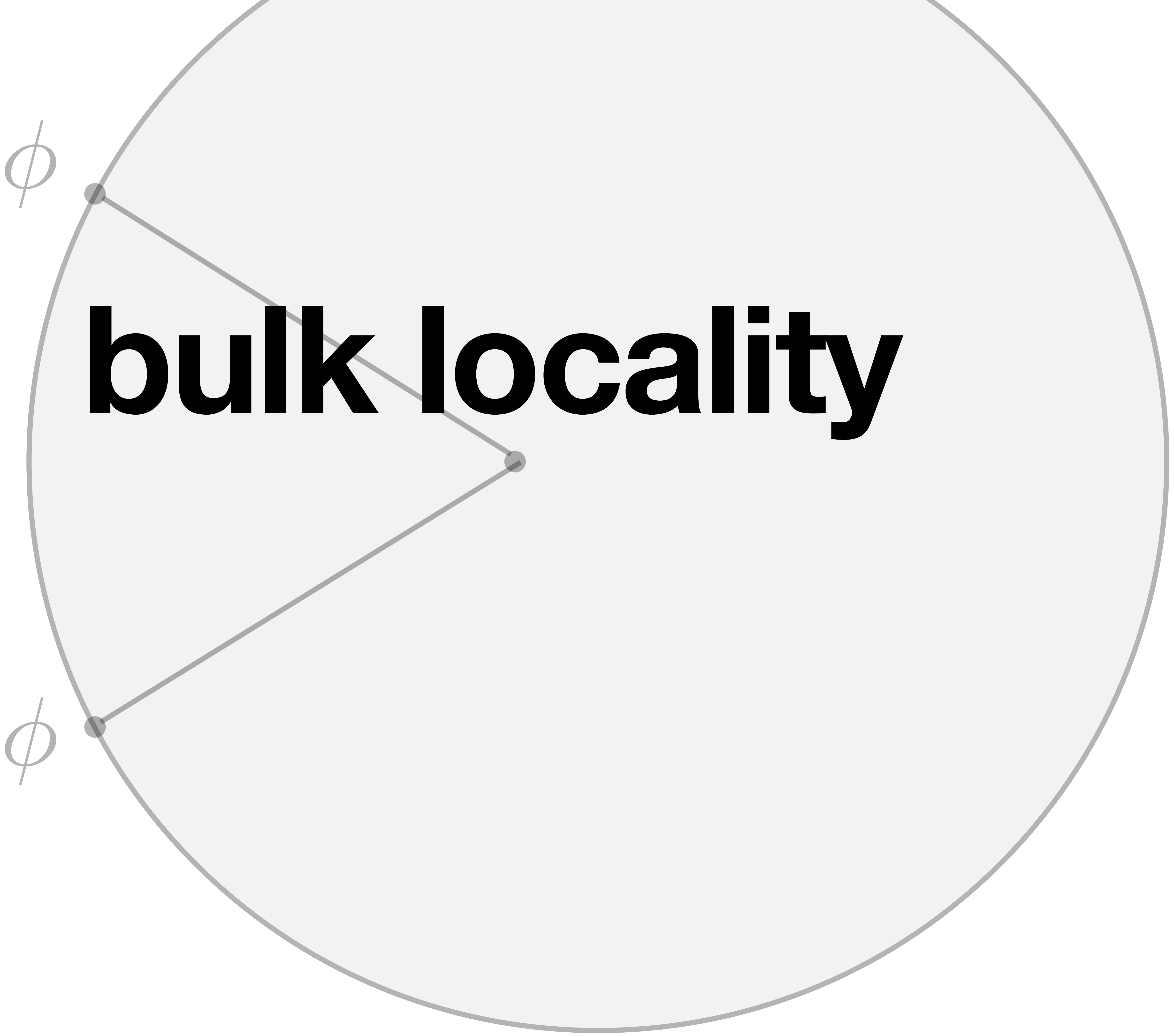
Bootstrapping bulk locality



Nat Levine

École Normale Supérieure

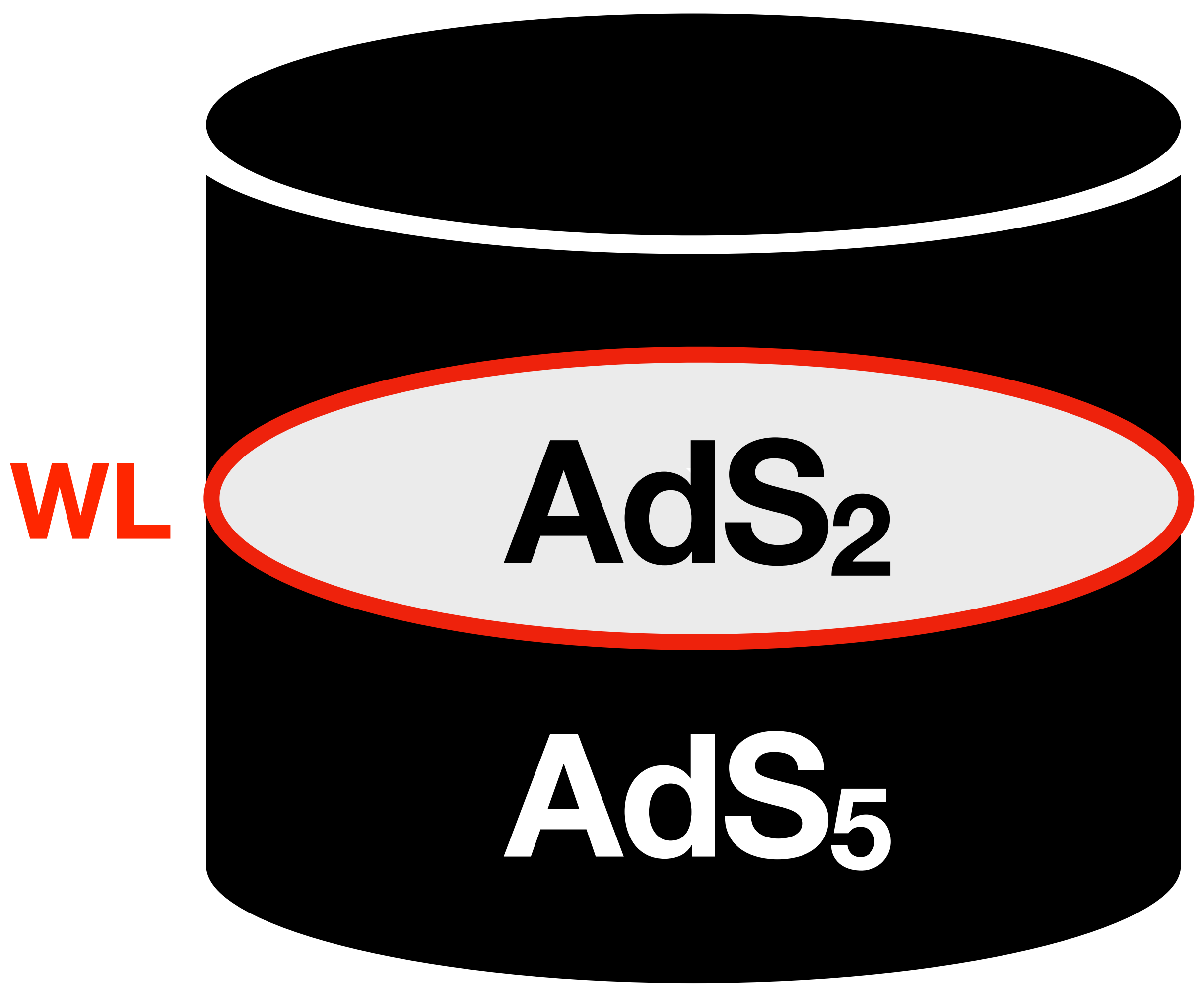
IGST 2023

[Part I: 2305.07078] with Miguel Paulos

(Distant) Motivation:
Integrable models on AdS_2 ?

