Dualities Amongst A_{N-1} Class-$\mathcal{S}$ Argyres-Douglas Theories

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The Seiberg-Witten curves of a large class of $\mathcal{N}=2$ superconformal can be calculated using the spectral curve of a Hitchin system with singularities. By considering solutions with regular and irregular singularities we can construct a set of non-Lagrangian theories known as Argyres-Douglas theories. We conjecture a set of dualities amongst these constructions.

The 6d Origins of 4d $\mathcal{N}=2$ SCFTs

It has been known since the work of Gaiotto, Moore and Neitzke [1] that a large class of 4d $\mathcal{N}=2$ superconformal field theories (SCFTs) can be constructed from 6d $(2,0)$ theories with 4d defect insertions. These theories are known collectively as class-$\mathcal{S}$. To do this one compactors $(2,0)$ onto $\mathcal{C}\times\mathbb{R}^4$ and performs a partial topological twist [2] on the Riemann surface $\mathcal{C}$. This, loosely speaking, identifies a sub-sector of $(2,0)$ whose degrees of freedom on $\mathcal{C}$ are topological. The SCFTs are recovered in the zero area limit of $\mathcal{C}$.

The Connection to Integrability

The dimensional reduction of an $\mathcal{N}=2$ SCFT on S^1 gives an 3d $\mathcal{N}=4$ sigma model whose target space is the moduli space of solutions of an integrable model on $\mathcal{C}$. In class-$\mathcal{S}$ this model is a Hitchin system [3, 4] whose equations of motion are:

$$F(A)=\phi\wedge\phi\,,\quad D_\phi=D\star\phi=0\,, \tag{1}$$

where A is a connection on principal $SL(N)$-bundle P over $\mathcal{C}$, and $\phi=\varphi dz+\bar{\varphi}d\bar{z}$ a section of the adjoint bundle $\mathrm{ad}(P)$ known as the Higgs field.
The Poisson commuting set of Hamiltonians of the Hitchin system appear in its spectral curve:

$$\det(\lambda-\varphi dz)=\lambda^N-\sum_{i=2}^{N}\lambda^{N-i}\mathrm{tr}(\varphi^i)(dz)^i=0\,, \tag{2}$$

where λ is a holomorphic one-form in $T^*\mathcal{C}$. The key insight of [1] is that after twisting the Coulomb branch operators of $(2,0)$ become identified with Hamiltonians of (2), allowing us to interpret (2) as the Seiberg-Witten curve of an $\mathcal{N}=2$.

Regular and Irregular Singularities

The 4d defects in $(2,0)$ that realise SCFTs in class-$\mathcal{S}$ sit at points in $\mathcal{C}$. In the Hitchin system these defects arise as both regular and irregular singularities in the field ϕ. Regular singularities fall into nilpotent orbits of $\mathfrak{sl}_N$, i.e. they are locally of the form:

$$\varphi dz=gT_1g^{-1}\frac{dz}{z}+\dots \tag{3}$$

where T_1 is a nilpotent matrix in Jordan normal form and $g\in SL(N)$. We label such punctures by partition of N denoted by Y, with the choice of Y corresponding to a choice of nilpotent orbit [5].

Irregular singularities of Hitchin systems were introduced in [6]. In our constructions these singularities are locally of the form:

$$\varphi dz=gT_mg^{-1}\frac{dz}{z^m}+\dots\,, \tag{4}$$

for $m>1$. Again, $g\in SL(N)$ with T_m a nilpotent element of $\mathfrak{sl}_n$ in Jordan normal form. We label such singularities by a partition $[n_1,n_2,\dots]$ whose elements are the dimensions of the Jordan blocks of T_m. We focus on irregular singularities labelled by partitions of the form $[n,1^{N-n}]$.

Superconformal Field Configurations

When we partially topologically twist $(2,0)$ the holonomy group of $\mathcal{C}$ is identified with the $U(1)$ R-symmetry of the $\mathcal{N}=2$ superconformal group. This restricts the field configurations of ϕ we can consider to those which are invariant under the $U(1)$ action $z\mapsto e^{i\theta}z$. This occurs only if $\mathcal{C}=\mathbb{C}P^1$ with two antipodal singularities [7] (see figure 1).

