# Regge and Bridging trajectories in ABJM theory & Quantum Spectral Curve

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## ABJM theory & Regge theory

ABJM theory is an *enriched Chern-Simons theory*, dual to type **IIA** superstring theory on $AdS_4 \times \mathbb{CP}^3$, with gauge group $U(N_1) \times U(N_2)$. The global symmetry group of the theory is the ortho-symplectic supergroup $OSp(6|4)$. The *bosonic* components of $OSp(6|4)$ are R-symmetry group $SO(6)$ and Lorentzian $d = 3$ *conformal group* $SO(3,2)$. The fermionic part of $OSp(6|4)$ generates the $\mathcal{N} = 6$ supersymmetry transformations. The theory can be restricted to the *planar sector* by taking 't Hooft limit

$$k, N_1 = N_2 \equiv N \to +\infty\,, \qquad \lambda := \frac{N}{k} \quad \text{fixed}$$

Let us consider the stress-tensor supermultiplet. Let $\hat{\mathcal{S}}$ be the lightest primary in the supermultiplet. The operator $\hat{\mathcal{S}}$ is a scalar, $S = 0$, with *protected* dimension $\Delta = 1$. First, we are interested in the four-point function

$$< \hat{\mathcal{S}}\,(x_1)\,\hat{\mathcal{S}}\,(x_2)\,\hat{\mathcal{S}}\,(x_3)\,\hat{\mathcal{S}}\,(x_4) > \,.$$

- Unprotected operators appearing in the OPE decomposition belong to a multiplet whose top transforms in the singlet **1** of $SO(6)_{\mathbf{R}}$.
- This operators organise themselves in fixed $\Delta - S$ trajectories. Maximum spin at fixed $\Delta$ identifies the leading one.

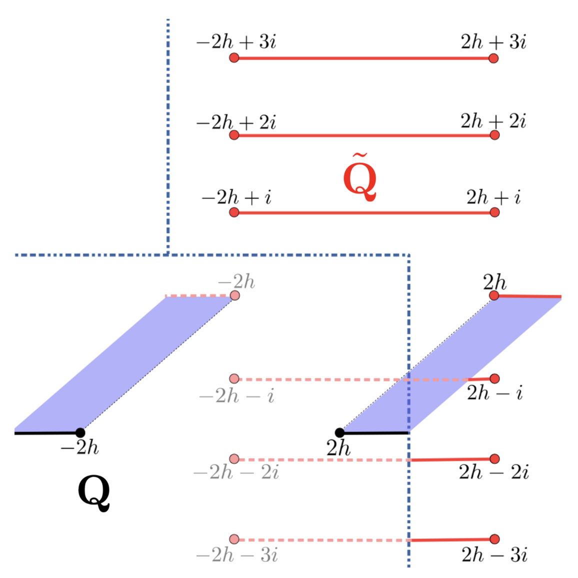
The analytic continuation in the spin, "interpolating" a trajectory of physical operators, takes the name of *Regge trajectory* [1].

- The value assumed by the leading Regge trajectory at the minimum, $\Delta = 3/2$, defines the behaviour of the correlator in a Lorentzian limit known as Regge limit. This limit is related to the high energy behaviour for the dual graviton **S**-matrix. The minimum can be interpreted as the *Lyapunov exponent* characterizing the out-of-time-ordered four-point function [2].

## The Quantum Spectral Curve

The QSC in AdS/CFT is an integrability-based framework which provides the exact spectrum of planar ABJM in terms of a solution of a *Riemann-Hilbert problem* for a finite set of functions [3].

- Functions **P** have short cuts as **Q** have long ones.
- The constant $h$ plays the role of *coupling constant*.
- The quantum numbers of the spectrum enter in the large-$u$ asymptotics in such a way the right $SO(6)$-irrep is taken into account.




$$\mathbf{P} \sim (u^{-1}, u^{-2}, u^2, u^1, 1, 1)\,, \qquad \mathbf{Q} \sim (u^{\Delta+S}, u^{\Delta-S-1}, u^{S-\Delta-1}, u^{-\Delta-S-1}, 1/u, 1/u)\,.$$

- We can *glue* $\tilde{\mathbf{Q}}(u)$ and $\mathbf{Q}(-u)$ by a matrix without cuts, the gluing matrix $\mathcal{K}(u)$.

$$\mathcal{K}(u|\Delta, S) \overset{fixed}{\longleftrightarrow} \begin{array}{l}\textit{analyticity, periodicity,}\\ \textit{algebraic constraints.}\end{array}$$

- Numerically *imposing* the gluing condition, we extract $\Delta$ [4].