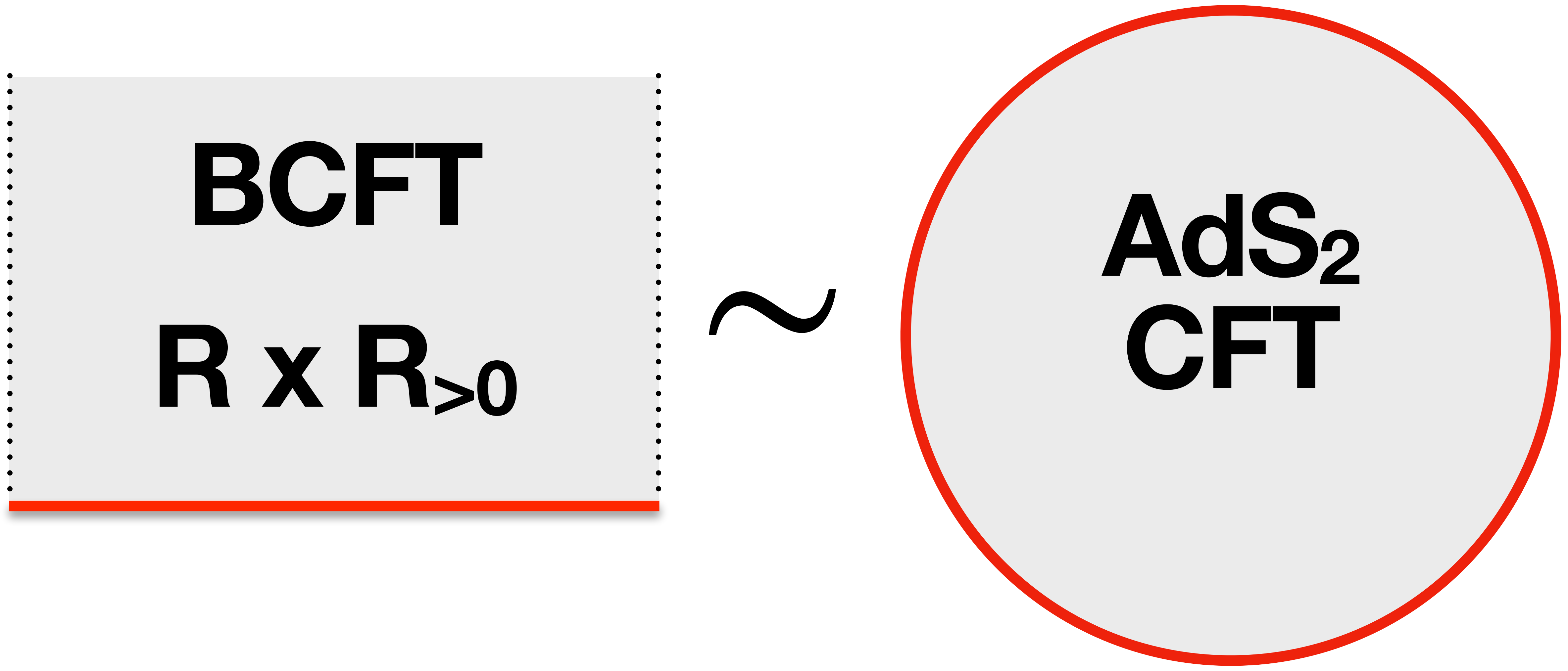
QFT on AdS_2 $\longleftrightarrow$ C(F)T1

E.g: Wilson line in $\mathcal{N} = 4$



[Drukker] [Giombi Komatsu] [Giombi Roiban Tseytlin]
[Liendo Meneghelli Mitev] [Cavaglia Gromov Julius Preti] …

Integrable boundary CFT_2



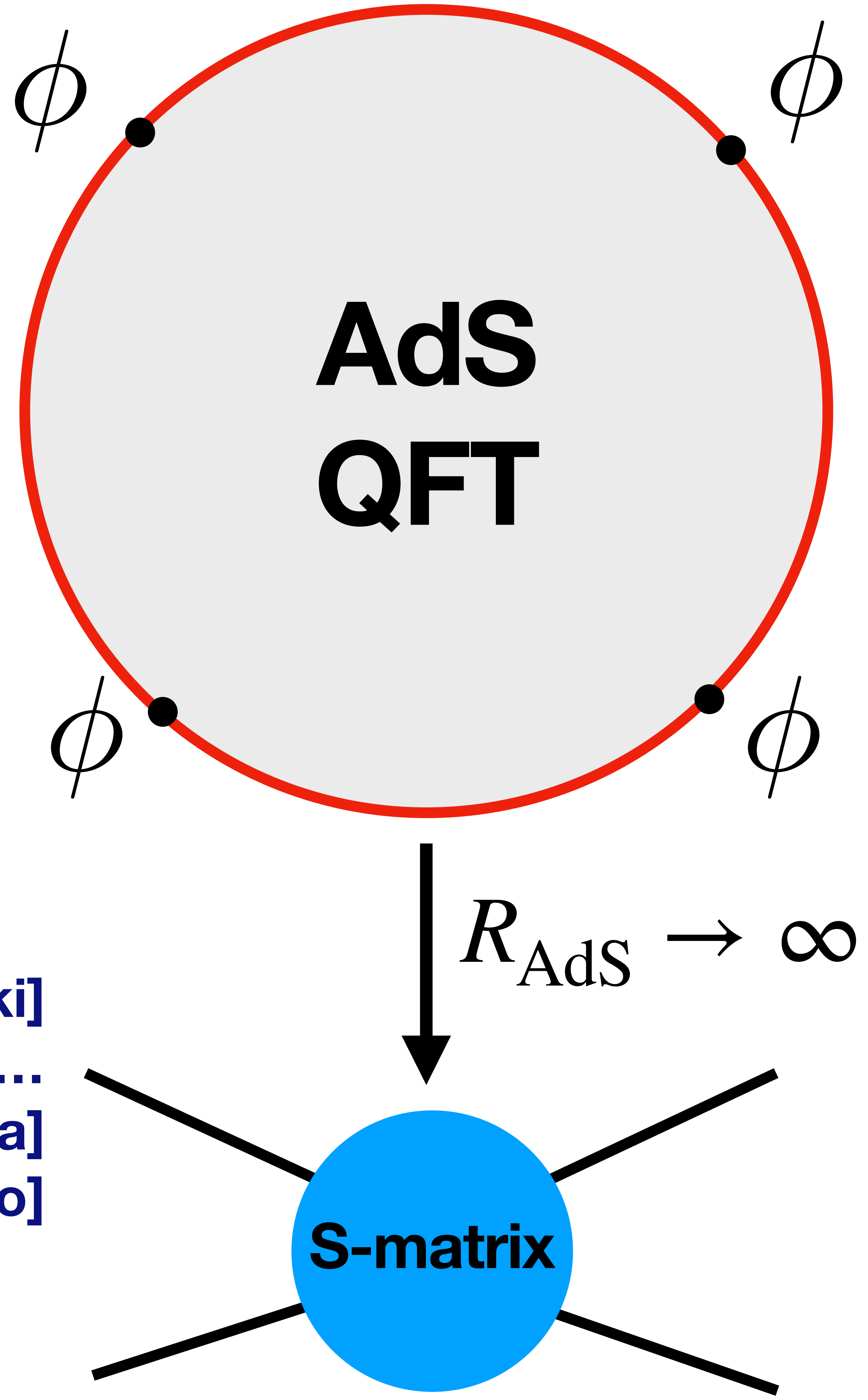
(Distant) Motivation:
Integrable models on AdS_2 ?

- Sine Gordon on AdS_2 ?

  discussed in [Antunes, Costa, Penedones, Sarlgrkar, van Rees]

- What is signature of integrability?

