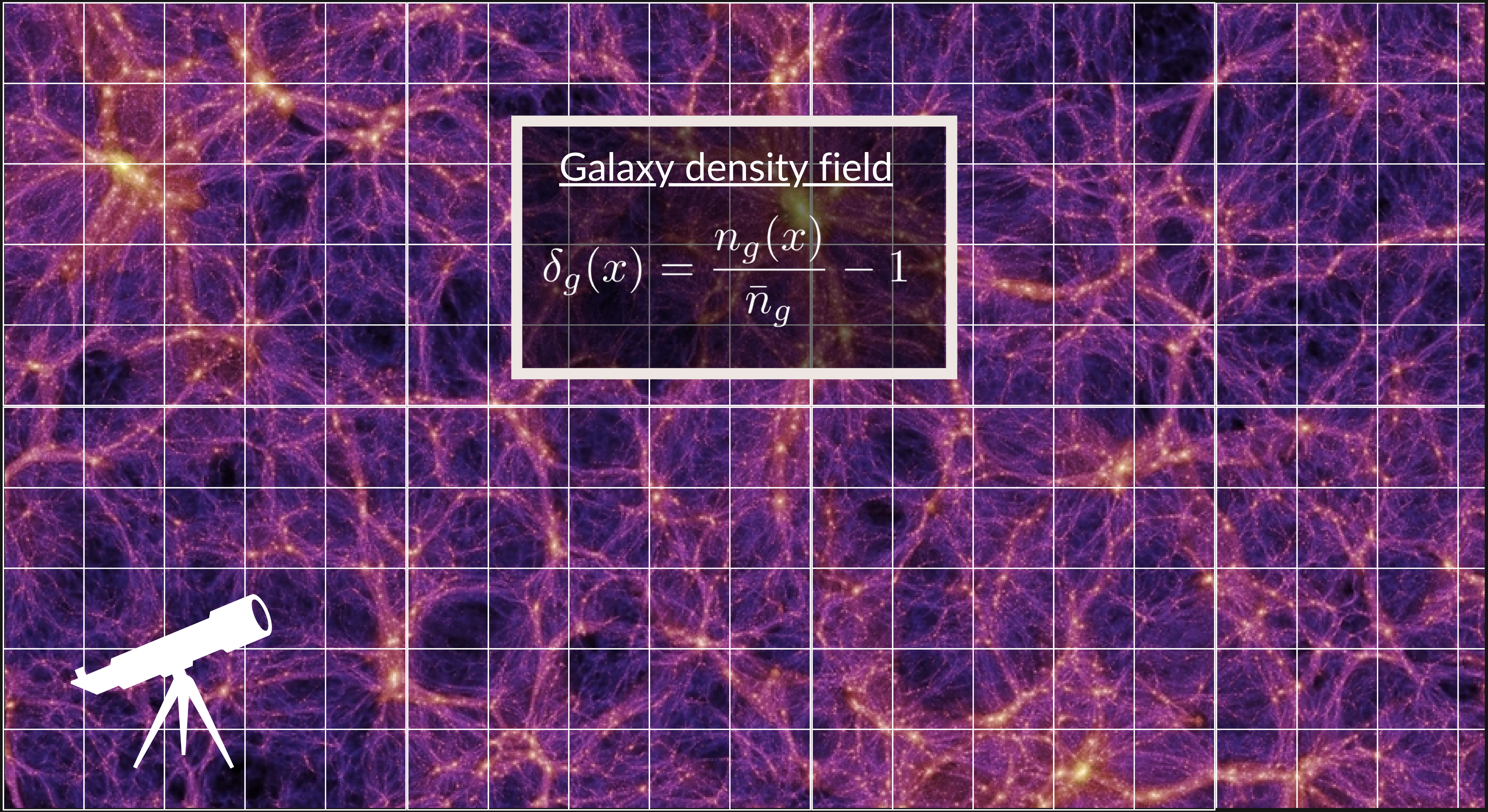


How can we extract cosmological information from the large-scale distribution of galaxies in the sky?

