Tensionless limit of AdS₃ /CFT₂ from integrability

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Introduction

The integrability methods have been proven to be successful in AdS_{d+1}/CFT_d at d=4, 5. Our next goal is the case of d=3 with RR-flux, coming from D1-D5 branes with 16 susy. This background has the moduli space of T^4, and the spin chain contains massless modes.

The spin chain proposal suffers from various ambiguities. Unlike higher-dimensional cases, or unlike the background with pure NS-NS flux, we do not know the CFT dual. However, recently the mirror TBA equations for AdS_3 have been proposed, which should predict the exact energy of the ground states.

OUR GOAL is to obtain the excited-state mirror TBA by contour deformation trick, simplify it in the small tension (h=0) limit, and solve the equations to obtain the exact energy of massless particles at general length L, and at the leading order of small h.

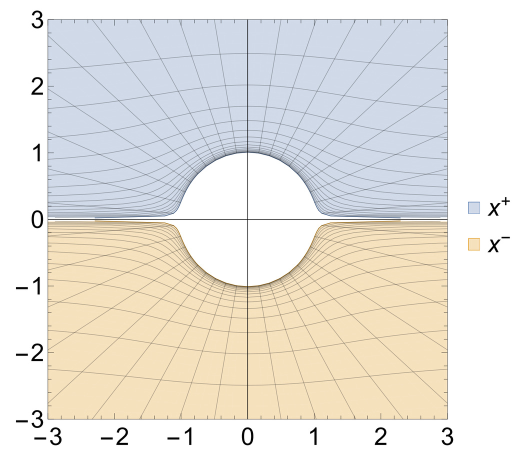
Superstring in $AdS_3 \times S^3 \times T^4$

$$\frac{PSU(1,1|2)^2}{SU(1,1)\times SU(2)} \supset AdS_3\times S^3$$

- GS in $AdS_3 \times S^3 \times T^4$ is classically integrable; coset κ-gauge → supercoset + free T^4 SCFTs
- GS in the LC gauge is no longer a supercoset, but massive and massless interact
- Kinetic term: $ds^2 = -\left(\frac{1+\frac{z^2}{4}}{1-\frac{z^2}{4}}\right)^2 dt^2 + \frac{d\vec z^2}{(1-\frac{z^2}{4})^2} + \left(\frac{1-\frac{y^2}{4}}{1+\frac{y^2}{4}}\right)^2 d\phi^2 + \frac{d\vec y^2}{(1+\frac{y^2}{4})^2} + \sum_{i=6}^{9} dX_i dX_i$
- Uniform LC gauge: $x^+ = \frac{1}{2}(\phi + t) = \tau$
- Decompactification limit → Central extension: $\mathfrak{psu}(1,1|2)_L \oplus \mathfrak{psu}(1,1|2)_R \to \mathfrak{psu}(1|1)^4 \to \mathfrak{psu}(1|1)^4_{c.e.}$

  $\{\mathfrak{Q}_L{}^{\dot a}, \mathfrak{S}_{L\dot b}\} = \frac{1}{2}\delta^{\dot a}_{\dot b}(H+M), \quad \{\mathfrak{Q}_L{}^{\dot a}, \mathfrak{Q}_{R\dot b}\} = \delta^{\dot a}_{\dot b} C, \quad \{\mathfrak{Q}_{R\dot a}, \mathfrak{S}_R{}^{\dot b}\} = \frac{1}{2}\delta_{\dot a}{}^{\dot b}(H-M), \quad \{\mathfrak{S}_{L\dot a}, \mathfrak{S}_R{}^{\dot b}\} = \delta_{\dot a}{}^{\dot b}\overline{C}$ — mass
- Symmetry of T^4: $\mathfrak{so}(4) = \mathfrak{su}(2)_\bullet \oplus \mathfrak{su}(2)_\circ$, $\mathfrak{su}(2)_\bullet \rightsquigarrow (\dot a = 1,2)$
- Shortening conditions for 1-particle states: $H^2 = M^2 + 4C\overline{C} \Rightarrow E(p) = \sqrt{m^2 + 4h^2 \sin^2\frac{p}{2}}, \quad (m\in\mathbb{Z})$
- Scattering between left (m>0) and right (m<0) is fixed by psu(1|1) + LR symmetry

[Babichenko, Stefański, Zarembo, 0912.1723] [Borsato, Ohlsson Sax, Sfondrini, Stefański, 1403.4543 & 1406.0453] [Lloyd, Ohlsson Sax, Sfondrini, Stefański, 1410.0866] [Beisert, Zwiebel, 0707.1031] [Borsato, Ohlsson Sax, Sfondrini, Torrielli, 1303.5995] [Rughoonauth, Sundin, Wulff, 1204.4742] [Borsato, Ohlsson Sax, Sfondrini,1211.5119]