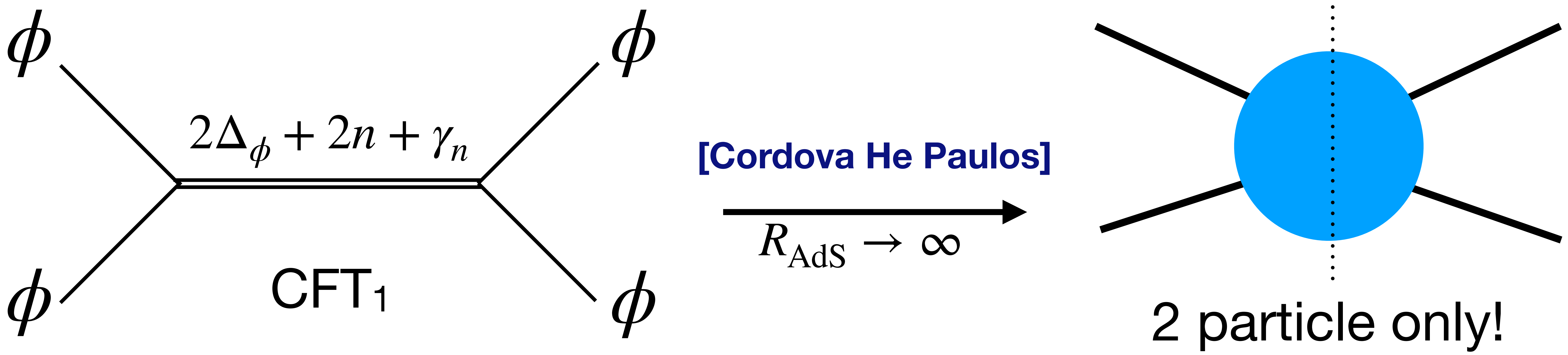
  (cf. factorized scattering)



[Polchinski]
...
[Paulos, Penedones, Toledo, van Rees, Vieira]
[Komatsu, Paulos, van Rees, Zhao]

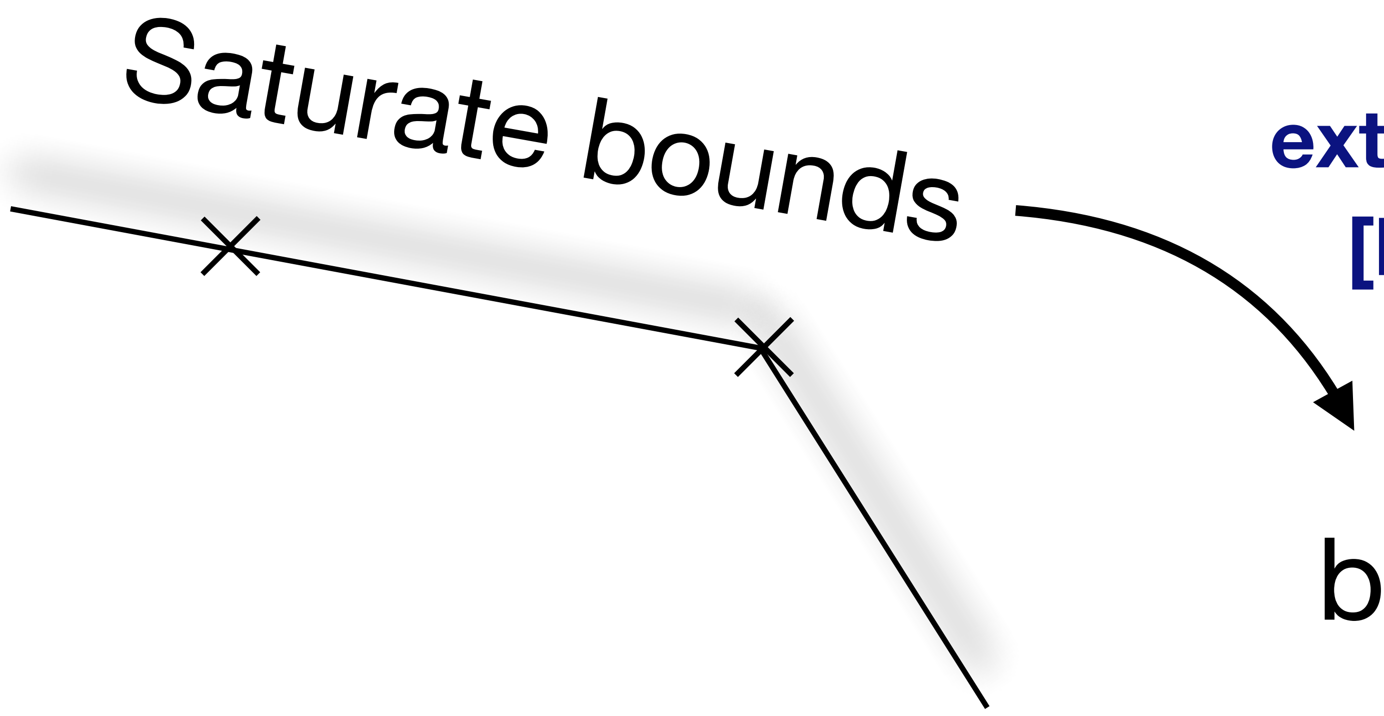
Integrable boundary correlators?