The Millennium Simulation Project



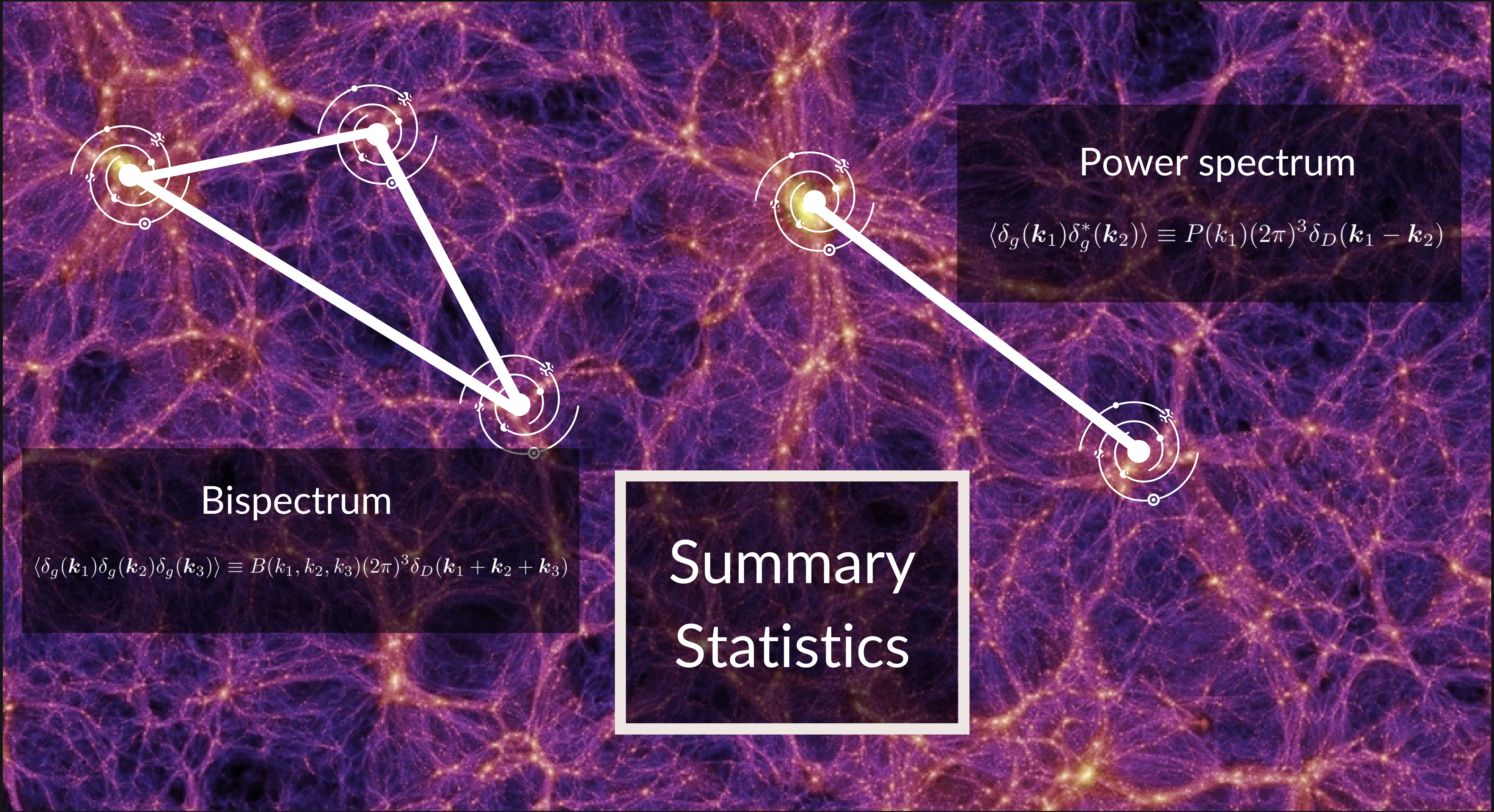
The Millennium Simulation Project



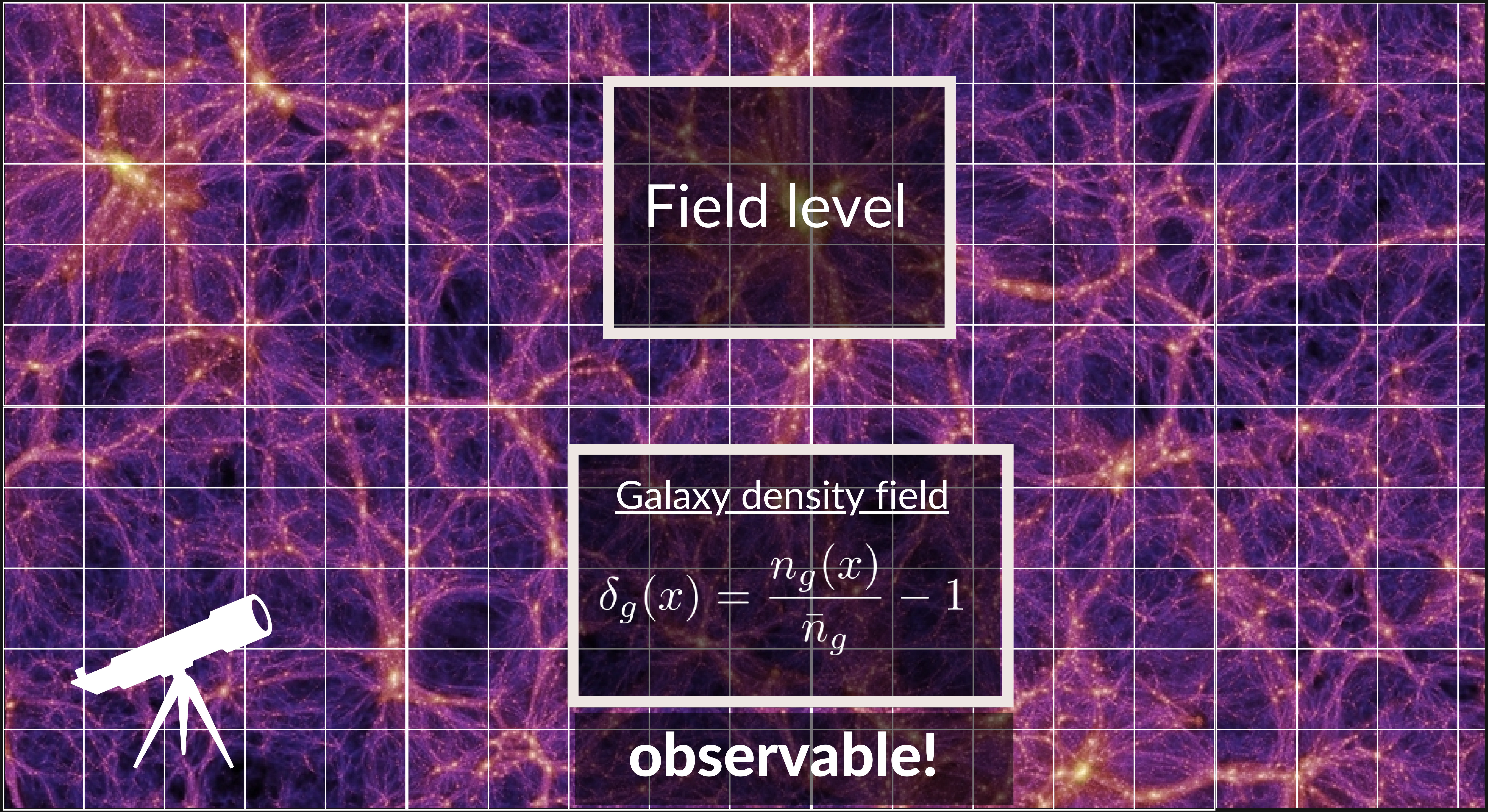
Galaxy density field

$$\delta_g(x) = \frac{n_g(x)}{\bar{n}_g} - 1$$

The Millennium Simulation Project



The Millennium Simulation Project



Field level

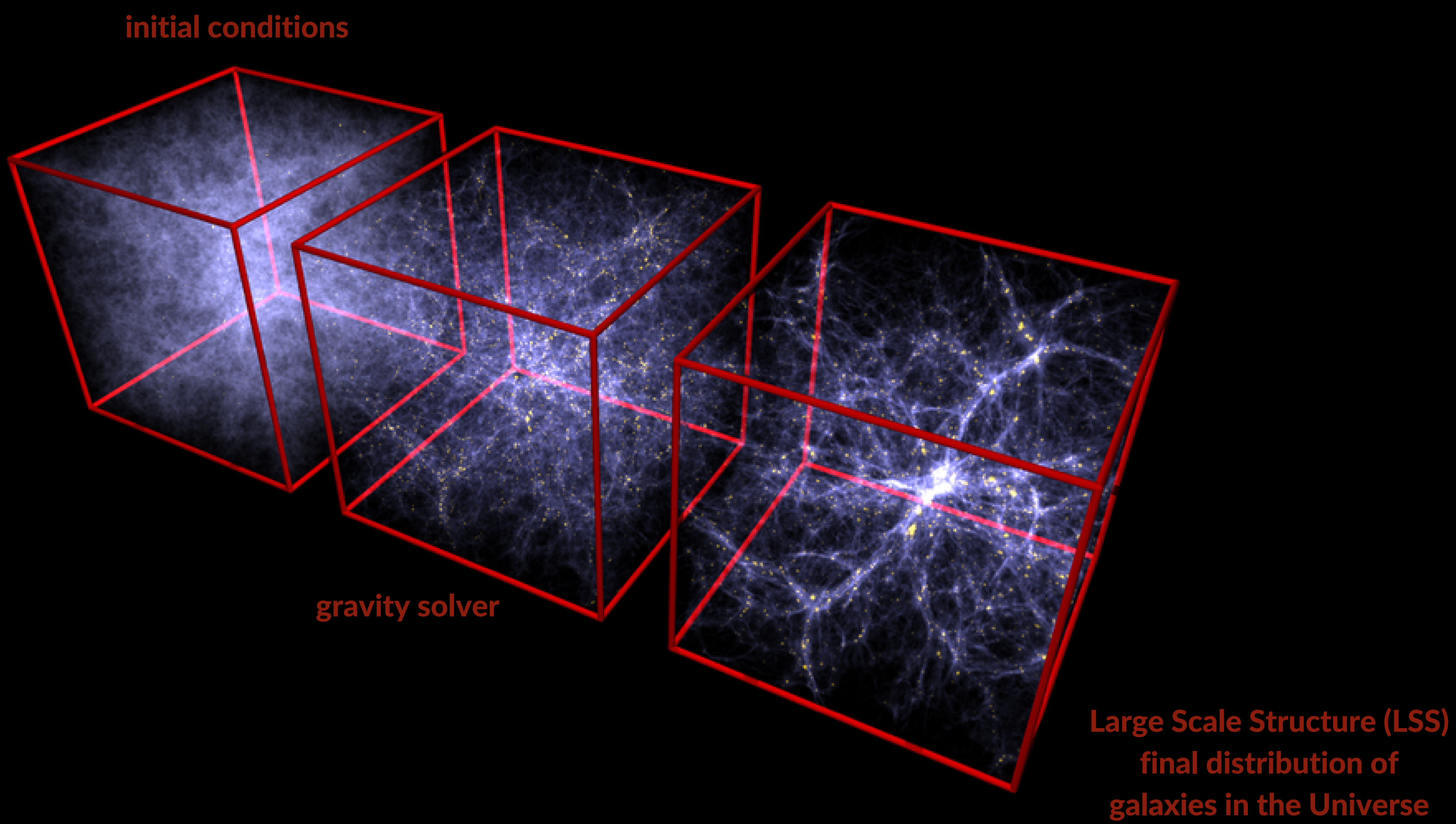
Galaxy density field

$$\delta_g(x) = \frac{n_g(x)}{\bar{n}_g} - 1$$

observable!

The Millennium Simulation Project

Forward modelling

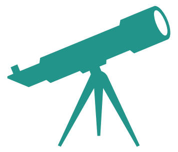


Credits: Volker Springel (MPA)

Bayesian inference in cosmology

Parameters posterior

$$\mathcal{P}(\boldsymbol{\theta}|\mathbf{D})$$



Observed data vector
(e.g., power-spectrum bins)

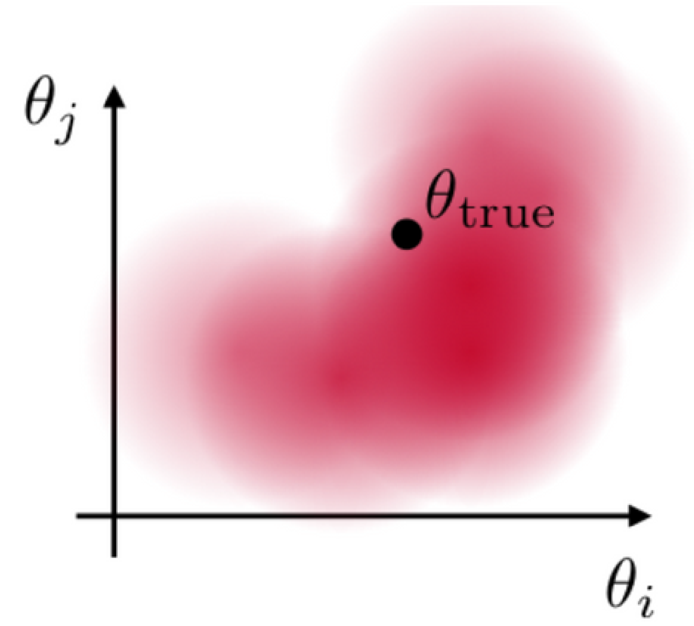
Bayesian inference in cosmology

Parameters posterior

$$\mathcal{P}(\boldsymbol{\theta}|\mathbf{D})$$

Cosmological and
"nuisance" parameters

$$\boldsymbol{\theta} = \{\boldsymbol{\Omega}, \{b_O\}\}$$



Bayesian inference in cosmology

Parameters posterior

Prior over parameters

Bayes' Theorem

$$\mathcal{P}(\boldsymbol{\theta}|\mathbf{D}) \propto \mathcal{L}(\mathbf{D}|\boldsymbol{\theta})\pi(\boldsymbol{\theta})$$

Likelihood

$$\boldsymbol{\theta} = \{\boldsymbol{\Omega}, \{b_O\}\}$$

Bayesian inference in cosmology

Bayes' Theorem

Parameters posterior — Prior over parameters

$$\mathcal{P}(\boldsymbol{\theta}|\mathbf{D}) \propto \mathcal{L}(\mathbf{D}|\boldsymbol{\theta})\pi(\boldsymbol{\theta})$$

Likelihood

E.g., assuming that the data vector is normally distributed:

$$-2\ln\mathcal{L}(\mathbf{D}|\boldsymbol{\theta}) = \left(\mathbf{D} - \mathbf{T}(\boldsymbol{\theta})\right) \cdot \mathbf{C}^{-1} \cdot \left(\mathbf{D} - \mathbf{T}(\boldsymbol{\theta})\right)$$

Theory

Covariance of the data vector

Bayesian inference in cosmology

Bayes' Theorem

Parameters posterior

Prior over parameters

$$\mathcal{P}(\boldsymbol{\theta}|\mathbf{D}) \propto \mathcal{L}(\mathbf{D}|\boldsymbol{\theta})\pi(\boldsymbol{\theta})$$

Likelihood

Prior knowledge over the parameters (e.g., from other observations such as Planck, or simulations)

Bayesian inference in cosmology

Parameters posterior

Prior over parameters

Bayes' Theorem

$$\mathcal{P}(\boldsymbol{\theta}|\mathbf{D}) \propto \mathcal{L}(\mathbf{D}|\boldsymbol{\theta})\pi(\boldsymbol{\theta})$$

Likelihood

Markov-chain Monte Carlo (MCMC) +
"Nuisance" parameters marginalization

$$\boldsymbol{\theta} = \{\Omega, \{b_O\}\}$$

$$\mathcal{P}(\boldsymbol{\Omega}|\mathbf{D})$$

Posterior of cosmological parameters given observed data

Bayesian inference in cosmology

Parameters posterior

Prior over parameters

Bayes' Theorem

$$\mathcal{P}(\boldsymbol{\theta}|\mathbf{D}) \propto \mathcal{L}(\mathbf{D}|\boldsymbol{\theta})\pi(\boldsymbol{\theta})$$

Likelihood

Markov-chain Monte Carlo (MCMC) +
"Nuisance" parameters marginalization

$$\boldsymbol{\theta} = \{\boldsymbol{\Omega}, \{b_O\}\}$$

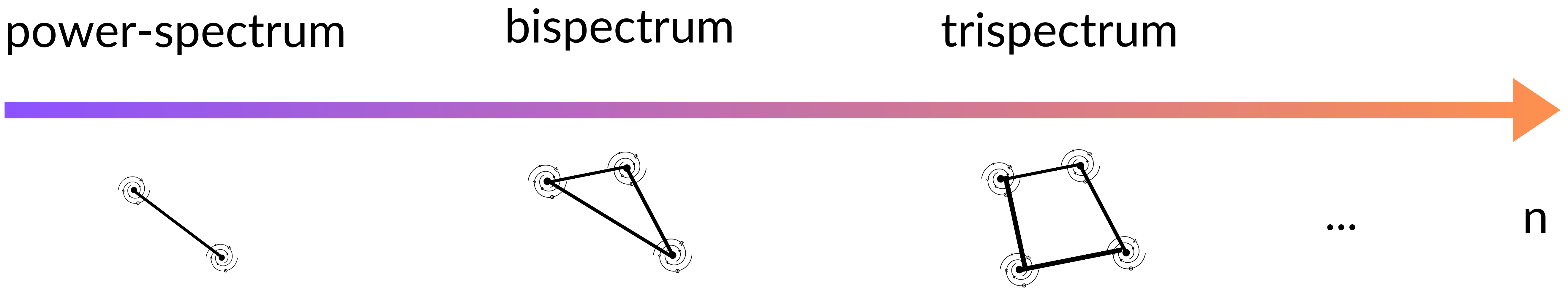
(new physics?!)

Better constraints on parameters!

$$\mathcal{P}(\boldsymbol{\Omega}|\mathbf{D})$$

Posterior of cosmological parameters given observed data

The n-point functions approach



Theory

$$\delta_g(\boldsymbol{\theta}) \rightarrow \mathbf{T}(\boldsymbol{\theta}) = \{\langle\delta_g\delta_g\rangle(\boldsymbol{\theta}), \langle\delta_g\delta_g\delta_g\rangle(\boldsymbol{\theta}), \ldots\}$$

$$\delta_g^{\text{obs}} \rightarrow \mathbf{D} = \{\langle\delta_g\delta_g\rangle^{\text{obs}}, \langle\delta_g\delta_g\delta_g\rangle^{\text{obs}}, \ldots\}$$

Data

Covariance

$$-2\ln\mathcal{L}(\mathbf{D}|\boldsymbol{\theta}) = \left(\mathbf{D} - \mathbf{T}(\boldsymbol{\theta})\right) \cdot \mathbf{C}^{-1} \cdot \left(\mathbf{D} - \mathbf{T}(\boldsymbol{\theta})\right)$$

Bayes theorem

$$\mathcal{P}(\boldsymbol{\theta}|\mathbf{D}) \propto \mathcal{L}(\mathbf{D}|\boldsymbol{\theta})\pi(\boldsymbol{\theta})$$

Markov-chain Monte Carlo (MCMC) +
"Nuisance" parameters marginalization

$$\boldsymbol{\theta} = \{\boldsymbol{\Omega}, \{b_O\}\}$$

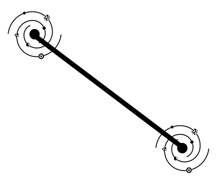
$$\mathcal{P}(\boldsymbol{\Omega}|\mathbf{D})$$

Posterior of cosmological parameters given observed data

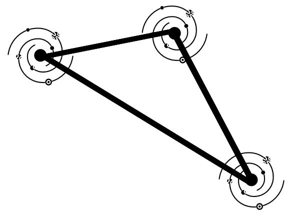
Pierre's talk today!

The n-point functions approach: issues

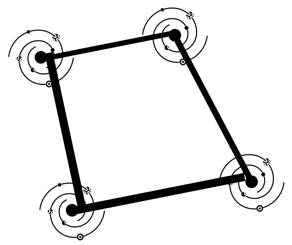
power-spectrum



bispectrum



trispectrum



Increasing complexity

...

n

At which n we stop?
To which n is it even feasible to go?

