The cusp anomalous dimension of ABJM from a TBA approach

Motivation


One can construct 1/2 BPS Wilson loops as

$$
W:=\frac{1}{2 N} \operatorname{Tr}\left[\mathcal{P} \exp \left(i \int_{\text {straight }} \mathcal{L}(\tau) d \tau\right)\right]
$$

where $\mathcal{L}$ is a superconnection covariant under $\operatorname{SU}(1,1 \mid 3)$.

## Wilson loop's integrable open spin chain

| Anomalous dimensions |
| :---: | :---: |
| of insertions in |
| $1 / 2$ BPS Wilson loop |$\quad \longleftrightarrow$| Energies of |
| :---: |
| integrable open |
| spin chain |

Vacuum state: insertion of

$$
\mathcal{V}_{\ell}=\left(\begin{array}{cc}
0\left(C_{1} \bar{C}^{2}\right)^{\ell} C_{1} \\
0 & 0
\end{array}\right)
$$

$\Longrightarrow S U(2 \mid 1)$ symmetry (consistent with string theory expectations).
Impurities: replacing $C_{1}$ or $\bar{C}^{2}$ by

$$
\begin{array}{ll}
\text { type } A \text { magnons: } & \left(C_{3}, C_{4} \mid \bar{\psi}_{+}^{2}, \bar{\psi}_{-}^{2}\right), \\
\text { type } B \text { magnons: } & \left(\bar{C}^{3}, \bar{C}^{4} \mid \psi_{1}^{+}, \psi_{1}^{-}\right) .
\end{array}
$$

Scattering matrix: same $S U(2 \mid 2)$ invariant matrix as for single-trace operators.

## Reflection matrix:

$$
\mathbf{R}=\left(\begin{array}{cc}
R_{A}^{0} \hat{R} & 0 \\
0 & R_{B}^{0} \hat{R}
\end{array}\right)
$$

with

$$
\hat{R}=\operatorname{diag}\left(1,1, e^{-i p / 2},-e^{i p / 2}\right),
$$

and where $R_{A}^{0}$ and $R_{B}^{0}$ are dressing phases (not fixed by symmetry).
The above matrix is compatible with the $S U(2 \mid 1)$ symmetry, the boundary Yang-Baxter equations and weakcoupling expansions.

## Dressing phases

## Crossing equation:

$$
R_{A}^{0}(p) R_{B}^{0}(\bar{p})=-\frac{\frac{1}{x^{+}}+x^{+}}{\frac{1}{x^{-}}+x^{-}} \frac{1}{\sigma(p,-\bar{p})} .
$$

where $\sigma$ is the BES phase.

## All loop solutions:

$$
\begin{aligned}
& R_{A}^{0}(p)=-\frac{1}{R_{0}(p)}\left(\frac{\frac{1}{x^{+}}+x^{+}}{\frac{1}{x^{-}}+x^{-}}\right)\left(\frac{x^{-}}{x^{+}}\right), \\
& R_{B}^{0}(p)=\frac{1}{R_{0}(p)}\left(\frac{x^{-}}{x^{+}}\right),
\end{aligned}
$$

where $R_{0}$ is the square root of the dressing phase proposed in [1] for the $\mathcal{N}=4 \mathrm{sYM}$ case.

Consistent with weak and strong coupling expectations.

Perturbative boundary bound states only for type A particles.

## $\Gamma_{\text {cusp }}$ from a TBA approach

The anomalous dimension of

can be computed with a TBA formula for small $\ell$. The limit with no insertions ( $\ell=-1 / 2$ ) gives $\Gamma_{\text {cusp }}$.
$E_{0}(\ell)=-\frac{1}{4 \pi} \sum_{a=1}^{\infty} \int_{0}^{\infty} d q \log \left[1+Y_{a, 0}^{I}(q)\right]-\frac{1}{4 \pi} \sum_{a=1}^{\infty} \int_{0}^{\infty} d q \log \left[1+Y_{a, 0}^{I I}(q)\right]$.

> We propose the same $Y$ system as for the periodic spin chain.

## Asymptotic solution to the $Y$-system

$$
Y_{a, 0}^{I}=Y_{a, 0}^{I I} \sim\left(\frac{z^{[-a]}}{z^{[+a]}}\right)^{2 L} \frac{\varphi\left(u-\frac{i a}{2}\right)}{\varphi\left(u+\frac{i a}{2}\right)} T_{a, 1} .
$$

T functions are constrained by symmetry:

$$
T_{a, 1}^{S U(12)} \equiv T_{1, a}^{S U(2 \mid 1)} .
$$

$\varphi$ is obtained from the leading Lüscher correction.
With this we recover

$$
\begin{aligned}
\Gamma_{\text {cusp }} & =-2 \lambda \sin ^{2} \frac{\theta}{2} \sum_{k=0}^{\infty} P_{k}^{(0,1)}(-\cos \theta)+\mathcal{O}\left(\lambda^{2}\right)= \\
& =-\lambda\left(\frac{1}{\cos \frac{\theta}{2}}-1\right)+\mathcal{O}\left(\lambda^{2}\right) .
\end{aligned}
$$

as computed in [2] with an expansion in Feynman diagrams.

## References