Proposal "minimal" exchanged spectrum



Bootstrap

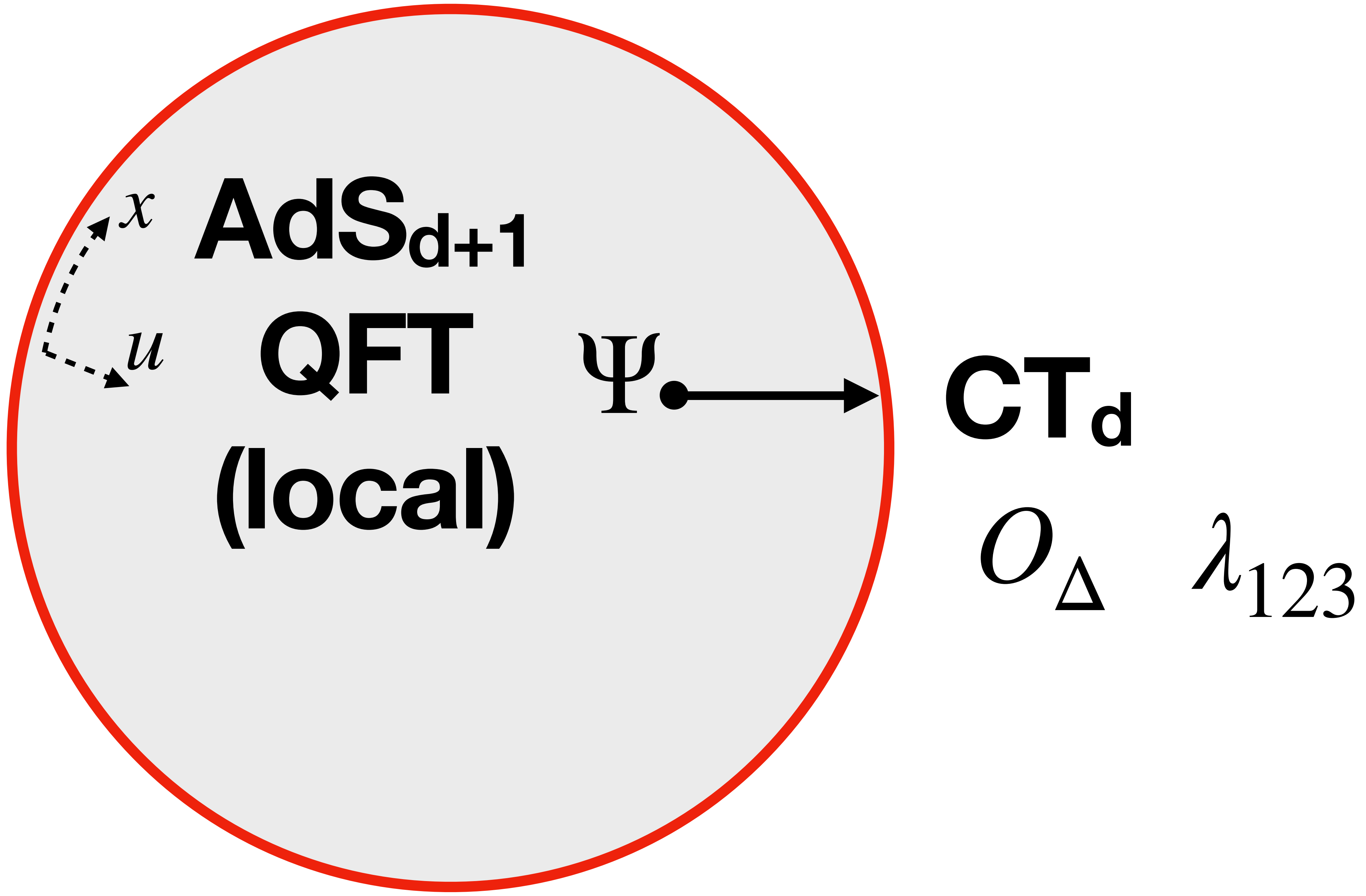
$$F = \sum_\Delta c_\Delta G_\Delta$$

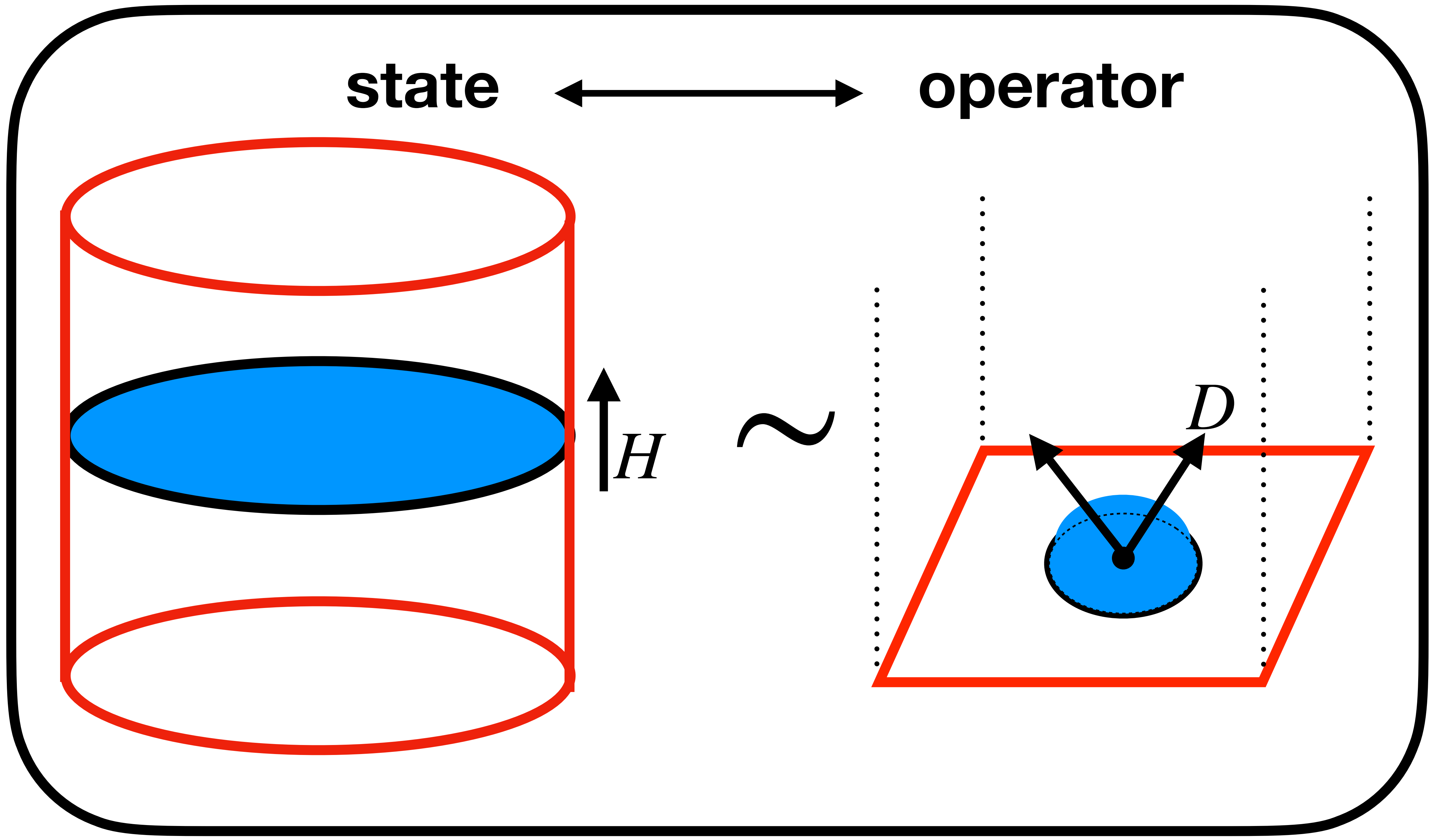


extremal functionals
[El-Showk Paulos]

bootstrap fixes c_Δ

Setup:
AdS bulk locality





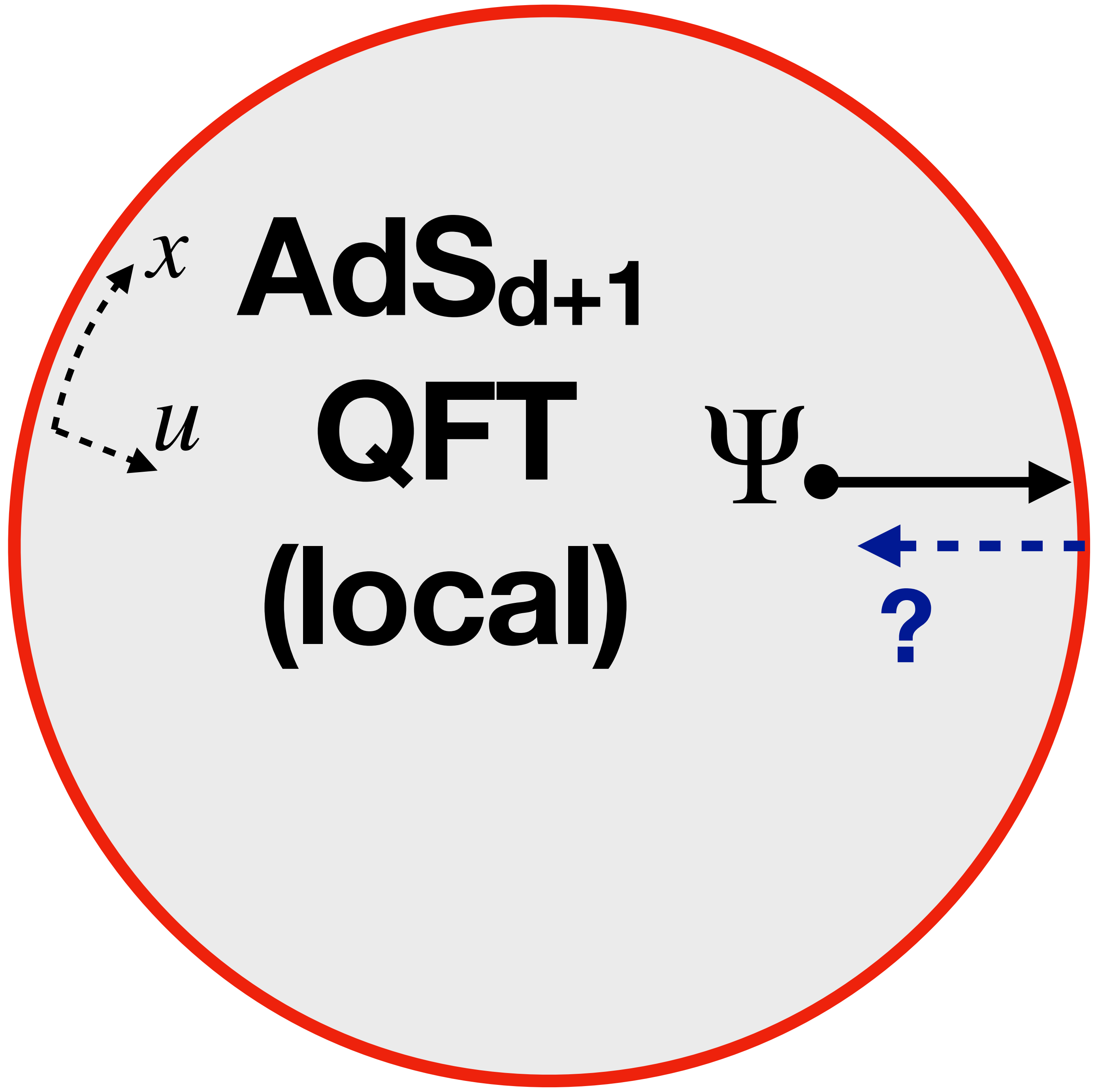
Boundary Operator Expansion (BOE)

$$\underbrace{\Psi(x,u)}_{\text{scalar}} = \sum_{\Delta} \mu_\Delta \, u^\Delta \, \hat{C}_\Delta \cdot \underbrace{O_\Delta(x)}_{\text{primaries}}$$

"BOE coeffs"

Setup:

AdS bulk locality



CT_d

$O_\Delta \quad \lambda_{123}$

- Crossing
- Unitarity

conformal bootstrap
[Rattazzi Rychkov Tonni Vichi] ...