Modelling

$$\delta_g(\boldsymbol{\theta}) \rightarrow \mathbf{T}(\boldsymbol{\theta}) = \{\langle\delta_g\delta_g\rangle(\boldsymbol{\theta}), \langle\delta_g\delta_g\delta_g\rangle(\boldsymbol{\theta}), \ldots\}$$

$$\delta_g^{\rm obs} \rightarrow \mathbf{D} = \{\langle\delta_g\delta_g\rangle^{\rm obs}, \langle\delta_g\delta_g\delta_g\rangle^{\rm obs}, \ldots\}$$

Measurements

$$-2\ln\mathcal{L}(\mathbf{D}|\boldsymbol{\theta}) = \left(\mathbf{D} - \mathbf{T}(\boldsymbol{\theta})\right) \cdot \mathbf{C}^{-1} \cdot \left(\mathbf{D} - \mathbf{T}(\boldsymbol{\theta})\right)$$

Estimation

Bayes theorem

$$\mathcal{P}(\boldsymbol{\theta}|\mathbf{D}) \propto \mathcal{L}(\mathbf{D}|\boldsymbol{\theta})\pi(\boldsymbol{\theta})$$

Markov-chain Monte Carlo (MCMC) +
“Nuisance” parameters marginalization

$$\boldsymbol{\theta} = \{\boldsymbol{\Omega}, \{b_O\}\}$$

More nuisance parameters

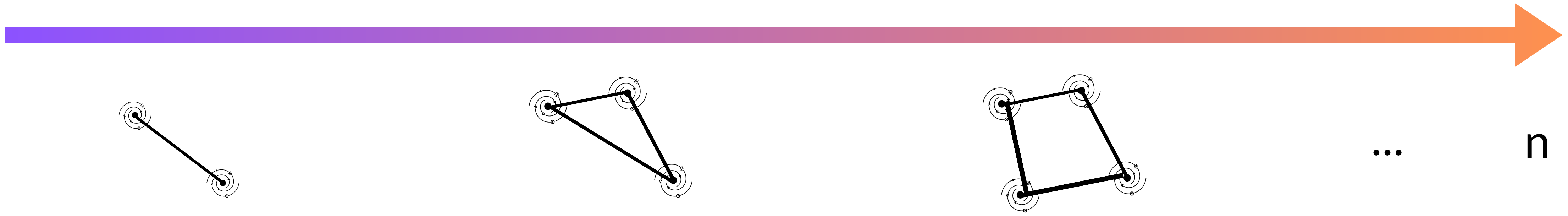
$$\mathcal{P}(\boldsymbol{\Omega}|\mathbf{D})$$

Posterior of cosmological parameters given observed data

Field-level inference

N-point functions

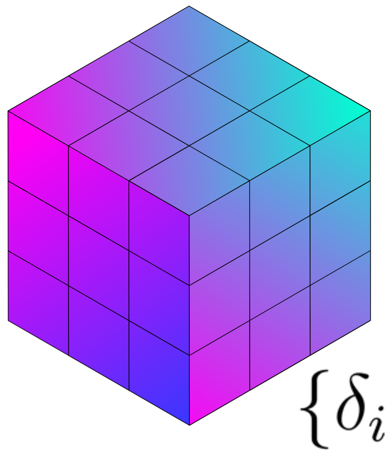
power-spectrum bispectrum trispectrum



$$\mathbf{D} = \{\langle\delta_g\delta_g\rangle^{\rm obs}, \langle\delta_g\delta_g\delta_g\rangle^{\rm obs}, \ldots\}$$

$$\boldsymbol{\theta} = \{\boldsymbol{\Omega}, \{b_O\}\}$$

Field-level



$\{\delta_i\}_{i=1}^{N_{\rm g}}$

amplitudes and phases of the galaxy density field

$$\mathbf{D} = \left\{\delta_g^1, \delta_g^2, \ldots, \delta_g^{N_g}\right\}$$

$$\boldsymbol{\theta} = \left\{\boldsymbol{\Omega}, \{b_O\}, \delta_\Lambda^{(1)}\right\}$$

$$\delta_\Lambda^{(1)} = \left\{\delta_\Lambda^{(1),i}\right\}_{i=1}^{N_g}$$

How to set our theory?

(Matt's talks)

$$\delta_g[\boldsymbol{\theta}]$$

1. Gravity
2. EFTofLSS for matter
3. The bias expansion

1. Gravity

Gaussian initial conditions
(CMB)

in the absensce of primordial non-Gaussianities (PNG)

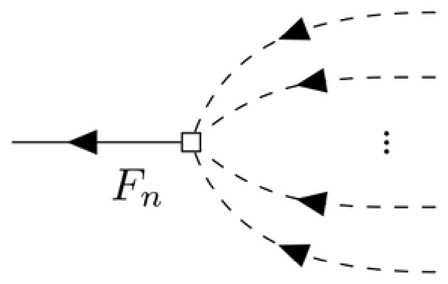
$$\mathcal{P}[\delta^{(1)}(\boldsymbol{k})] = \mathcal{N}(0, P_L(k)[\boldsymbol{\theta}])$$

↓ **Gravitational evolution**

$$\delta(\boldsymbol{k}) = \sum_{\ell=1}^{\infty} \int_{\boldsymbol{p}_1,\ldots,\boldsymbol{p}_\ell} \delta_{\mathrm{D}}\left(\boldsymbol{k} - \boldsymbol{p}_{1\ldots\ell}\right) F_\ell\left(\boldsymbol{p}_1, \ldots \boldsymbol{p}_\ell\right) \delta^{(1)}\left(\boldsymbol{p}_1\right) \cdots \delta^{(1)}\left(\boldsymbol{p}_\ell\right)$$

PT kernels

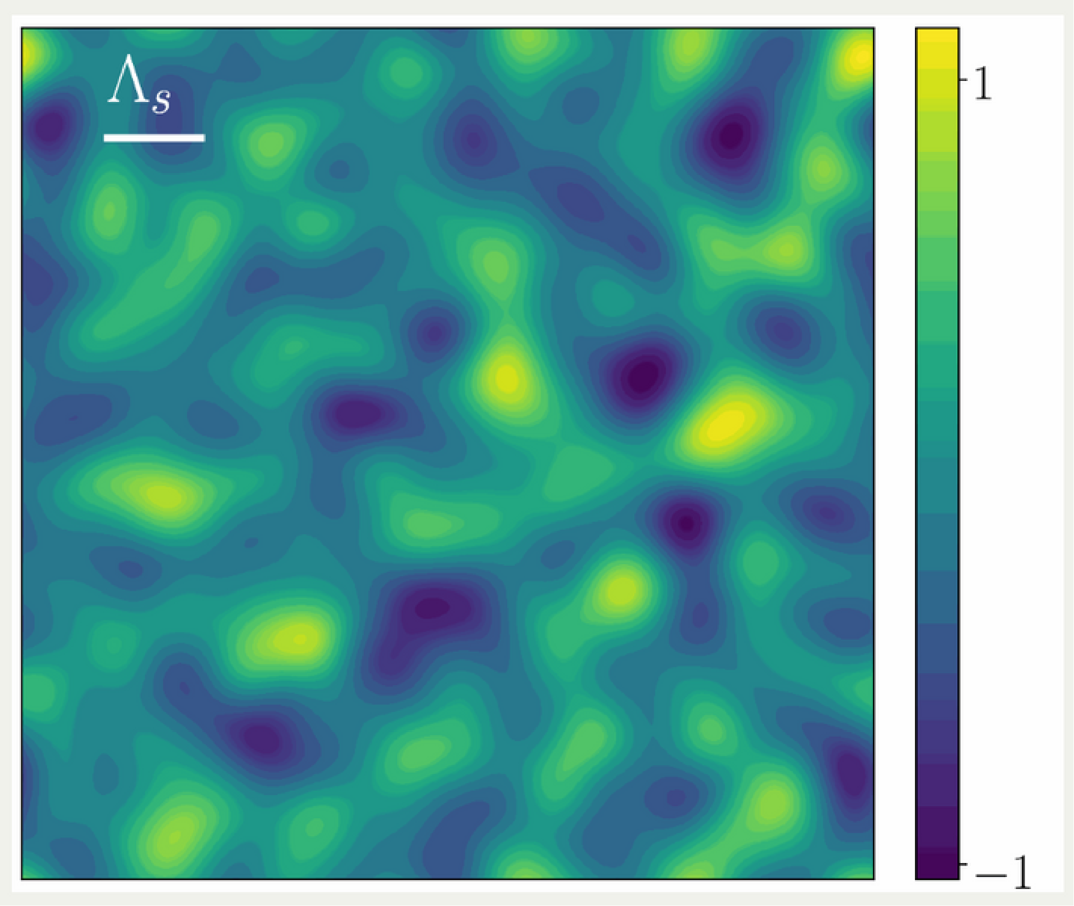
We know the full perturbative solution for the gravitational evolution given Euler and continuity equations for matter



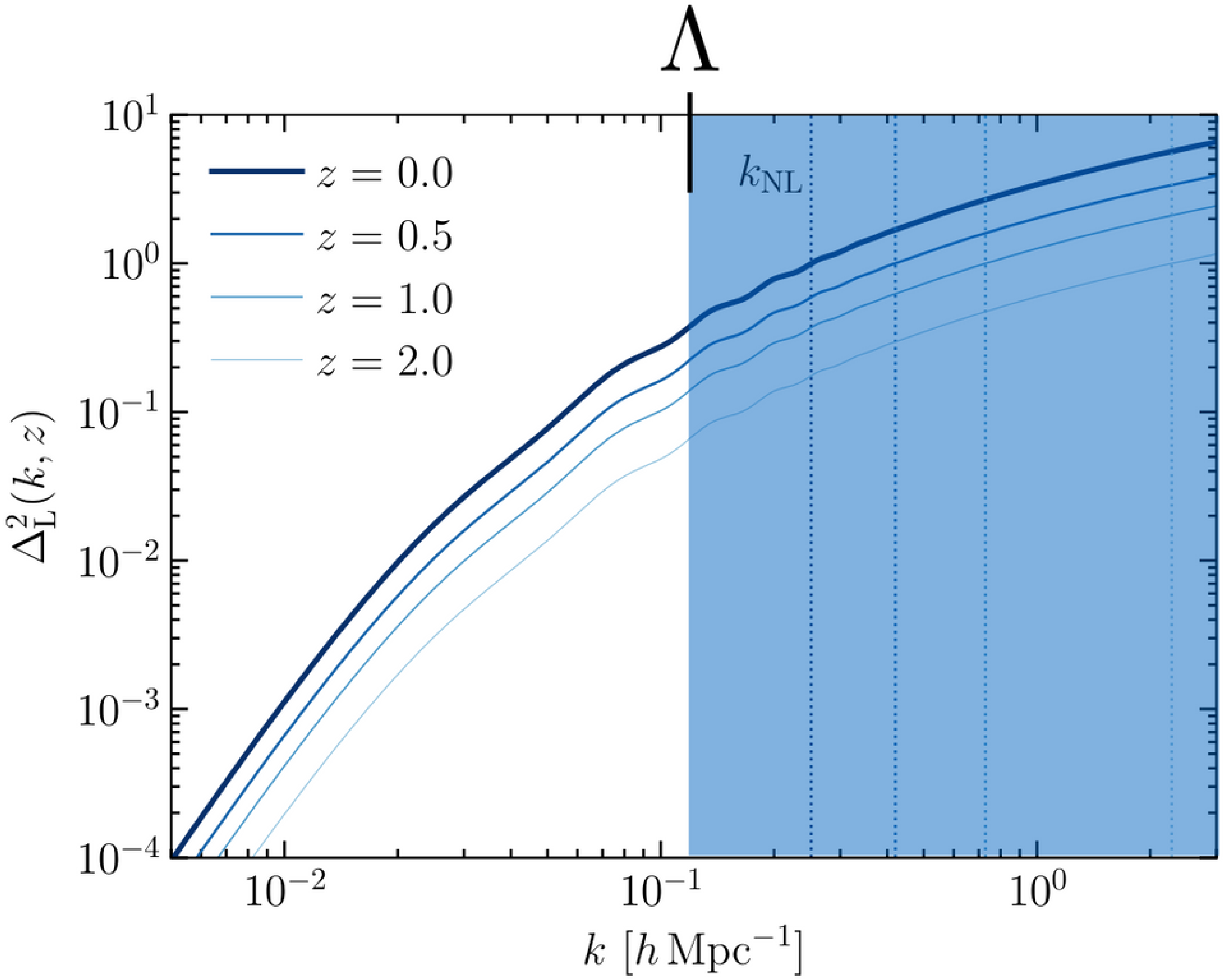
2. The EFTofLSS for matter

"coarse-graining"

$$\delta_\Lambda^{(1)}(\boldsymbol{k}) = W_\Lambda(k)\delta^{(1)}(\boldsymbol{k})$$