Massive & massless kinematics

- Massive Zhukovski: $x^\pm(p) = e^{\pm ip/2}\frac{|m| + \sqrt{m^2 + 4h^2\sin^2\frac{p}{2}}}{2h\sin\frac{p}{2}}$
- Massless (boundary of massive): $\lim_{m\to 0+} x^\pm(p) = z^\pm(p), \quad z^+z^- = 1 \Rightarrow z^\pm(p) = (z(p))^{\pm 1}$
- Energy and momentum: $E = i\tilde p = \frac{h}{2i}\left(x^+ - \frac{1}{x^+} - x^- + \frac{1}{x^-}\right), \quad ip = -\tilde E \equiv \ln\frac{x^+}{x^-}$

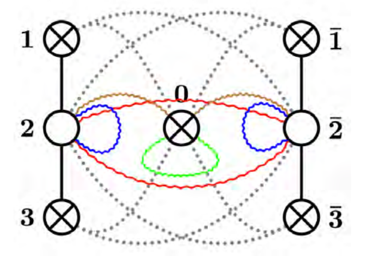
  $u \equiv x + \frac{1}{x}, \quad f^\pm(u) = f\left(u \pm \frac{i\,|m|}{h}\right)$
- Rapidity:
- Relativistic parameter (γ rapidity): $z(\gamma) \equiv \frac{i - e^\gamma}{i + e^\gamma}, \quad \gamma(z) = \log\left(\frac{z-1}{z+1}\right) - \frac{\pi i}{2}$

  Crossing: $z(\gamma + \pi i) = \frac{1}{z(\gamma)}$, Periodicity: $z(\gamma + 2\pi i) = z(\gamma)$

γ rapidity is more important than Zhukovski; the (sG) dressing phase is not periodic under 2πi shift of γ

Asymptotic Bethe Ansatz Equations

$\mathfrak{psu}(2|2)$, not $\mathfrak{psu}(1|1)$:



2, for left massive $x^\pm$
$\bar 2$, for right massive $\bar x^\pm$
0, for massless $z^\pm$
1, 3, for left auxiliary $y_1^{(1)}, y_3^{(1)}$
$\bar 1, \bar 3$, for right auxiliary $y_1^{(2)}, y_3^{(2)}$

Massless Bethe Ansatz (without $\mathfrak{su}(2)_\circ$ indices):

$$\left(\frac{z_k^+}{z_k^-}\right)^L = \prod_{j=1, j\neq k}^{N_0} e^{-ip_k/2} e^{+ip_j/2}\frac{z_k^+ - z_j^-}{z_k^- - z_j^+}(\sigma_{kj}^{\circ\circ})^2 \quad \text{massless - massless}$$
$$\times \prod_{j=1}^{N_1} e^{+ip_k/2}\frac{z_k^- - y_{1,j}}{z_k^+ - y_{1,j}} \prod_{j=1}^{N_3} e^{+ip_k/2}\frac{z_k^- - y_{3,j}}{z_k^+ - y_{3,j}} \quad \text{massless - auxiliary}$$
$$\times \prod_{j=1}^{N_2} e^{-ip_k} e^{+ip_j/2}\frac{z_k^+ - x_j^-}{z_k^- - x_j^+} z_k^+ (\sigma_{kj}^{\circ\bullet})^2 \quad \text{massless - massive (L)}$$
$$\times \prod_{j=1}^{N_{\bar 2}} e^{+ip_j/2}\frac{1 - \frac{1}{z_k^+ \bar x_j^+}}{1 - \frac{1}{z_k^- \bar x_j^-}}\frac{1}{z_k^+}(\sigma_{kj}^{\circ\bullet})^2 \quad \text{massless - massive (R)}$$

[Borsato, Ohlsson Sax, Sfondrini, Stefański, Torrielli, 1607.00914] [Frolov, Sfondrini, 2112.08896]

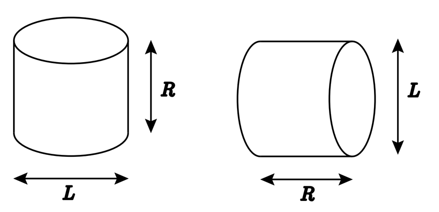
Mirror TBA Equations

- At small h, massive wrapping corrections are small, massless wrapping corrections are $O(1)$

$$E_J(p) = \sum_i \sqrt{m_i^2 + 4h^2\sin^2\frac{p_i}{2}} + \delta E_J$$

- Equivalence of Euclidean torus partition functions

Ground state at finite volume evolving for a long period of time



Gas of all types of particles at large volume and finite temperature

$$e^{-RE_0(L)} = \lim_{R\to\infty} e^{-\tilde F(R)}$$

- Massless contributions to the mirror free energy at the large R should be negligible
- The saddle-point equations for densities = TBA

$$-\log Y_A(u) = L\tilde E_A(u) - i\gamma_A - \sum_B \int dv \log(1 + Y_B(v))\,K_{BA}(v,u) \qquad K_{BA}(v,u) = \frac{1}{2\pi i}\frac{\partial}{\partial v}\log S_{BA}(v,u)$$