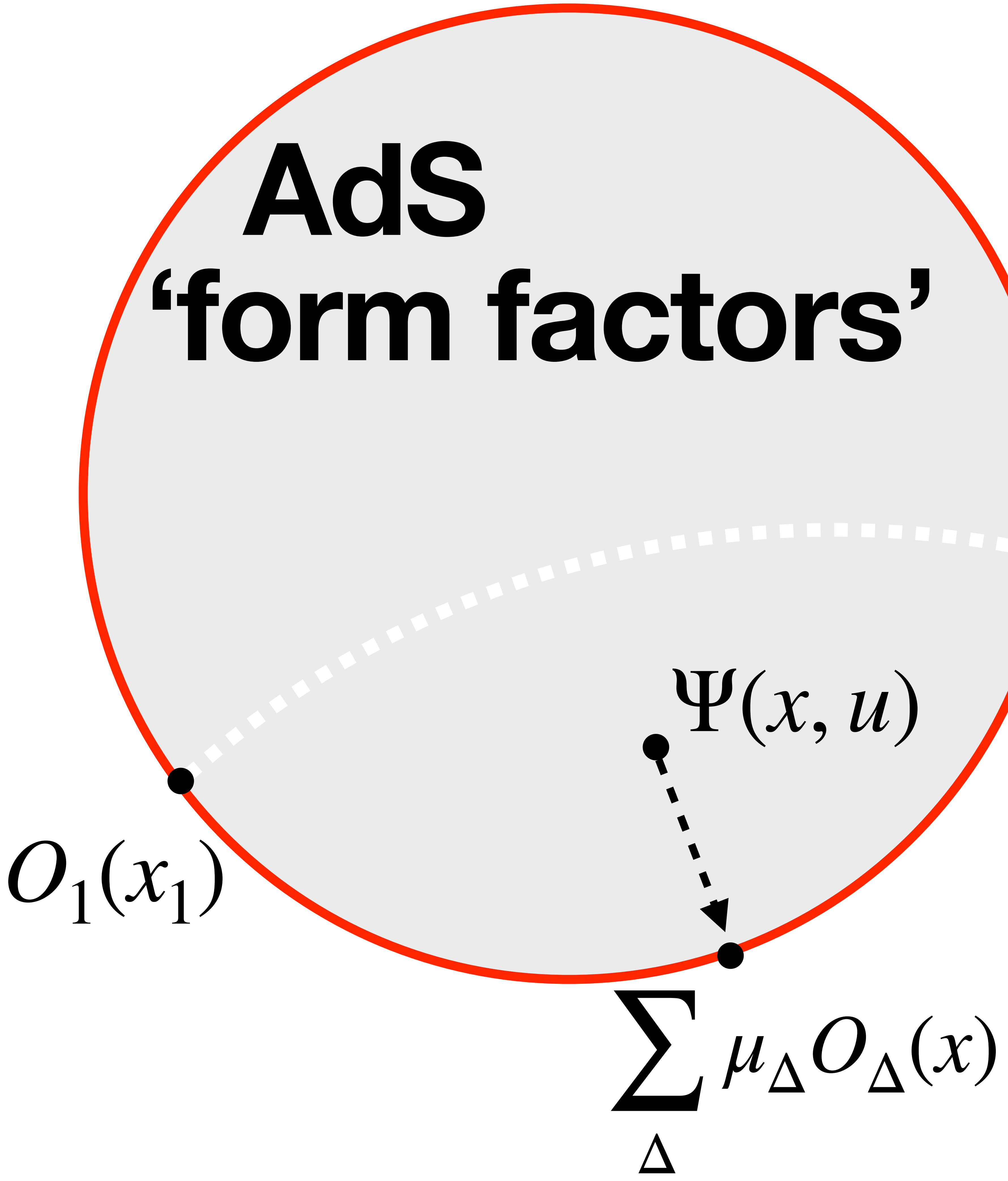
- **Bulk locality !**

Boundary Operator Expansion (BOE)

$$\underbrace{\Psi(x,u)}_{\text{scalar}} = \sum_{\Delta} \mu_\Delta \, u^\Delta \, \hat{C}_\Delta \cdot \underbrace{O_\Delta(x)}_{\text{primaries}}$$

μ_Δ: "BOE coeffs"

cf. smearing kernels
[Bena] [Hamilton Kabat Lifschytz Lowe]



$$\langle \Psi \,|\, O_1 O_2 \rangle = P_1^{B\partial}\, P_2^{B\partial}\, F(z)$$

$$F(z) = \sum_\Delta \mu_\Delta \lambda_\Delta^{12}\, G_\Delta^{12}(z)$$

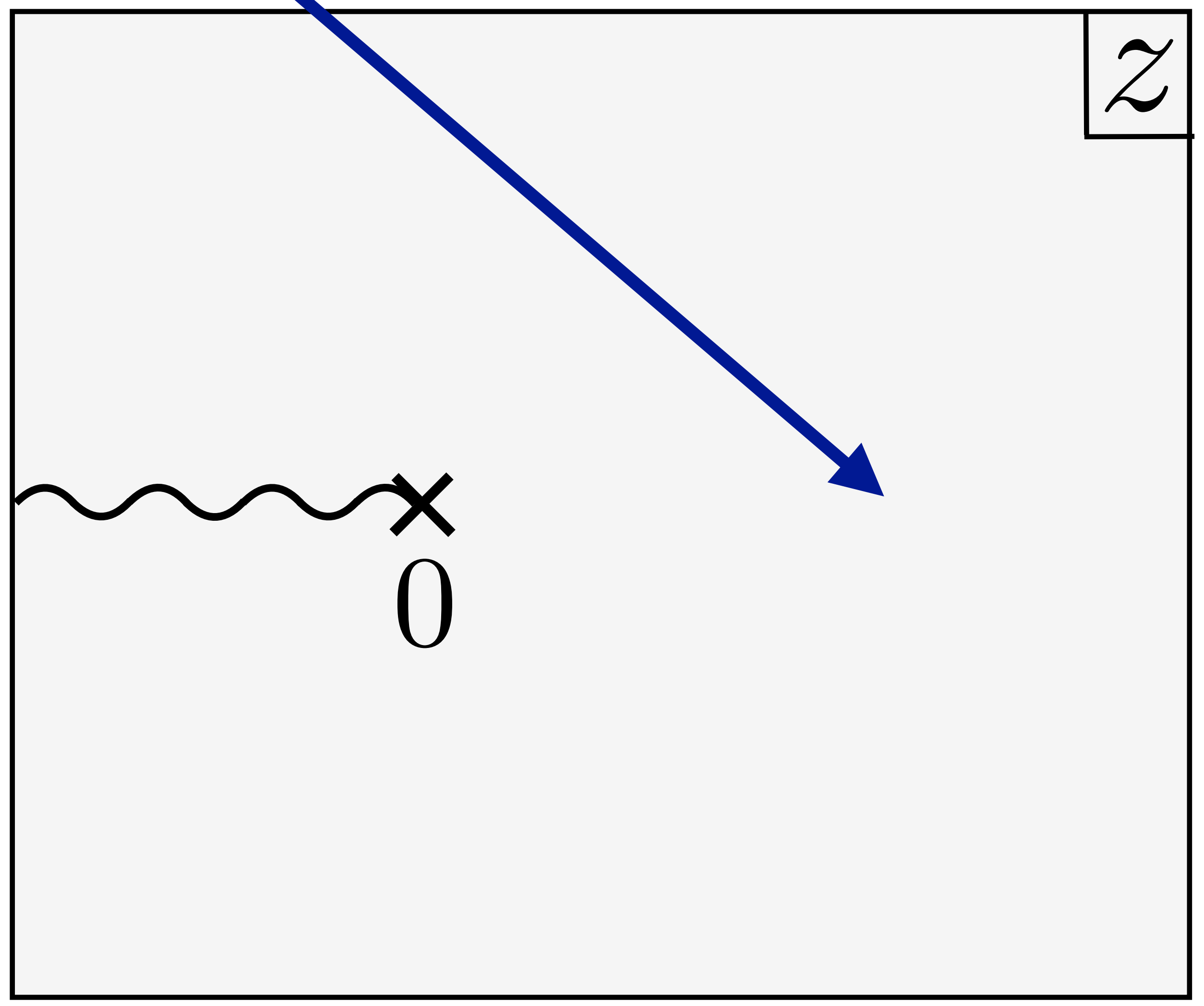
"Boundary blocks"

$$G_\Delta^{12}(z) = z^{\Delta/2}\, {}_2F_1\left(\tfrac{1}{2}(\Delta + \Delta_{12}), \tfrac{1}{2}(\Delta - \Delta_{12}); \Delta + 1 - \tfrac{d}{2};\ z\right)$$