Borrowed from Pierre Zhang

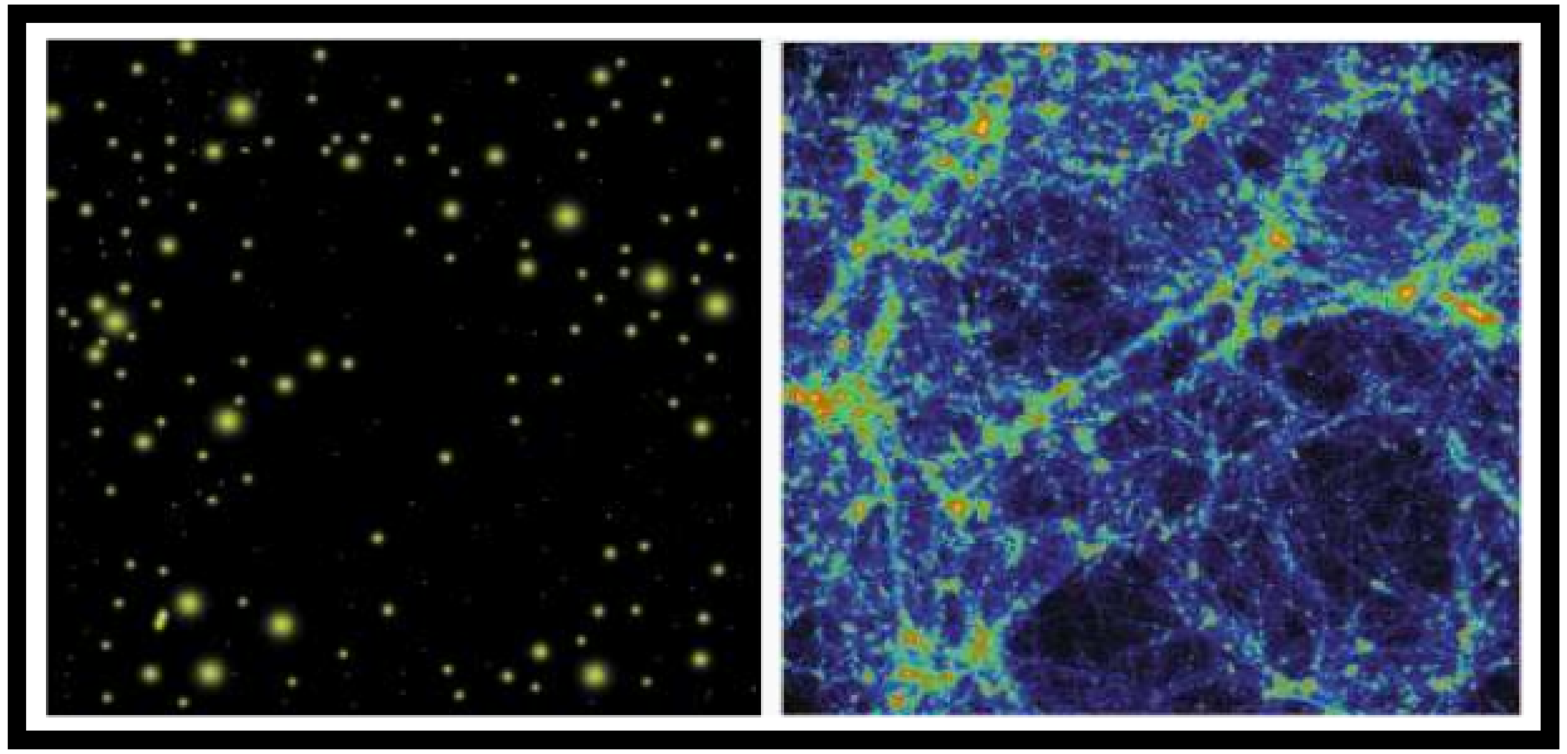


Borrowed from Fabian Schmidt

3. The bias expansion

Cooray & Sheth (2002)

Cosmological tracers



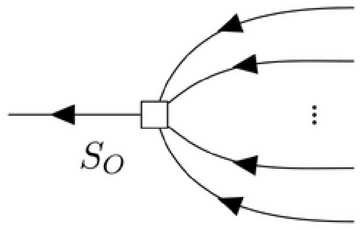
Matter distribution

For a review, see: Desjacques, Jeong & Schmidt (2016)

$$\delta_g(\mathbf{x},\tau) = \sum_{\mathcal{O}} b_{\mathcal{O}}(\tau)\mathcal{O}(\mathbf{x},\tau) + \varepsilon(\mathbf{x},\tau) + \sum_{\mathcal{O}} \varepsilon_{\mathcal{O}}(\mathbf{x},\tau)\mathcal{O}(\mathbf{x},\tau)$$

$$O[\delta](\boldsymbol{k}) = \int_{\boldsymbol{p}_1,\ldots,\boldsymbol{p}_n} \delta_{\mathrm{D}}\left(\boldsymbol{k} - \boldsymbol{p}_{1\ldots n}\right) S_O\left(\boldsymbol{p}_1,\ldots \boldsymbol{p}_n\right) \delta\left(\boldsymbol{p}_1\right)\cdots\delta\left(\boldsymbol{p}_n\right)$$

operator "convolution"



How to set our likelihood?

$$\mathcal{P}[\delta_g^{\mathrm{obs}}|\boldsymbol{\theta}]$$

Generating functional of galaxies

The generating functional of matter

Carroll et al. (2013)

Matter field measure

$$\mathcal{Z}^{\mathrm{m}}[J_\Lambda] = \int \mathcal{D}\delta_\Lambda^{(1)} \mathcal{P}[\delta_\Lambda^{(1)}] \exp\left(\int_{\boldsymbol{k}} J_\Lambda(\boldsymbol{k}) \delta_{\mathrm{fwd}}[\delta_\Lambda^{(1)}](-\boldsymbol{k}) \right)$$

Matter field PDF

For Gaussian initial conditions:

$$\mathcal{P}[\delta_\Lambda^{(1)}] = \left(\prod_k^\Lambda 2\pi P_L^\Lambda(k) \right)^{-1/2} \exp\left[-\frac{1}{2} \int_{\boldsymbol{k}}^\Lambda \frac{|\delta_\Lambda^{(1)}|^2}{P_L^\Lambda(k)} \right]$$

$$P_{\mathrm{L}}^\Lambda(k) = \left\langle \delta_\Lambda^{(1)}(\boldsymbol{k}) \delta_\Lambda^{(1)}(\boldsymbol{k}') \right\rangle'$$

The generating functional of galaxies

Cabass & Schmidt (2020)
Rubira & Schmidt (2023)

deterministic bias expansion

$$\mathcal{Z}\left[J_\Lambda\right] = \int \mathcal{D}\delta_\Lambda^{(1)} \mathcal{P}\left[\delta_\Lambda^{(1)}\right] \exp\left(\int_{\boldsymbol{k}} J_\Lambda(\boldsymbol{k}) \left[\sum_O b_O^\Lambda O\left[\delta_\Lambda^{(1)}\right](-\boldsymbol{k}) \right] + \frac{1}{2} P_\epsilon^\Lambda \int_{\boldsymbol{k}} J_\Lambda(\boldsymbol{k}) J_\Lambda(-\boldsymbol{k}) + \mathcal{O}\left[J_\Lambda^2 \delta_\Lambda^{(1)}, J_\Lambda^3 \right] \right)$$

Gaussian stochastic term

higher-order stochastic terms

$$\langle \varepsilon(\boldsymbol{k}) \varepsilon(\boldsymbol{k}') \rangle = (2\pi)^3 \delta_D(\boldsymbol{k} + \boldsymbol{k}') P_\varepsilon(k)$$

The generating functional of galaxies

Cabass & Schmidt (2020)
Rubira & Schmidt (2023)

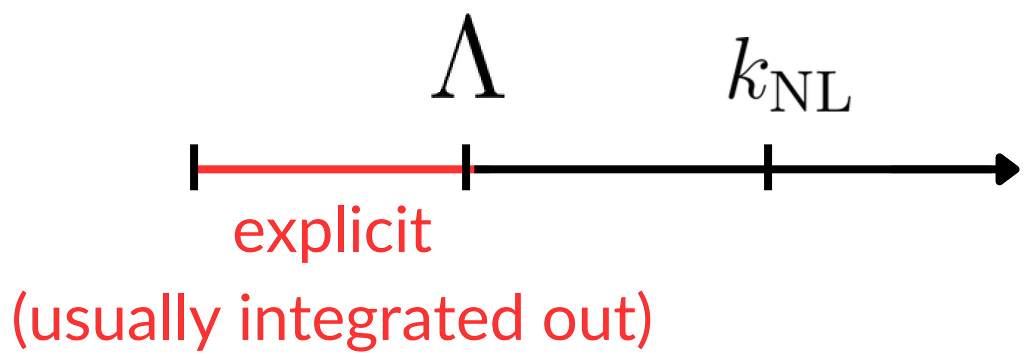
deterministic bias expansion

$$\mathcal{Z}\left[J_\Lambda\right] = \int \mathcal{D}\delta_\Lambda^{(1)} \mathcal{P}\left[\delta_\Lambda^{(1)}\right] \exp\left(\int_{\boldsymbol{k}} J_\Lambda(\boldsymbol{k}) \left[\sum_O b_O^\Lambda O\left[\delta_\Lambda^{(1)}\right](-\boldsymbol{k}) \right] + \frac{1}{2} P_\epsilon^\Lambda \int_{\boldsymbol{k}} J_\Lambda(\boldsymbol{k}) J_\Lambda(-\boldsymbol{k}) + \mathcal{O}\left[J_\Lambda^2 \delta_\Lambda^{(1)}, J_\Lambda^3\right] \right)$$

Gaussian stochastic term

higher-order stochastic terms

$$\langle \varepsilon(\boldsymbol{k}) \varepsilon(\boldsymbol{k}') \rangle = (2\pi)^3 \delta_D(\boldsymbol{k} + \boldsymbol{k}') P_\varepsilon(k)$$



The generating functional of galaxies

Cabass & Schmidt (2020)
Rubira & Schmidt (2023)

$$\mathcal{Z}\left[J_\Lambda\right] = \int \mathcal{D}\delta_\Lambda^{(1)} \mathcal{P}\left[\delta_\Lambda^{(1)}\right] \exp\left(\int_{\boldsymbol{k}} J_\Lambda(\boldsymbol{k}) \left[\sum_O b_O^\Lambda O\left[\delta_\Lambda^{(1)}\right](-\boldsymbol{k})\right] + \frac{1}{2} P_\epsilon^\Lambda \int_{\boldsymbol{k}} J_\Lambda(\boldsymbol{k}) J_\Lambda(-\boldsymbol{k}) + \mathcal{O}\left[J_\Lambda^2 \delta_\Lambda^{(1)}, J_\Lambda^3\right] \right)$$

$$\delta_g = \delta_{g,\mathrm{det}} + \delta_{g,\mathrm{stoch}}$$

$$\delta_g = \sum_O b_O\, O + \varepsilon$$

at LO in stochasticity

$$\langle \delta(\boldsymbol{k}) \delta(\boldsymbol{k}') \rangle = (2\pi)^3 \delta_D(\boldsymbol{k} + \boldsymbol{k}') P_L(k)$$

propagator

$$\langle \varepsilon(\boldsymbol{k}) \varepsilon(\boldsymbol{k}') \rangle = (2\pi)^3 \delta_D(\boldsymbol{k} + \boldsymbol{k}') P_\varepsilon(k)$$

The generating functional of galaxies

Cabass & Schmidt (2020)
Rubira & Schmidt (2023)

$$\mathcal{Z}[J_\Lambda] = \int \mathcal{D}\delta_\Lambda^{(1)} \mathcal{P}\left[\delta_\Lambda^{(1)}\right] \exp\left(\int_{\boldsymbol{k}} J_\Lambda(\boldsymbol{k}) \left[\sum_O b_O^\Lambda O\left[\delta_\Lambda^{(1)}\right](-\boldsymbol{k})\right] + \frac{1}{2} P_\epsilon^\Lambda \int_{\boldsymbol{k}} J_\Lambda(\boldsymbol{k}) J_\Lambda(-\boldsymbol{k}) + \mathcal{O}\left[J_\Lambda^2 \delta_\Lambda^{(1)}, J_\Lambda^3\right]\right)$$

N-point functions

e.g., $b_1^2 P_\mathrm{L}(k)$

$$\langle \delta_g(\boldsymbol{k}) \delta_g(\boldsymbol{k}') \rangle' = \left(\frac{\partial^2 \ln\left(\mathcal{Z}[J_\Lambda]/\mathcal{Z}[0]\right)}{\partial J_\Lambda(\boldsymbol{k}) \partial J_\Lambda(\boldsymbol{k}')} \bigg|_{J_\Lambda = 0} \right)' = \sum_{O,O'} b_O b_{O'} P_{OO'}(k) + P_\varepsilon(k)$$

$$\delta_g = \delta_{g,\mathrm{det}} + \delta_{g,\mathrm{stoch}}$$

$$\delta_g = \sum_O b_O\, O + \varepsilon$$

at LO in stochasticity

$$\langle \delta(\boldsymbol{k}) \delta(\boldsymbol{k}') \rangle = (2\pi)^3 \delta_D(\boldsymbol{k} + \boldsymbol{k}') P_L(k)$$ propagator

$$\langle \varepsilon(\boldsymbol{k}) \varepsilon(\boldsymbol{k}') \rangle = (2\pi)^3 \delta_D(\boldsymbol{k} + \boldsymbol{k}') P_\varepsilon(k)$$

$$\langle O(\boldsymbol{k}) O'(\boldsymbol{k}') \rangle = (2\pi)^3 \delta_D(\boldsymbol{k} + \boldsymbol{k}') P_{OO'}(k)$$