- Solution of TBA → Exact ground-state energy $\quad E(L) = -\sum_A \int \frac{d\tilde p_A(u)}{2\pi}\log(1 + Y_A(u))$

[Frolov, Sfondrini, 2112.08898]

Small tension limit of excited-state TBA

- TBA for excited states ← Contour deformation trick

$$-\log Y_A(u) = L\tilde E_A(u) - i\gamma_A - \sum_j \log S_{BA}(v_{*j}, u) - \sum_B \log(1 + Y_B) \star K_{BA}(u)$$

- Exact Bethe equations $\quad Y_A(v_{*j}) = -1 \;\to\; \log Y_A(v_{*j}) = (2n_j + 1)\pi i$ — mode number
- Exact excited-state energy $\quad E(L) = \sum_j E_A(u_{*j}) - \sum_A \int \frac{d\tilde p_A(u)}{2\pi}\log(1 + Y_A(u))$

In our setup, we choose length L = even, excite a pair of massless particles, specify states by solving asymptotic BAE ($L/2$ distinct mode numbers for each L), and take the small coupling (h=0) limit. It turns out that the mirror TBA simplify a lot at small h; two Y-functions convoluted by Cauchy kernel

- Massive Y_Q functions decouple, because they are $O(h^{2L})$ at small h
- Dressing phase kernels of all particle types decouple at the leading order of small h

Solving TBA

$$s(\gamma) = \frac{1}{2\pi i}\frac{\partial \log S(\gamma)}{\partial\gamma} = \frac{1}{2\pi\cosh(\gamma)}$$

- Mirror TBA equations for massless & auxiliary particles

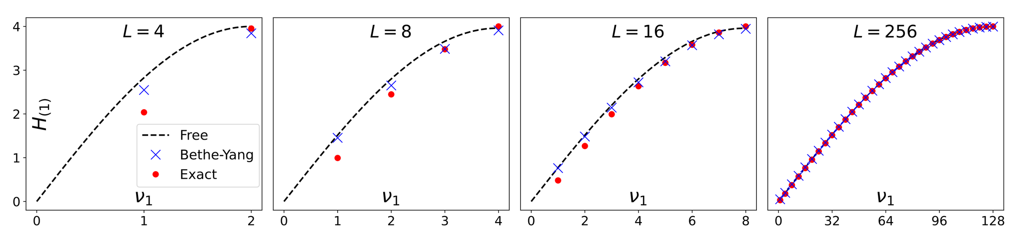
$$\log Y_0 = -L\tilde E_0 + \log[(1 + Y_0)^2(1 - Y)^4] * s - \sum_j \log S(\gamma_{*j} - \gamma)$$
$$\log Y = \log[(1 + Y_0)^2] * s - \sum_j \log S(\gamma_{*j} - \gamma)$$

- Solve TBA for 2-particle states numerically, and compute the exact energy

$$E = \sum_j \left|2h\sin\frac{p_{*j}}{2}\right| - \int \frac{d\tilde p_0}{2\pi}\log(1 + Y_0)$$

Both terms are $O(h)$

- Wrapping corrections are parametrically small, $O(1/L)$



- In the BMN limit (large L, small p), the spectrum is approximated by the free particles
- We can confirm this expectation by quasi-analytic solution of TBA

Conclusion and Outlook

- Integrability methods in AdS_3/CFT_2
- Numerically solved excited-state TBA with massless particles
- $O(1/L)$ contributions to the energy

- How to fully justify TBA / dressing phase / QSC / etc
- Identify the dual CFT
- Moduli and integrable deformation, like $AdS_3 \times S^3 \times S^3 \times S^1$
- How to resolve mismatches (no problems in the massive sector, similar to $AdS_5 \times S^5$)

- Massless integrable field theories can have non-zero particle productions
- Energy of massless particles at 2-loop in string theory
- Twisted ground state energy is 1/2 of the TBA prediction

[Cavaglià, Gromov, Stefański, Torrielli, 2109.05500] [Ekhammar, Volin, 2109.06164]
[Sundin, Wulff; 1411.4662, 1605.01632] [Hoare, Levine, Tseytlin; 1812.02549] [Frolov, Pribytok, Sfondrini; 2305.17128]