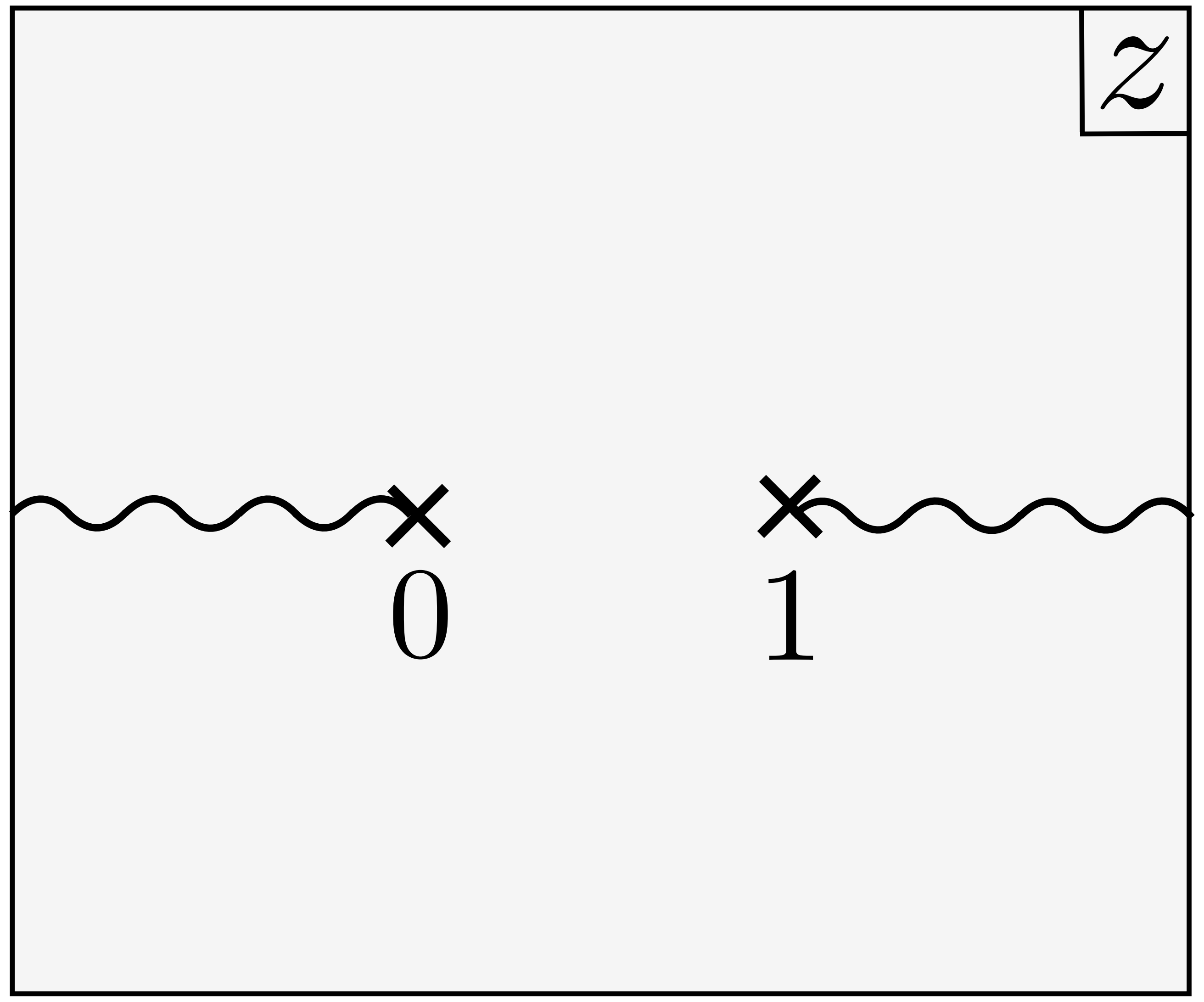
cross-ratio $$z = \frac{(x_1 - x_2)^2 u^2}{[(x - x_1)^2 + u^2][(x - x_2)^2 + u^2]}$$

Locality

$$F(z) = \sum_{\Delta} \mu_\Delta \lambda_\Delta^{12} \, G_\Delta^{12}(z)$$



$$= \sum_{\Delta} \mu_\Delta \lambda_\Delta^{12}$$



Locality bootstrap:

[Kabat Lifschytz]

$$\operatorname*{Im}_{z\geq 1} F(z) = \operatorname*{Im}_{z\geq 1} \sum_{\Delta} \mu_\Delta \lambda_\Delta^{12} G_\Delta^{12}(z) = 0$$

- Non-positive
- Special case: BCFT

Result

Complete set of sum rules for bulk locality

$$\sum_{\Delta} \mu_{\Delta} \lambda_{\Delta}^{12} \, \theta_n^{12}(\Delta) = 0 \qquad (n = 1, 2, \, \ldots)$$

Plan

1. Sum rules

2. Applications

3. Flat space limit

Sum rules

$$\operatorname*{Im}_{z\geq 1} F(z) = 0$$

$\updownarrow$

Functionals

$$\theta_n[F] := \int_1^\infty dz\ f_n(z)\ \mathrm{Im}\, F(z) = 0$$

f_n "complete set" of functions

BOE

$$F(z) = \sum_\Delta (\mu\lambda)_\Delta G_\Delta(z)$$

$\updownarrow$ "Swapping"