We are interested in the $\mathfrak{sl}_2$-like symmetric sub-sector to which the leading trajectory belongs.

### Physical spectrum

The simplest assumption is a constant gluing matrix. In such a situation $\mathcal{K}$ is uniquely fixed in the sector and the spin dependence is quantised, for our purposes $S \in \mathbb{Z}$. E.g.

$$\begin{aligned}\tilde{\mathbf{Q}}_1(u) &= \xi_{11}\,\mathbf{Q}_1(-u) + \xi_{13}\,\mathbf{Q}_3(-u)\,,\\ \tilde{\mathbf{Q}}_2(u) &= \xi_{22}\,\mathbf{Q}_2(-u) + \xi_{24}\,\mathbf{Q}_4(-u)\,,\end{aligned}$$

### Regge and bridging trajectories

Now, our goal is to classify the admissible gluing with a more general assumption, i.e. $e^{\pi u}$-like behaviour.

Surprisingly, we found two consistent options. Each one reduces to the physical one, $u$-independent, for integer spin values. Two analytic continuations in the spectrum are therefore available. The first one corresponds to the expected shape of Regge trajectories. The second one, henceforth called *Bridging trajectory*, crosses the first one on the physical operators.

Regge

$$\begin{aligned}\tilde{\mathbf{Q}}_1 = {} & \varphi_{11}\,\mathbf{Q}_1(-u) + \left(\tilde{\varphi}_{12} + \varphi_{12}\,e^{2\pi u} + \hat{\varphi}_{12}\,e^{-2\pi u}\right)\,\mathbf{Q}_2(-u) + \varphi_{13}\,\mathbf{Q}_3(-u) + \\ & + \left(\tilde{\varphi}_{14} + \varphi_{14}\,e^{2\pi u} + \hat{\varphi}_{14}\,e^{-2\pi u}\right)\,\mathbf{Q}_4(-u) + \left(\varphi_{15}\,e^{\pi u} + \hat{\varphi}_{15}\,e^{-\pi u}\right)\,\mathbf{Q}_5(-u)\,,\\ \tilde{\mathbf{Q}}_2 = {} & \varphi_{22}\,\mathbf{Q}_2(-u) + \varphi_{24}\,\mathbf{Q}_4(-u)\,.\end{aligned}$$