The galaxy field-level likelihood

Cabass & Schmidt (2020)

$$\mathcal{P}[\delta_g^{\rm obs}|\boldsymbol{\theta}] = \left\langle \delta_D^{(\infty)} \left[\delta_g^{\rm obs} - \delta_g(\boldsymbol{\theta})\right] \right\rangle_{\delta_\Lambda^{(1)}}$$

$$\langle \cdots \rangle_{\delta_\Lambda^{(1)}} \equiv \int \mathcal{D}\delta_\Lambda^{(1)} \mathcal{P}\left[\delta_\Lambda^{(1)}\right] \cdots$$

The galaxy field-level likelihood

Cabass & Schmidt (2020)

$$\mathcal{P}[\delta_g^{\rm obs}|\boldsymbol{\theta}] = \left\langle \delta_D^{(\infty)}\left[\delta_g^{\rm obs} - \delta_g(\boldsymbol{\theta})\right] \right\rangle_{\delta_\Lambda^{(1)}}$$

$$= \int \mathcal{D}X \left\langle e^{iX[\delta_g^{\rm obs} - \delta_g(\boldsymbol{\theta})]} \right\rangle_{\delta_\Lambda^{(1)}}$$

The galaxy field-level likelihood

Cabass & Schmidt (2020)

$$\mathcal{P}[\delta_g^{\rm obs}|\boldsymbol{\theta}] = \left\langle \delta_D^{(\infty)}\left[\delta_g^{\rm obs} - \delta_g(\boldsymbol{\theta})\right]\right\rangle_{\delta_\Lambda^{(1)}}$$

$$= \int \mathcal{D}X \left\langle e^{iX[\delta_g^{\rm obs} - \delta_g(\boldsymbol{\theta})]}\right\rangle_{\delta_\Lambda^{(1)}}$$

$$= \int \mathcal{D}X e^{iX\delta_g^{\rm obs}} \left\langle e^{-iX\delta_g(\boldsymbol{\theta})}\right\rangle_{\delta_\Lambda^{(1)}}$$

$$\boldsymbol{\theta} = \left\{\boldsymbol{\Omega}, \{b_O\}, \delta_\Lambda^{(1)}\right\}$$

The galaxy field-level likelihood

Cabass & Schmidt (2020)

$$\mathcal{P}[\delta_g^{\rm obs}|\boldsymbol{\theta}] = \left\langle \delta_D^{(\infty)}\left[\delta_g^{\rm obs} - \delta_g(\boldsymbol{\theta})\right]\right\rangle_{\delta_\Lambda^{(1)}}$$

$$= \int \mathcal{D}X \left\langle e^{iX[\delta_g^{\rm obs} - \delta_g(\boldsymbol{\theta})]}\right\rangle_{\delta_\Lambda^{(1)}}$$

$$= \int \mathcal{D}X e^{iX\delta_g^{\rm obs}} \left\langle e^{-iX\delta_g(\boldsymbol{\theta})}\right\rangle_{\delta_\Lambda^{(1)}}$$

$$= \int \mathcal{D}X e^{iX\delta_g^{\rm obs}} \mathcal{Z}[-iX]$$

$$\mathcal{Z}\left[J_\Lambda\right] = \int \mathcal{D}\delta_\Lambda^{(1)} \mathcal{P}\left[\delta_\Lambda^{(1)}\right] \exp\left(\int_{\boldsymbol{k}} J_\Lambda(\boldsymbol{k}) \left[\sum_O b_O^\Lambda O\left[\delta_\Lambda^{(1)}\right](-\boldsymbol{k})\right] + \frac{1}{2} P_\epsilon^\Lambda \int_{\boldsymbol{k}} J_\Lambda(\boldsymbol{k}) J_\Lambda(-\boldsymbol{k}) + \mathcal{O}\left[J_\Lambda^2 \delta_\Lambda^{(1)}, J_\Lambda^3\right]\right)$$

Gaussian integral

The galaxy field-level likelihood

Cabass & Schmidt (2020)

$$\begin{aligned}
\mathcal{P}[\delta_g^{\rm obs}|\boldsymbol{\theta}] &= \left\langle \delta_D^{(\infty)} \left[\delta_g^{\rm obs} - \delta_g(\boldsymbol{\theta})\right] \right\rangle_{\delta_\Lambda^{(1)}} \\
&= \int \mathcal{D}X \left\langle e^{iX[\delta_g^{\rm obs} - \delta_g(\boldsymbol{\theta})]} \right\rangle_{\delta_\Lambda^{(1)}} \\
&= \int \mathcal{D}X e^{iX\delta_g^{\rm obs}} \left\langle e^{-iX\delta_g(\boldsymbol{\theta})} \right\rangle_{\delta_\Lambda^{(1)}} \\
&= \int \mathcal{D}X e^{iX\delta_g^{\rm obs}} \mathcal{Z}[-iX]
\end{aligned}$$

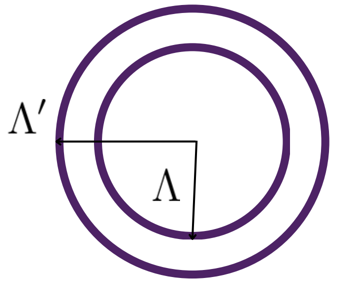
$$\ln \mathcal{P}[\delta_g^{\rm obs}|\boldsymbol{\theta}] = -\frac{1}{2} \int_{k<\Lambda} \frac{|\delta_g^{\rm obs}(\boldsymbol{k}) - \delta_{g,\rm det}[\boldsymbol{\theta}](\boldsymbol{k})|^2}{P_\varepsilon(k)} + \text{const.}$$

Wilson-Polchinski for bias running

Rubira & Schmidt (2023)

$$\delta^{(1)}_{\Lambda'}(\boldsymbol{k}) = \delta^{(1)}_{\Lambda}(\boldsymbol{k}) + \underbrace{\delta^{(1)}_{\text{shell}}(\boldsymbol{k})}_{|\boldsymbol{k}| \in [\Lambda, \Lambda')}$$

integrate out



$$\mathcal{Z}\left[J_\Lambda\right] = \int \mathcal{D}\delta^{(1)}_\Lambda \mathcal{P}\left[\delta^{(1)}_\Lambda\right] \exp\Bigg(\int_{\boldsymbol{k}} J_\Lambda(\boldsymbol{k}) \left[\sum_O b^\Lambda_O O\left[\delta^{(1)}_\Lambda\right](-\boldsymbol{k})\right] + \frac{1}{2} P^\Lambda_\epsilon \int_{\boldsymbol{k}} J_\Lambda(\boldsymbol{k}) J_\Lambda(-\boldsymbol{k})\Bigg)$$

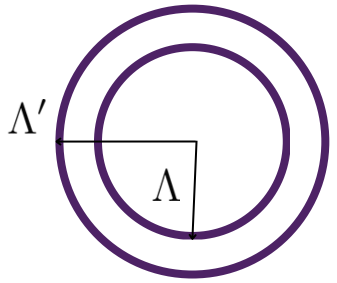
$$\mathcal{Z}_{\text{eff}}\left[J_\Lambda\right] = \int \mathcal{D}\delta^{(1)}_\Lambda \mathcal{P}\left[\delta^{(1)}_\Lambda\right] \int \mathcal{D}\delta^{(1)}_{\text{shell}} \mathcal{P}\left[\delta^{(1)}_{\text{shell}}\right] \exp\Bigg(\int_{\boldsymbol{k}} J_\Lambda(\boldsymbol{k}) \left[\sum_O b^{\Lambda'}_O O\left[\delta^{(1)}_\Lambda + \delta^{(1)}_{\text{shell}}\right](-\boldsymbol{k})\right] + \frac{1}{2} P^{\Lambda'}_\epsilon \int_{\boldsymbol{k}} J_\Lambda(\boldsymbol{k}) J_\Lambda(-\boldsymbol{k})\Bigg)$$

Wilson-Polchinski for bias running

Rubira & Schmidt (2023)

$$\delta^{(1)}_{\Lambda'}(\boldsymbol{k}) = \delta^{(1)}_{\Lambda}(\boldsymbol{k}) + \underbrace{\delta^{(1)}_{\text{shell}}(\boldsymbol{k})}_{|\boldsymbol{k}| \in [\Lambda, \Lambda')}$$

integrate out



$$\mathcal{Z}\left[J_\Lambda\right] = \int \mathcal{D}\delta^{(1)}_\Lambda \mathcal{P}\left[\delta^{(1)}_\Lambda\right] \exp\Big(\int_{\boldsymbol{k}} J_\Lambda(\boldsymbol{k}) \left[\sum_O b^\Lambda_O O\left[\delta^{(1)}_\Lambda\right](-\boldsymbol{k})\right] + \frac{1}{2} P^\Lambda_\epsilon \int_{\boldsymbol{k}} J_\Lambda(\boldsymbol{k}) J_\Lambda(-\boldsymbol{k})\Big)$$

$$\mathcal{Z}_{\text{eff}}\left[J_\Lambda\right] = \int \mathcal{D}\delta^{(1)}_\Lambda \mathcal{P}\left[\delta^{(1)}_\Lambda\right] \int \mathcal{D}\delta^{(1)}_{\text{shell}} \mathcal{P}\left[\delta^{(1)}_{\text{shell}}\right] \exp\Big(\int_{\boldsymbol{k}} J_\Lambda(\boldsymbol{k}) \left[\sum_O b^{\Lambda'}_O O\left[\delta^{(1)}_\Lambda + \delta^{(1)}_{\text{shell}}\right](-\boldsymbol{k})\right] + \frac{1}{2} P^{\Lambda'}_\epsilon \int_{\boldsymbol{k}} J_\Lambda(\boldsymbol{k}) J_\Lambda(-\boldsymbol{k})\Big)$$

No dependence on cuttoff:

$$\sum_O b^\Lambda_O \, O\left[\delta^{(1)}_\Lambda\right] = \sum_O b^{\Lambda'}_O \left(O\left[\delta^{(1)}_\Lambda\right] + \mathcal{S}^2_O\left[\delta^{(1)}_\Lambda\right]\right) \longrightarrow \boxed{\frac{db_O}{d\Lambda}}$$

Galaxy bias RG equations

bias = Wilson coefficients

“Callan-Symanzik equation”

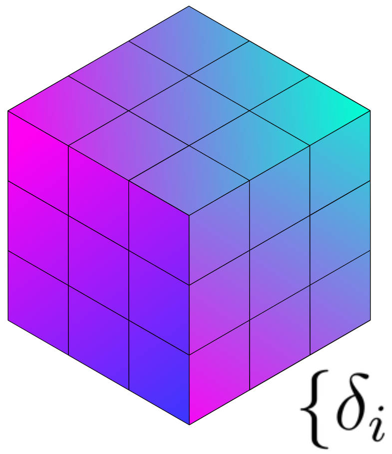
Field-level inference in practice

$$\delta_g[\boldsymbol{\theta}]$$
$$\mathcal{P}[\delta_g^{\rm obs}|\boldsymbol{\theta}]$$

Cosmological constraints?

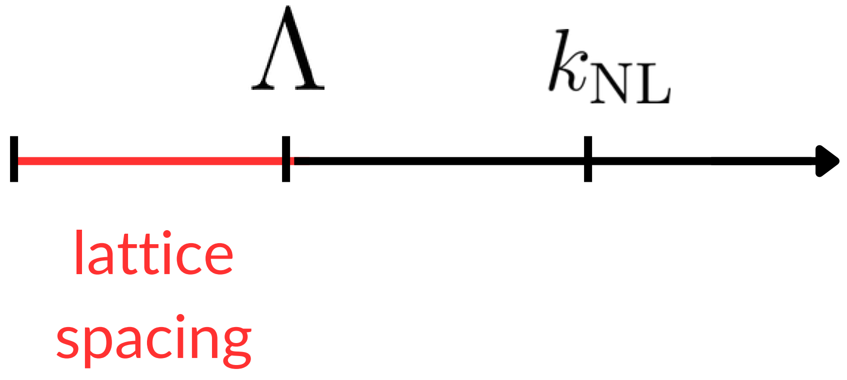
Is there any alternative for loops?