Drop 1,2 indices from now on

$$\sum_\Delta (\mu\lambda)_\Delta \theta_n(\Delta) = 0$$

$(\theta_n(\Delta) \equiv \theta_n[G_\Delta])$

Local blocks

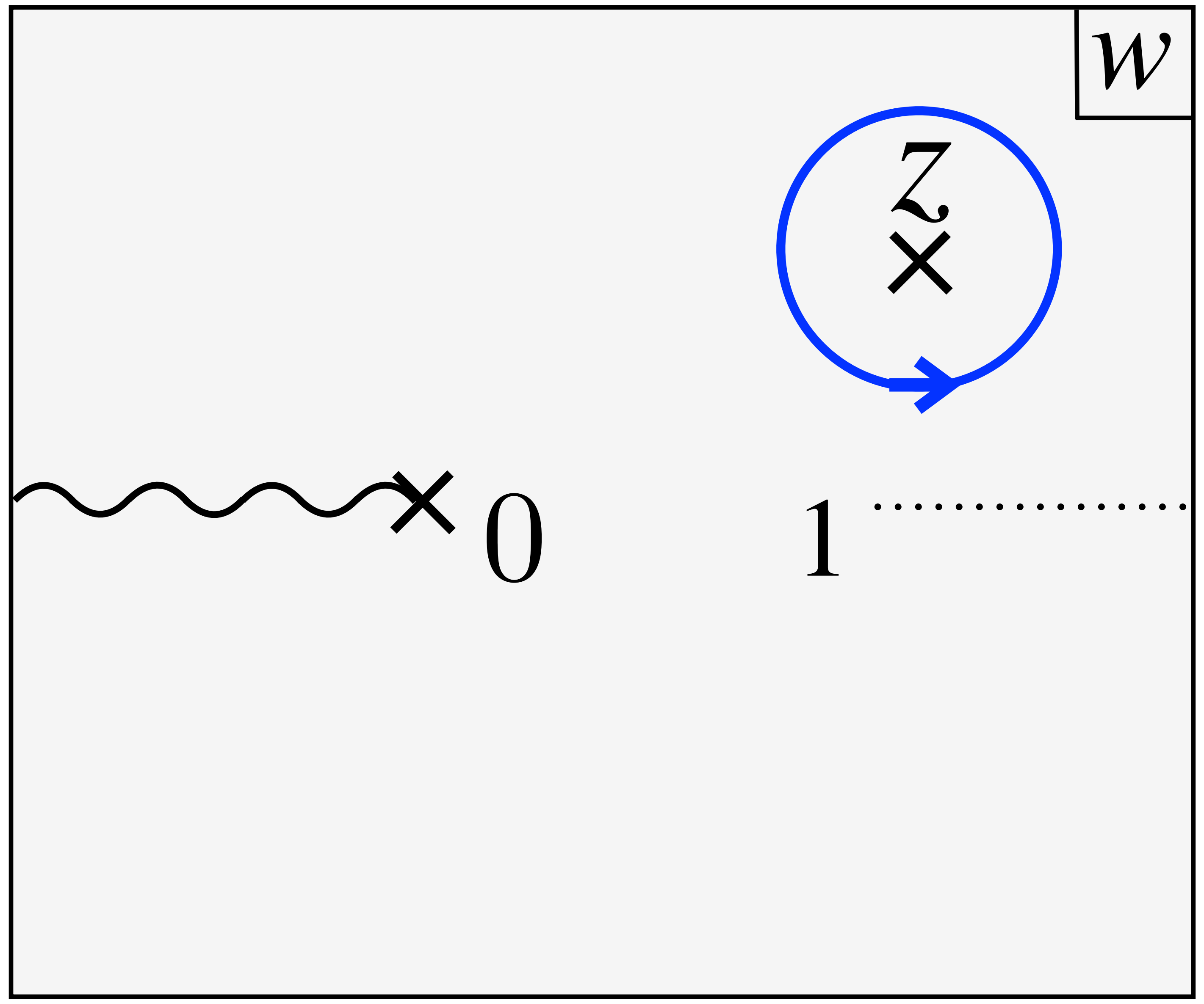
Assume: polynomial bdd

$$|F(z)| \overset{z\to\infty}{\sim} |z|^{\alpha_F}$$

naturally comes from assumptions on UV
[Meineri Penedones Spirig]

Dispersion relation

$$F(z) = \oint_z \frac{dw}{2\pi i} \qquad \frac{F(z)}{z-w}$$



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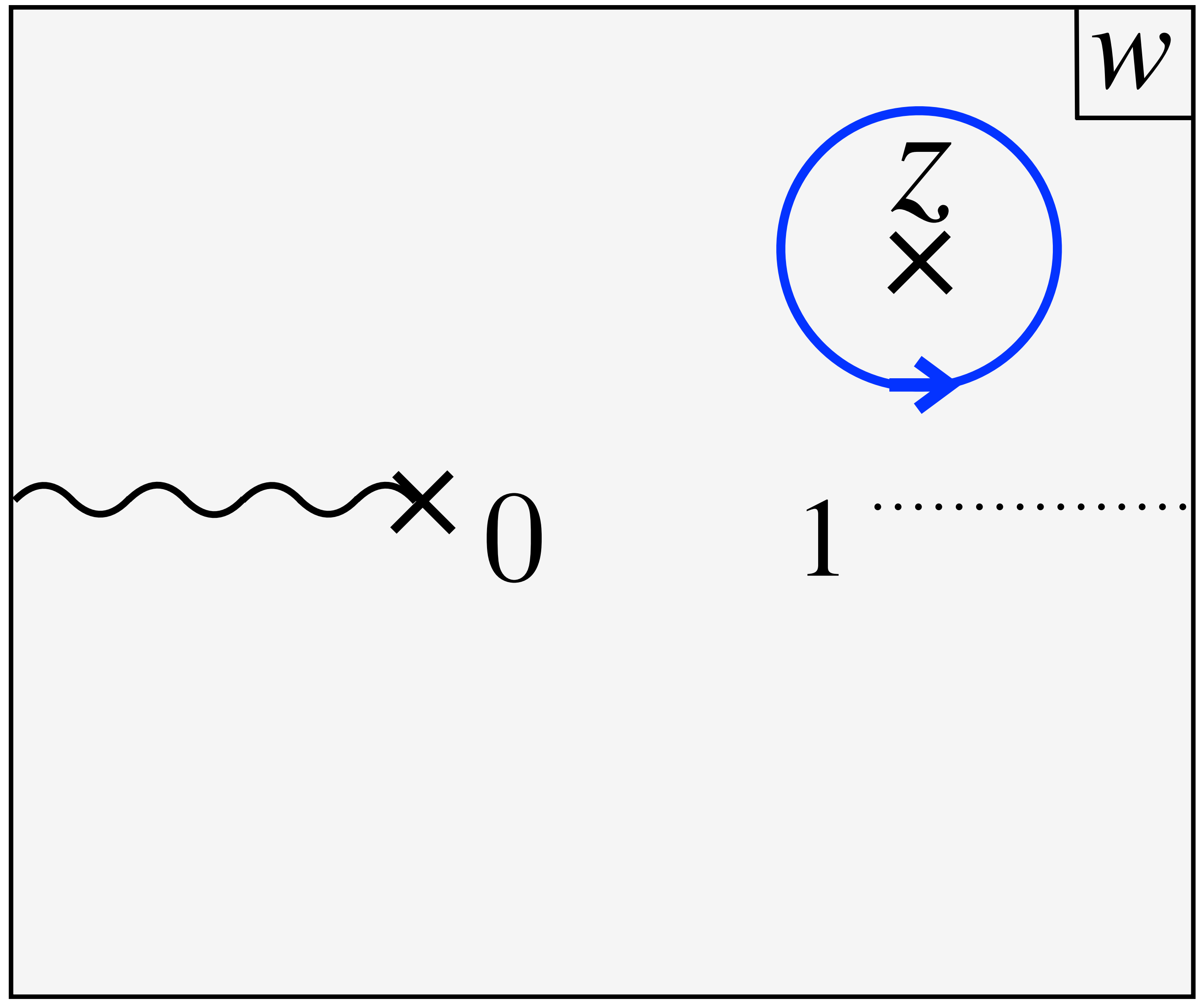
[Meineri Penedones Spirig]

Dispersion relation

$$F(z) = \oint_z \frac{dw}{2\pi i}\left(\frac{z}{w}\right)^{\tilde{\alpha}+1}\frac{F(z)}{z-w}$$



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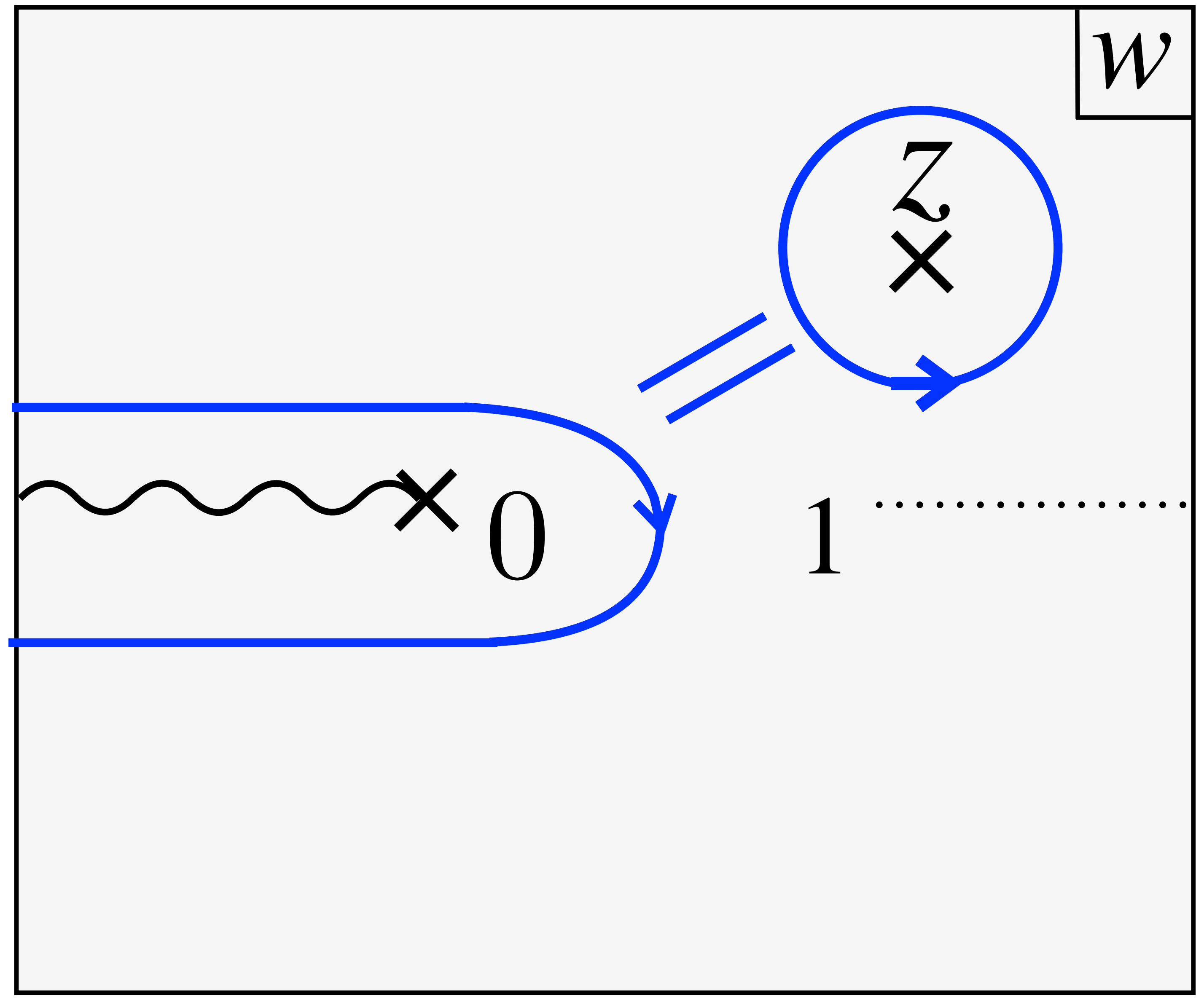
Dispersion relation

