AN ANALOGUE SPINNING BLACK HOLE

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In a jiffy

- We consider a generalized fluid dynamical model incorporating non-commutative corrections generated by Berry curvature terms.
- In an Euler framework, this is manifested as an extended algebra between the fluid variables.
- Here we study the dynamics of 1^{st} order sonic fluctuations that live in an analogous space-time structure.
- The effective metric resembles that of a spinning Black Hole; the spin is induced by the underlying Non-Commutative structure.
- The effective mass and spin parameters of the Black Hole, in terms of fluid parameters, are identified.
- The connection of our model with anomalous Hall systems may lead to observable signatures of the analogue black hole in physical systems.

Introduction

What is analogue gravity?

- Analogue gravity is a field of research that explores the existence of analogies or similarities between gravitational phenomena in general relativity and other physical systems, such as condensed matter systems. By studying these analogues, researchers aim to gain a deeper understanding of the corresponding problems in gravity [3].

Our study related to analogue gravity

- Our study investigates the acoustic metric obtained from our fluid model and its connection to analogue gravity and Berry curvature.
- To investigate this connection, we utilized the Hamiltonian formalism and developed an anomalous fluid model. By incorporating quantum effects through generalized Poisson bracket structures, we modify the fluid equations and obtain an acoustic metric. Interestingly, this metric exhibits resemblances to the Kerr metric in EF (Eddington–Finkelstein)coordinates. Additionally, it contains an extra term that can be managed under specific conditions.

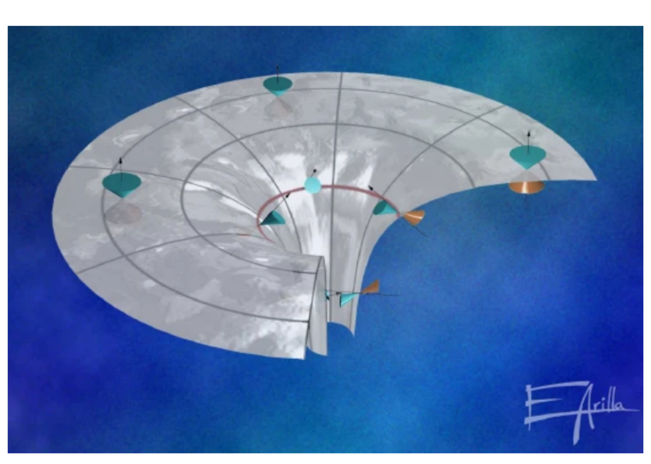

Figure 1: Artistic impression of cascading sound cones (in the geometrical acoustics limit) forming an acoustic black hole when supersonic flow tips the sound cones past the vertical.[3]

Methodology

To start with, we have Berry curvature corrected fluid dynamic equations

$$\textbf{Continuity:}\quad \dot{\rho} + \nabla.\mathbf{J}^{an} = e\rho\mathcal{F}.(\nabla\times\mathbf{E}) \tag{1}$$

where the anomalous current is $\mathbf{J}^{an}$ mention in [1]

$$\begin{aligned}\textbf{Euler:}\quad \dot{\mathbf{v}} + \frac{(\mathbf{v}.\nabla)\mathbf{v}}{\mathcal{A}} &= -\frac{\nabla P}{\rho\mathcal{A}} + e\frac{\rho\mathbf{v}\times\mathbf{B}}{\rho\mathcal{A}} - e\frac{\mathbf{B}.\nabla\mathbf{P}}{\rho}\mathcal{F} - e(\mathbf{v}.\mathcal{F})\\ &(\mathbf{B}.\nabla)\mathbf{v} + \{(\frac{\nabla P}{\rho}\times\mathcal{F}).\nabla - \frac{1}{\rho}\nabla v^2.(\nabla\times(\mathcal{F}æ)) + 2v^2(\mathcal{F}\\ &\times\frac{\nabla\rho}{\rho}).\nabla - \mathcal{F}.(\frac{\nabla\rho}{\rho}\times\nabla v^2))\}\mathbf{v}\\ &-e\{\frac{\mathbf{E}}{\mathcal{A}} + e(\mathbf{E}.\mathbf{B})\mathcal{F} - (\mathbf{E}\times\mathcal{F}).\nabla\mathbf{v}\}\end{aligned} \tag{2}$$

The equations (1) and (2) are unperturbed versions of the continuity and Euler equations. After assuming certain conditions, we simplify the fluid equations to the following forms:

$$\dot{\rho} = -\nabla(\frac{\rho\mathbf{v}}{A}) \tag{3}$$

$$\dot{\mathbf{v}} + \frac{(\mathbf{v}.\nabla)\mathbf{v}}{\mathcal{A}} = -\frac{\nabla P}{\rho\mathcal{A}} \tag{4}$$

Here, $\mathcal{A}$ is a function defined as $\mathcal{A}(\mathbf{x},\mathbf{k}) = 1 + e\mathbf{B}(\mathbf{x}).\mathbf{\Omega}(\mathbf{k})$.