Field-level



$\{\delta_i\}_{i=1}^{N_g}$

amplitudes and phases of the galaxy density field

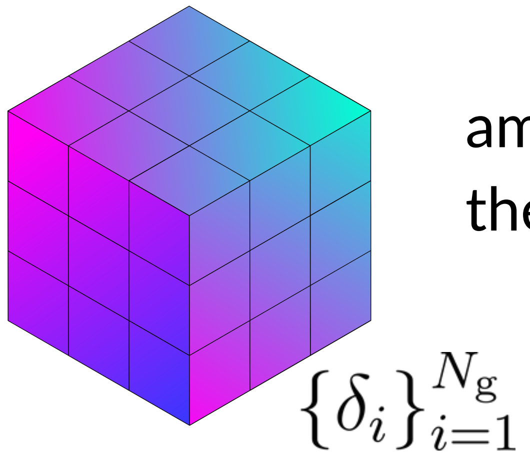


LEFTfield: a fast forward model that solves the gravitational evolution of all modes in a lattice

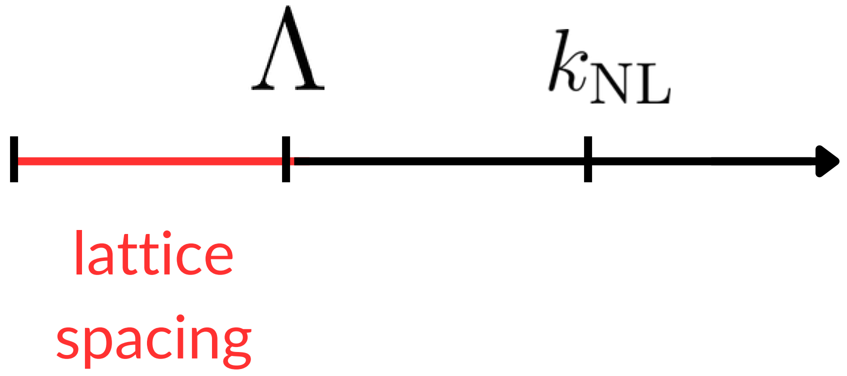
Forward modelling:
easier to deal with redshift space, masks and systematic effetcs

Is there any alternative for loops?

Field-level



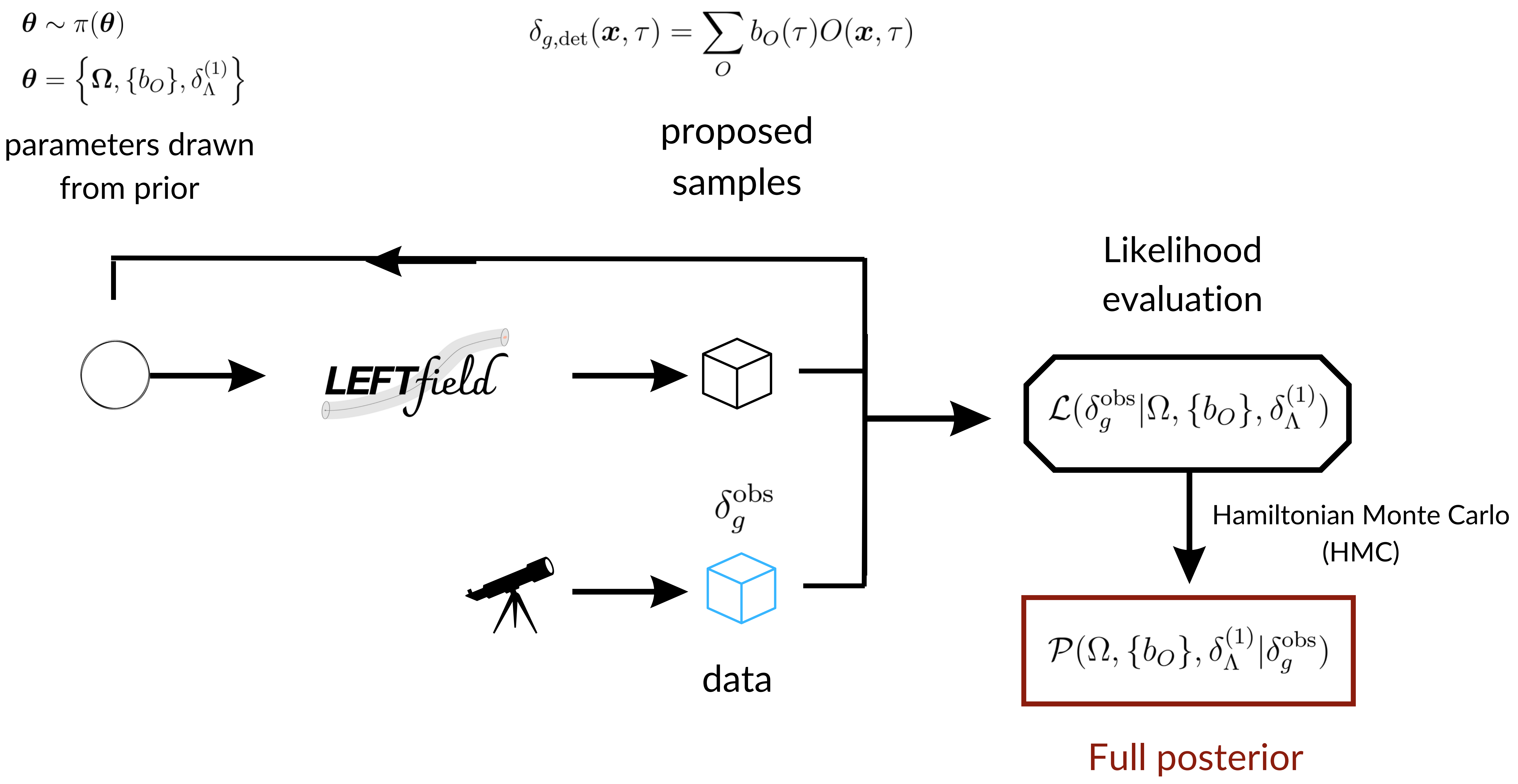
amplitudes and phases of the galaxy density field



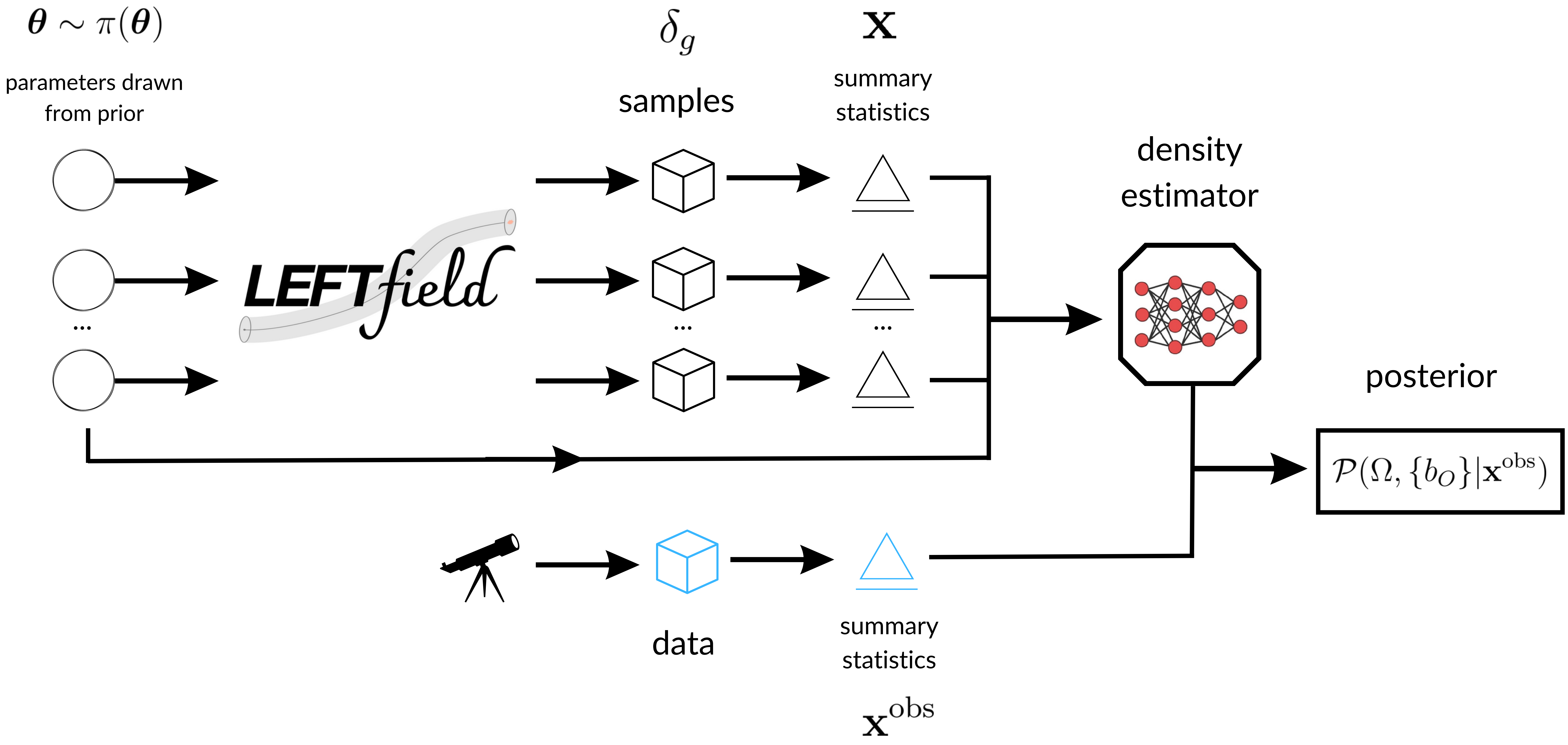
LEFTfield: a fast forward model that solves the gravitational evolution of all modes in a lattice

- Field-level inference
- Simulation-based inference

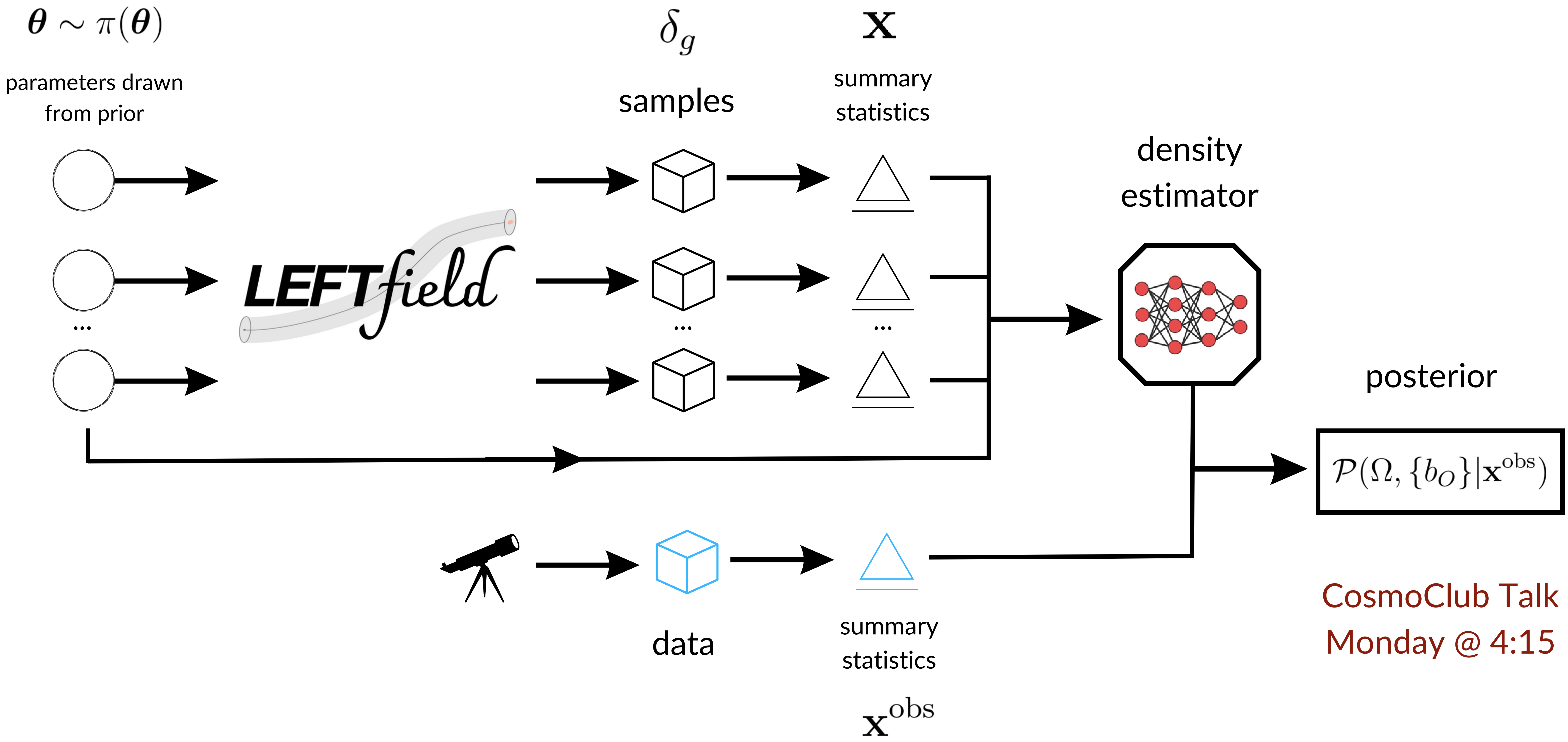
Field-level inference



Simulation-based inference



Simulation-based inference



CosmoClub Talk
Monday @ 4:15

Lagrangian Perturbation Theory (LPT)