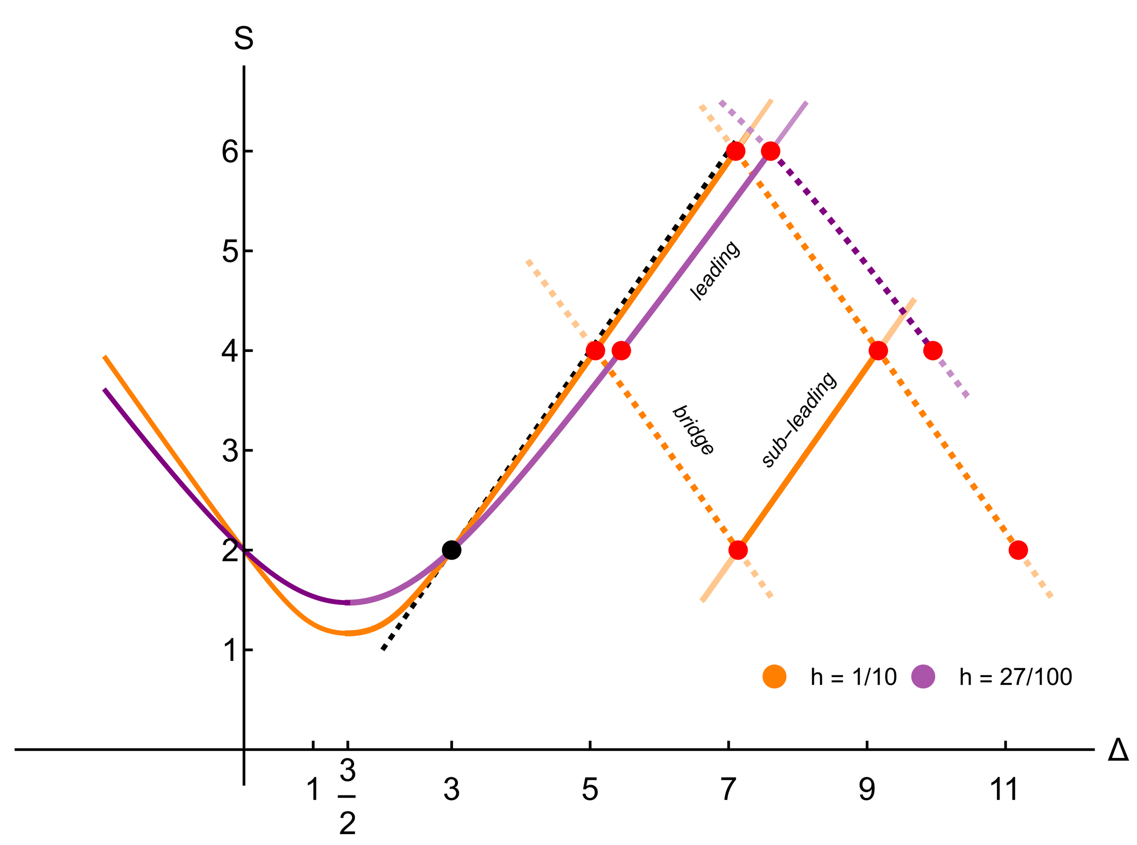
Bridging

$$\begin{aligned}\tilde{\mathbf{Q}}_1(u) = {} & \varphi_{12}\,\mathbf{Q}_1(-u) + \varphi_{14}\,\mathbf{Q}_3(-u)\,,\\ \tilde{\mathbf{Q}}_2(u) = {} & \varphi_{24}\,\mathbf{Q}_4(-u)\left(\tilde{\varphi}_{21} + \varphi_{21}\,e^{2\pi u} + \hat{\varphi}_{21}\,e^{-2\pi u}\right)\,\mathbf{Q}_1(-u) + \varphi_{22}\,\mathbf{Q}_2(-u) + \\ & + \left(\tilde{\varphi}_{23} + \varphi_{23}\,e^{2\pi u} + \hat{\varphi}_{23}\,e^{-2\pi u}\right)\,\mathbf{Q}_3(-u) + \left(\varphi_{25}\,e^{\pi u} + \hat{\varphi}_{25}\,e^{-\pi u}\right)\,\mathbf{Q}_5(-u)\,.\end{aligned}$$

## Numerical results

The leading Regge trajectory intercepts the series of unprotected operators of twist-1 appearing in the spectrum. The first derivative at the BPS point $(3, 2)$ verifies the result of the *slope function* in [5]. The novel Bridging trajectory cuts the Regge ones on physical states.

- The new Bridging trajectory provides a new way to pass from leading to subleading trajectories, an alternative to continuation around branch points in the complex spin plane [4].
- The leading trajectory at weak coupling seems to show a different shape from the profile of the *BFKL behaviour* of $\mathcal{N} = 4$ SYM [6].




Regge and Bridging trajectories are marked in the $(\Delta, S)$-plane, solid and dashed line, respectively. Physical operators are highlighted in red and the *unitarity bound* in dashed black. The BPS point is marked in black.

- We found trajectories interpolating odd spins can be similarly treated within our set-up with two distinct analytic continuations. On the other hand, the analytic continuations of even/odd spins are disjoint.

We found that a novel family of interpolating Bridging trajectories also exists in $\mathcal{N} = 4$ SYM Quantum Spectral Curve [7].

## Open questions & outlooks

What is the interpretation of the Bridging trajectories, which give an alternative analytic continuation of the spectrum? Are they relevant in studying physical processes? Can we analytically study what replaces the BFKL behaviour? Furthermore, can our set-up get involved in the *bootstrability program* [8]?

## References

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