The next step involves introducing a velocity potential ψ and defining the velocity of sonic disturbance c_s and the system enthalpy ∇h. With these variables, the simplified form of the Euler equation (4) becomes:

$$-\nabla\dot{\psi} + \nabla(\frac{(\nabla\psi)^2}{2A}) = -\nabla\frac{h}{A} \tag{5}$$

The perturbation terms of the continuity equation (3) and the perturbation terms of (1) are derived and combined to obtain the wave equation for the first-order perturbation ψ_1:

the effective background metric $g_{\mu\nu}$ is obtained. The acoustic metric looks like:

$$g_{\mu\nu}=\begin{pmatrix} -\frac{{u_1}^2+{u_2}^2+{u_3}^2-c_s^2}{A\Gamma c_s^2} & \frac{u_1}{\Gamma c_s^2} & \frac{u_2}{\Gamma c_s^2} & \frac{u_3}{\Gamma c_s^2}\\ \frac{u_1}{\Gamma c_s^2} & -\frac{A}{\Gamma c_s^2} & 0 & 0\\ \frac{u_2}{\Gamma c_s^2} & 0 & -\frac{A}{\Gamma c_s^2} & 0\\ \frac{u_3}{\Gamma c_s^2} & 0 & 0 & -\frac{A}{\Gamma c_s^2}\end{pmatrix}$$

where $\Gamma = \rho_0/c_s$.

The resulting acoustic metric resembles the Kerr solution in Painlevé-Gullstrand coordinates.

Acoustic Metric in PG coordinates

Our obtained acoustic metric structure in PL co-ordinate is the following:

$$\begin{aligned} ds^2 = \frac{\rho_0}{c_s}[&\frac{1}{\mathcal{A}^2}(c_s^2 - \frac{v_r^2}{\sin^2\theta\cos^2\phi})(dt + \frac{1}{c_s}dr)^2\\ &+2\frac{v_r}{\mathcal{A}}dr(dt+\frac{1}{c_s}dr) + 2\frac{v_r r\cot\theta}{\mathcal{A}}d\theta(dt+\frac{dr}{c_s})\\ &-2\frac{v_r r\tan\phi}{\mathcal{A}}(d\phi+\beta dr)(dt+\frac{1}{c_s}dr) - dr^2\\ &-r^2\sin^2\theta(d\phi+\beta dr)^2]\end{aligned} \tag{6}$$

Results

Comparisn between Kerr metric and acoustic metric

the Kerr metric written in EF coordinates and the acoustic metric. The Kerr metric in EF coordinates is the following (considering the signature (+,-,-,-)) :[2]

Now, the acoustic metric in the PL coordinates is given in the following (at $\theta = \pi/2$):

$$\begin{aligned} ds_{ac}^2 = \frac{c_s L\rho_0}{\mu}\Bigg[&\frac{1}{\mathcal{A}^2}\left(c_s^2 - \frac{v_r^2}{\cos^2\phi}\right)\left(dt+\frac{1}{c_s}dr\right)^2\\ &+2\frac{v_r}{\mathcal{A}}\left\{dr\left(dt+\frac{1}{c_s}dr\right) - \frac{r\sin\phi}{\cos\phi}\left(d\phi+\beta dr\right)\left(dt+\frac{1}{c_s}dr\right)\right\}\\ &-dr^2 - r^2\sin^2\theta\left(d\phi+\beta dr\right)^2\Bigg]\end{aligned} \tag{7}$$

The effective mass and effective spin parameter of an analogue spinning blackhole

We compared each of the metric coefficients between the Kerr metric written in EF coordinates[2]and the acoustic metric 7.

- effective mass parameter we obtained as

$$\boxed{m_{\text{eff}} \equiv \frac{L^3\rho_0 v_r^2}{\mathcal{A}^2 c_s^2}} \tag{8}$$

- In the similar procedure as earlier we obtained the effective spin parameter of the Kerr BH (at $\theta = \pi/2$) as

$$\begin{aligned} a_{\text{eff}} &= \frac{\mathcal{A}c_s^2 r}{v_r}\\ \Rightarrow\ \boxed{a_{\text{eff}} = \frac{\mathcal{A}c_s^2 L}{v_r}} &\qquad \text{(where } r \text{ is replaced by } r = Lr\text{)}\end{aligned} \tag{9}$$

References

[1] Krishna Mitra, A., & Ghosh, S. (2021). Divergence Anomaly and Schwinger Terms: Towards a Consistent Theory of Anomalous Classical Fluid. arXiv e-prints, arXiv-2111.

[2] Dalui, S., & Majhi, B. R. (2022). Horizon thermalization of Kerr black hole through local instability. Physics Letters B, 826, 136899.

[3] C. Barceló, S. Liberati, and M. Visser, *Analogue gravity*, Living Reviews in Relativity **14**, 1–159 (2011).