Advection equation

$$\boldsymbol{x}(\boldsymbol{q},\tau) = \boldsymbol{q} + \boldsymbol{s}(\boldsymbol{q},\tau)$$

Displacement

Geodesic + Poisson + Continuity:

$$1+\delta(\boldsymbol{x},\tau) = |\mathbf{1} + \boldsymbol{M}(\boldsymbol{q},\tau)|^{-1}$$

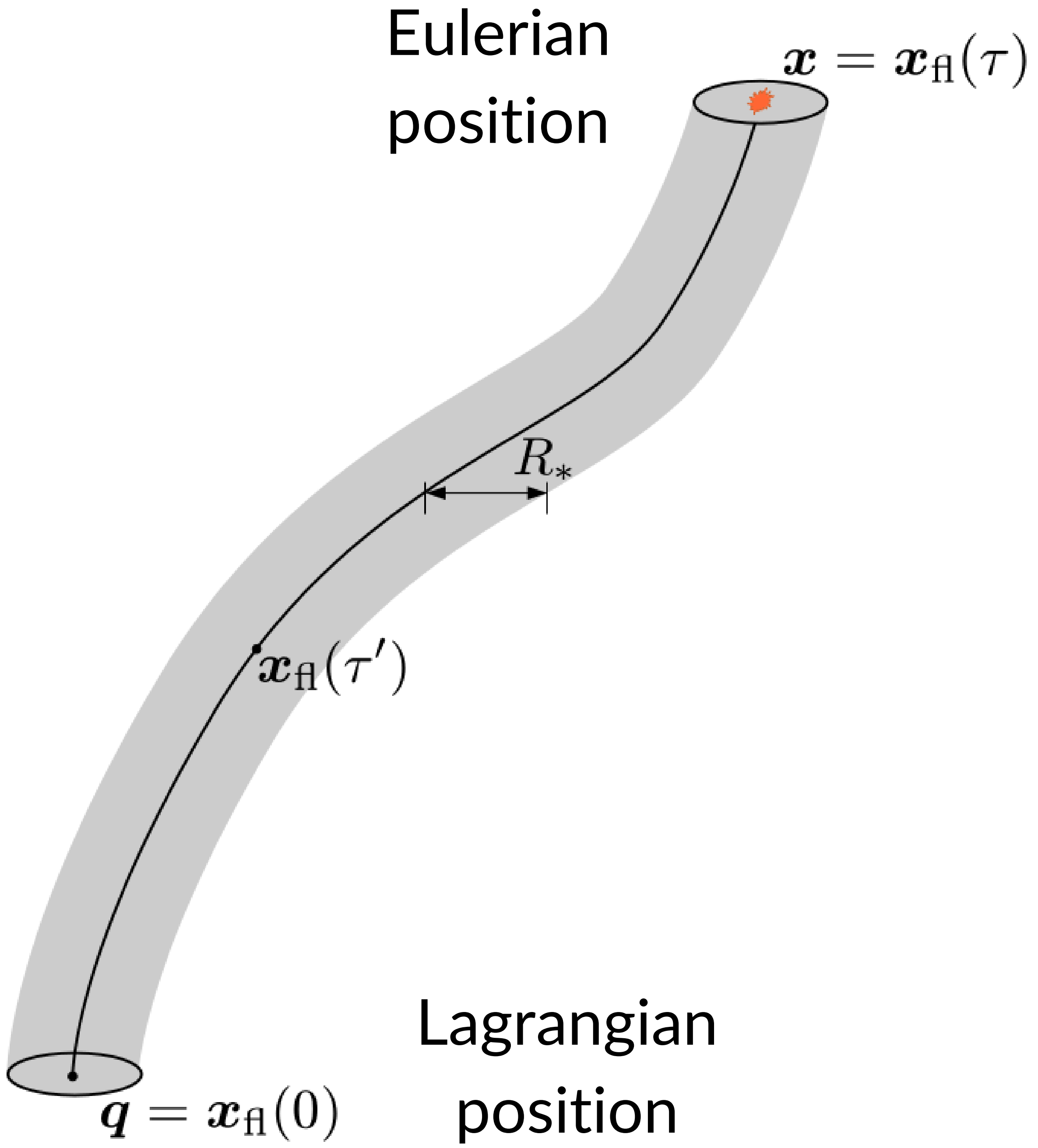
Deformation tensor

$$M_{ij} \equiv \partial s_j / \partial q_i$$

LPT

$$\boldsymbol{s}(\boldsymbol{q},\tau) = \sum_{n=1}^{\infty} \boldsymbol{s}^{(n)}(\boldsymbol{q},\tau)$$

Matsubara (2015)



Desjacques, Jeong & Schmidt (2016)

Lagrangian bias expansion

EFTofLSS:

$$\delta^L_{g,\mathrm{det}}(\boldsymbol{q},\tau) = \int_0^\tau d\tau' F_g\left[\boldsymbol{M}(\boldsymbol{q},\tau'),\tau',\tau\right]$$

Expand in F in M

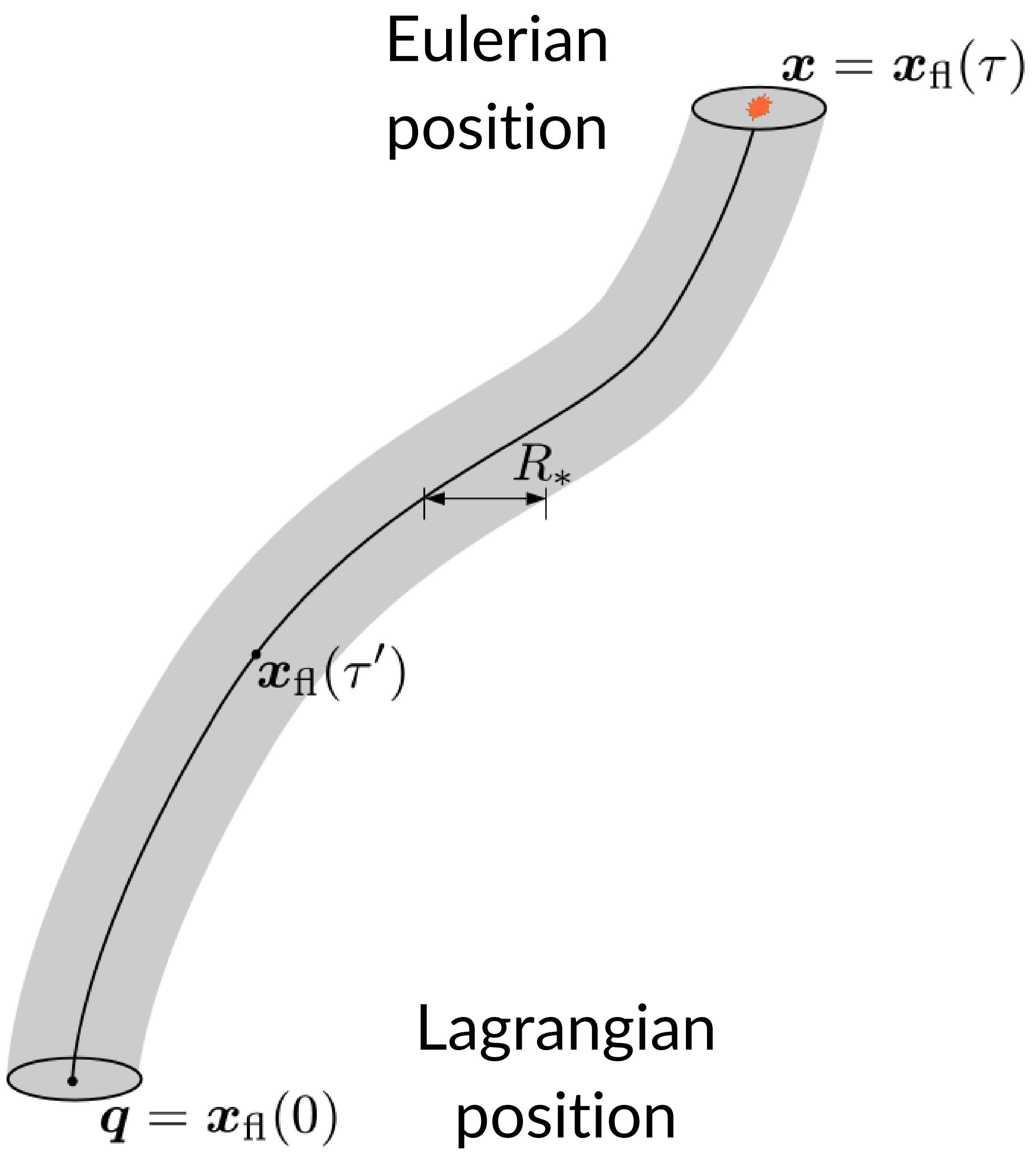
$$\delta^L_{g,\mathrm{det}}(\boldsymbol{q},\tau) = \sum_{\mathcal{O}^L} b_{\mathcal{O}^L}(\tau)\mathcal{O}^L(\boldsymbol{q},\tau)$$

all rotational invariants of M

1^{st} $\quad \mathrm{tr}[\boldsymbol{M}^{(1)}]$

2^{nd} $\quad \mathrm{tr}[\boldsymbol{M}^{(1)}\boldsymbol{M}^{(1)}],\ (\mathrm{tr}[\boldsymbol{M}^{(1)}])^2.$

Mirbabayi, Schmidt & Zaldarriaga (2015)



Desjacques, Jeong & Schmidt (2016)

Forward model

$$\alpha \equiv \sigma_8/\sigma_8^{\rm fid} \qquad \hat{s}(\boldsymbol{x}) \sim \mathcal{N}(0,1)$$

$$\delta_\Lambda^{(1)}(\boldsymbol{k},z) = W_\Lambda(k)\sqrt{\alpha^2 P_L(k,z)}\,\hat{s}(\boldsymbol{k})$$

$1+\delta(\boldsymbol{x},\tau) = |\mathbf{1}+\boldsymbol{M}(\boldsymbol{q},\tau)|^{-1}$ $\qquad M_{ij} \equiv \partial_i s_j$

$$\mathrm{tr}[\boldsymbol{M}_\Lambda^{(1)}] = -\delta_\Lambda^{(1)}$$

Lagrangian Bias Operators

1st	$\mathrm{tr}[\boldsymbol{M}_\Lambda^{(1)}]$
2nd	$\mathrm{tr}[\boldsymbol{M}_\Lambda^{(1)}\boldsymbol{M}_\Lambda^{(1)}],\ (\mathrm{tr}[\boldsymbol{M}_\Lambda^{(1)}])^2$

LPT recursion relations

$$\boldsymbol{s}^{(n)}$$

nLPT

$$\delta^L_{g,\rm det}(\mathbf{q},\tau) = \sum_{\mathcal{O}^L} b_{\mathcal{O}^L}(\tau)\mathcal{O}^L(\mathbf{q},\tau)$$

$$\boldsymbol{x}(\boldsymbol{q},\tau) = \boldsymbol{q} + \boldsymbol{s}(\boldsymbol{q},\tau)$$

$$\delta_{g,\rm det}(\boldsymbol{x},\tau)$$

An n-th order Lagrangian Forward Model for Large-Scale Structure
Schmidt (2021)

Forward model

$$\alpha \equiv \sigma_8/\sigma_8^{\text{fid}} \qquad \hat{s}(\boldsymbol{x}) \sim \mathcal{N}(0,1)$$

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Lagrangian Bias Operators

1^{st}	$\mathrm{tr}[\boldsymbol{M}_\Lambda^{(1)}]$
2^{nd}	$\mathrm{tr}[\boldsymbol{M}_\Lambda^{(1)}\boldsymbol{M}_\Lambda^{(1)}]$, $(\mathrm{tr}[\boldsymbol{M}_\Lambda^{(1)}])^2$

LPT recursion relations

$$\boldsymbol{s}^{(n)}$$

nLPT

$$\delta^L_{g,\text{det}}(\mathbf{q},\tau) = \sum_{\mathcal{O}^L} b_{\mathcal{O}^L}(\tau)\mathcal{O}^L(\mathbf{q},\tau)$$

$\boldsymbol{x}(\boldsymbol{q},\tau) = \boldsymbol{q} + \boldsymbol{s}(\boldsymbol{q},\tau)$