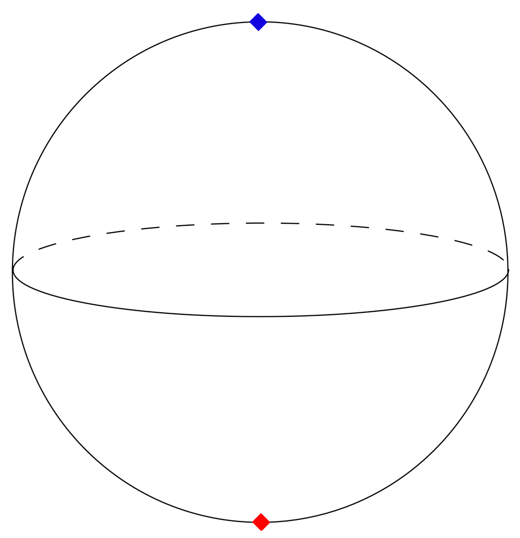

Figure 1: A representation of the allowed field configurations of ϕ.

In $\mathcal{N}=2$ theories the scaling dimensions of a relevant operator $\mathcal{O}$ and coupling constant γ are paired such that $\Delta(\mathcal{O})+\Delta(\gamma)=2$ [8]. This forces us to consider field configurations of ϕ with at most one regular and irregular singularity each. When diagonalised and written as a power series expansion around the irregular singularity φ is of the form:

$$\varphi=\frac{u_0}{z^{2+\frac{k}{n}}}+\frac{u_1}{z^{1+\frac{k}{n}}}+\dots \tag{5}$$

where each u_i is in a Cartan of $\mathfrak{sl}_N$. Using the Lie algebra A_{N-1}, partition $[Y]$, and numbers n and k we label each SCFT by:

$$(A^n_{N-1}[k],[Y])\,. \tag{6}$$

Argyres-Douglas Theories and SCFT Data

Argyres-Douglas (AD) theories, first found in [9], are non-Lagrangian $\mathcal{N}=2$ SCFTs with Coulomb branch operators of fractional scaling dimension. These operators appear as the coefficients of the Seiberg-Witten curve. If we take ϕ to have an irregular singularity and use (2) to calculate the Seiberg-Witten curve one always finds AD theories [7].

Using the Seiberg-Witten curves we can read off the scaling dimensions of the Coulomb branch operators $\{\Delta_i\}$ (where $\Delta_i>1$) and chiral deformations, as well as the numbers of marginal and mass deformations. Using this data the a_{4d} and c_{4d} anomalies are determined by [10, 11]:

$$c_{4d}=(2r+rk(\mathfrak{g}_F))\frac{\Delta_{max}}{12}+\frac{r}{6}\,,\quad 2a_{4d}-c_{4d}=\frac{1}{4}\sum_i(2\Delta_i-1)\,, \tag{7}$$

where Δ_{max} is the largest of the Coulomb branch scaling dimensions, r the number of Coulomb branch operators, and $rk(\mathfrak{g})_F$ the rank of the flavour symmetry group. We collectively call the set of scaling dimensions and anomalies the SCFT data.

New Dualities

In our paper we conjecture a new set of Dualities amongst class-$\mathcal{S}$ constructions of AD theories:

$$\begin{aligned}(A^n_{N-1}[k],[Y])\equiv\ & A^{(N-n)(n+k-1)+N}_{(N-n)(n+k-1)+N-1}[k-(N-n)(n+k)],[(n+k-1)^{N-n},Y])\\ &+H_{free}\text{ free hypermultiplets}\,,\end{aligned} \tag{8}$$

where

$$H_{free}=(N-n)\sum_{i=n+k}^{N}(i-n-k+1)l_i\quad,\text{ for }[Y]=[N^{l_N},\dots,1^{l_1}]\,. \tag{9}$$

We have three pieces of evidence for this conjecture:

- The SCFT data on either side of $\cong$ is the same,
- The compactifications on S^1 realise the same 3d $\mathcal{N}=4$ sigma model,
- The theories with exactly marginal couplings can be constructed via a conformal gauging from those without exactly marginal couplings.

There are two interesting consequences of this conjecture: 1) there exists a set of isomorphism amongst the vertex operator algebras associated to dual $\mathcal{N}=2$ SCFT; and 2) the moduli spaces of naively different Hitchin systems are in fact the same. In the latter case, this is because the moduli space of the Hitchin system is the Coulomb branch of the 3d $\mathcal{N}=4$ theory.

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