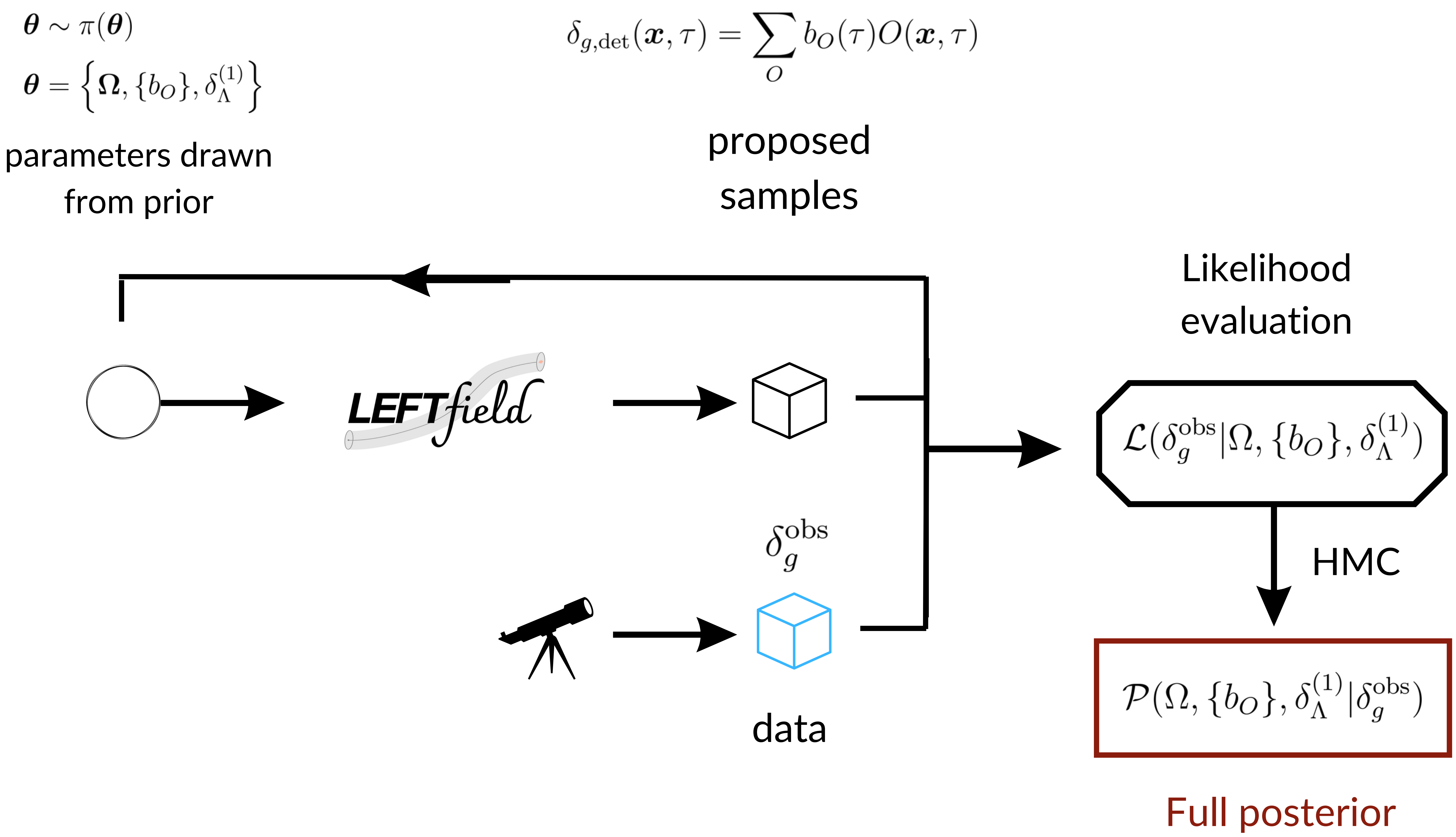
$$\delta_{g,\text{det}}(\boldsymbol{x},\tau)$$

$$\delta_{g,\text{det}}(\boldsymbol{k},\tau) = \int \frac{d\boldsymbol{q}}{(2\pi)^3} e^{i\boldsymbol{k}\cdot[\boldsymbol{q}+\boldsymbol{s}(\boldsymbol{q},\tau)]}\left[1 + \delta_g^L(\boldsymbol{q},\tau)\right]$$

$$M_{\text{CIC}}^{\mathcal{O}}(\boldsymbol{q}) = \mathcal{O}^L(\boldsymbol{q})$$

An n-th order Lagrangian Forward Model for Large-Scale Structure
Schmidt (2021)

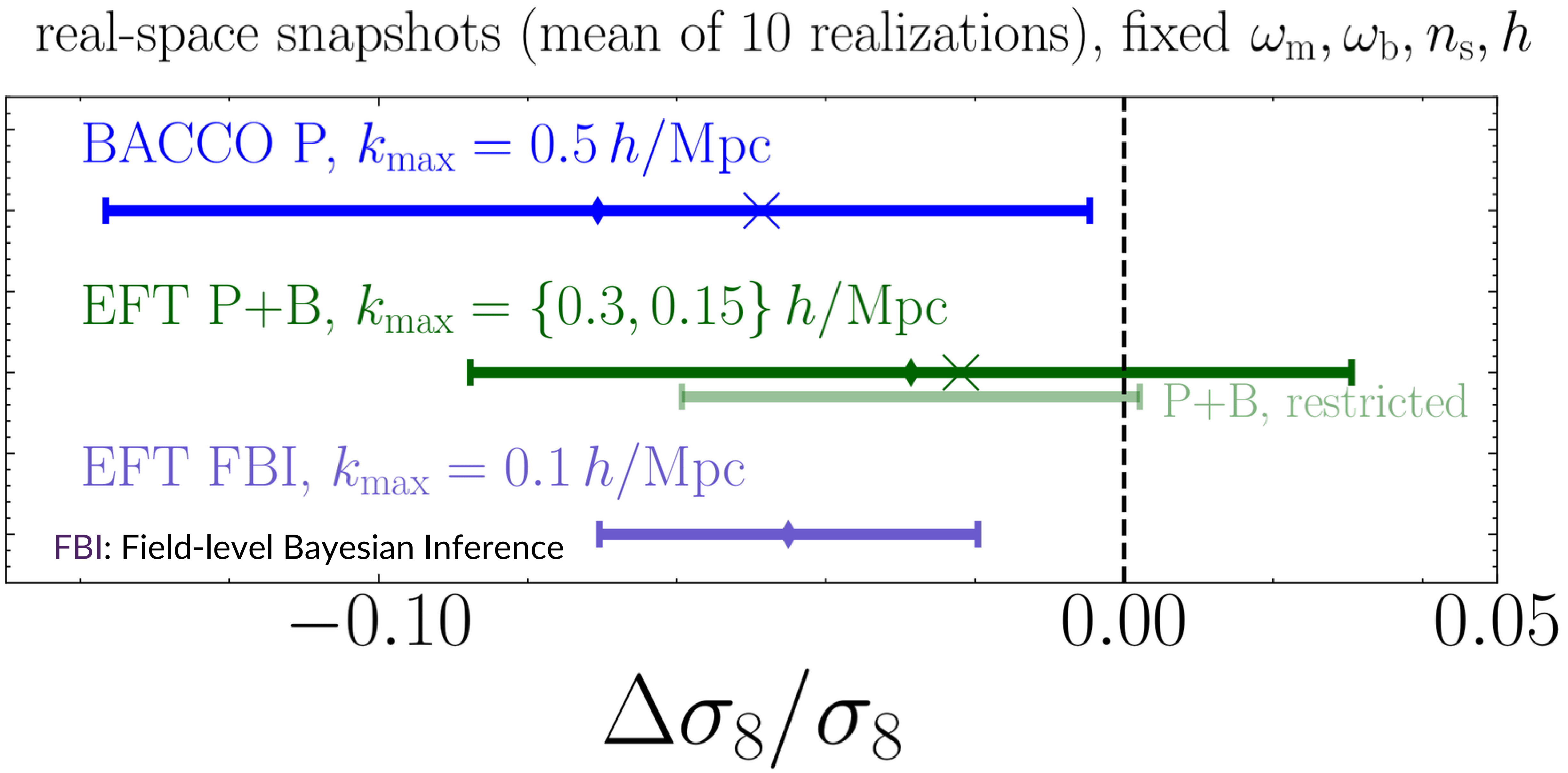
Field-level inference



Some results

M. Nguyen, Y. Kobayashi, A. Salcedo, E. Krause, M. Ivanov, M. Pellejero

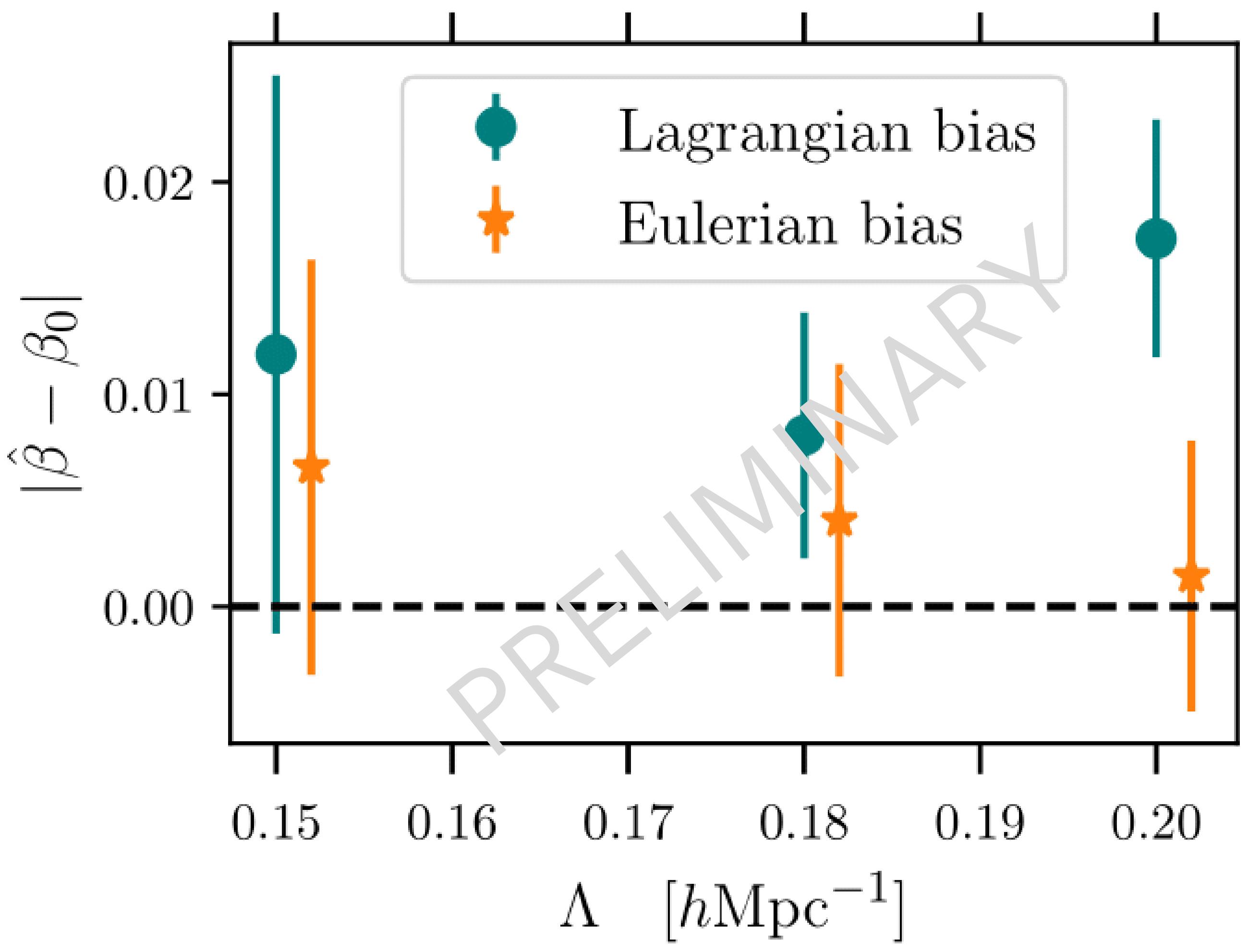
EFT-based full field-level inference on blind catalogs from beyond 2-pt blind challenge



Some results

Babic, Schmidt & Tucci (2022)
Babic, Schmidt & Tucci (in prep)

Fixed and free initial conditions of BAO scale and bias parameters on rest frame (tested on mock data and Nbody halos)



Conclusion & Next Steps

Field-level inference with LEFTfield has proven to be a powerful tool for galaxy clustering analysis and offers several advantages over standard analysis.

LEFTfield goals (with Fabian Schmidt, HMC field-level inference and SBI with summary statistics):

- Rest-frame Nbody halos (Nhat-Minh Nguyen)
- Redshift space, survey mask and systematic effects (Julia Stadler)
- BAO scale inference (Ivana Babić)
- Renormalization group approach for bias running (Henrique Rubira, Charalampos Nikolis)

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