$$F(z) = \oint_z \frac{dw}{2\pi i}\left(\frac{z}{w}\right)^{\tilde\alpha+1}\frac{F(z)}{z-w}$$

$\tilde\alpha > \alpha_F - 1$

$$\overset{\text{LOCALITY}}{=} \int_{-\infty}^{0}\frac{dw}{\pi}\frac{z^{\tilde\alpha}}{w(w-z)}\,\mathrm{Im}\left[w^{-\tilde\alpha}F(w)\right]$$



Local blocks

Dispersion

$$F(z) = \int_{-\infty}^{0} \frac{dw}{\pi} \frac{z^{\tilde{\alpha}+1}}{w(w-z)} \mathrm{Im}\,[w^{-\tilde{\alpha}} F(w)]$$

Apply BOE

Locality ⟷

$$F(z) = \sum_{\Delta} (\mu\lambda)_{\Delta}\, G_{\Delta}(z) = \sum_{\Delta} (\mu\lambda)_{\Delta}\, \mathscr{L}_{\Delta}^{\tilde{\alpha}}(z)$$

✓conformal ✓locality

by swapping

"Local block"

$$\mathscr{L}_{\Delta}^{\tilde{\alpha}}(z) := \int_{-\infty}^{0} \frac{dw}{\pi} \frac{z^{\tilde{\alpha}+1}}{w(w-z)} \mathrm{Im}\,[w^{-\tilde{\alpha}} G_{\Delta}(w)]$$

- Local: $\mathrm{Im}_{z\geq 1}\, \mathscr{L}_{\Delta}^{\tilde{\alpha}}(z) = 0$
- $\mathscr{L}_{\Delta}^{\tilde{\alpha}}(z) \overset{z\to 0}{\sim} G_{\Delta}(z) + O(z^{\tilde{\alpha}+\epsilon})$

Functionals from local blocks

Move contour

$$\mathscr{L}^{\tilde{\alpha}}_{\Delta}(z) = G_{\Delta}(z) - \int_1^{\infty} \frac{dw}{\pi} \left(\frac{z}{w}\right)^{\tilde{\alpha}+1} \frac{1}{w-z} \, \mathrm{Im}\, G_{\Delta}(w)$$

$$= G_{\Delta}(z) - \sum_{n=1}^{\infty} \theta_n^{\tilde{\alpha}}(\Delta)\; G_{2\tilde{\alpha}+2n}(z)$$

$$\sum_{\Delta} (\mu\lambda)_{\Delta}\, G_{\Delta}(z) = \sum_{\Delta} (\mu\lambda)_{\Delta}\, \mathscr{L}^{\tilde{\alpha}}_{\Delta}(z)$$

Locality

$$\sum_{\Delta} (\mu\lambda)_{\Delta}\, \theta_n^{\tilde{\alpha}}(\Delta) = 0$$

Subtleties

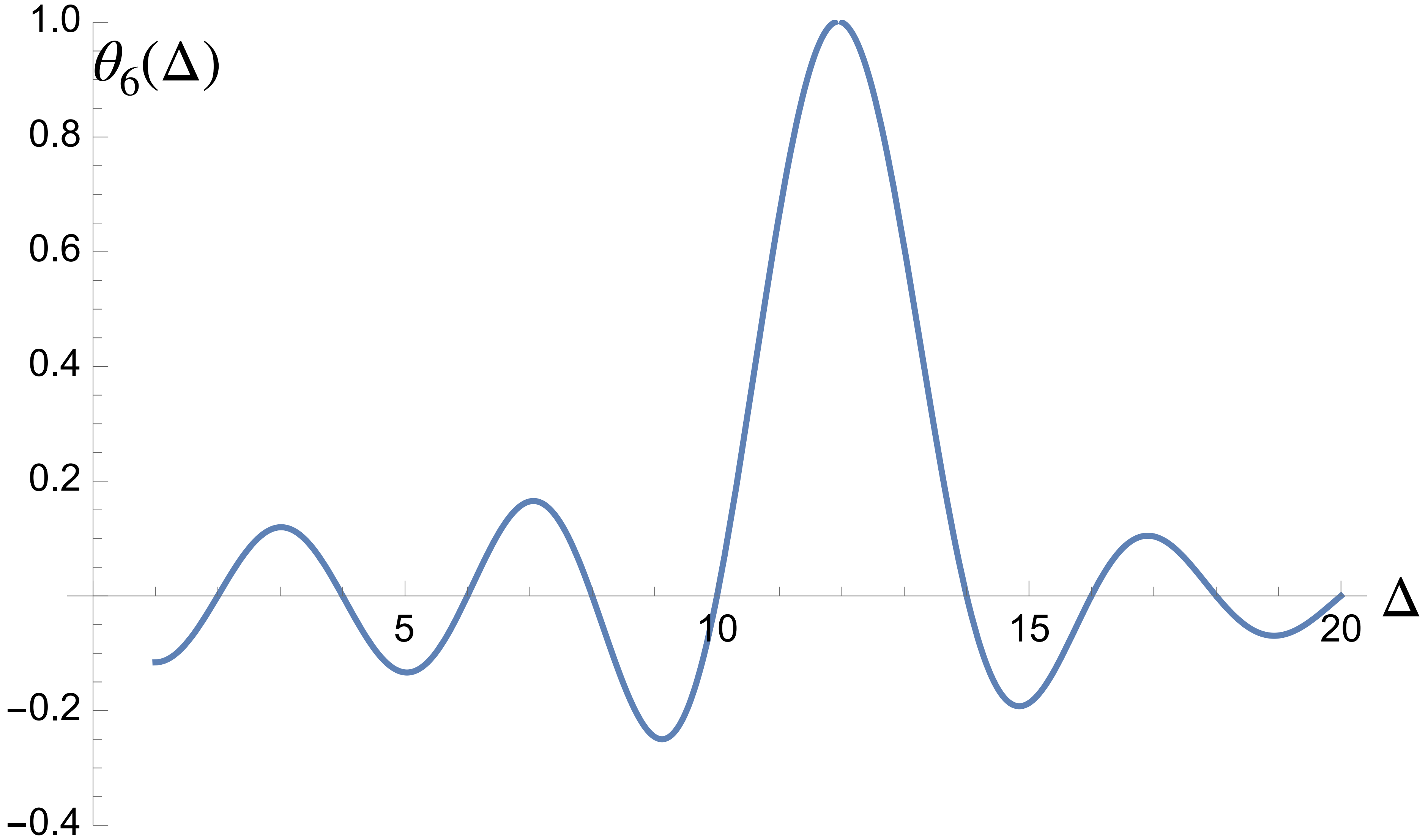
- **Completeness:** **Locality** $\longleftarrow$ $\displaystyle\sum_{\Delta} (\mu\lambda)_\Delta\, \theta_n^{\tilde\alpha}(\Delta) = 0$

$$\theta_n^{\tilde\alpha}(\Delta) = \int_1^\infty \frac{dz}{\pi}\, f_n^{\tilde\alpha}(z)\ \mathrm{Im}\, G_\Delta(z) \qquad f_n^{\tilde\alpha}(z) = z^{-2-\tilde\alpha} \underbrace{p_n^{\tilde\alpha}(1/z)}_{\text{complete polynomials}}$$

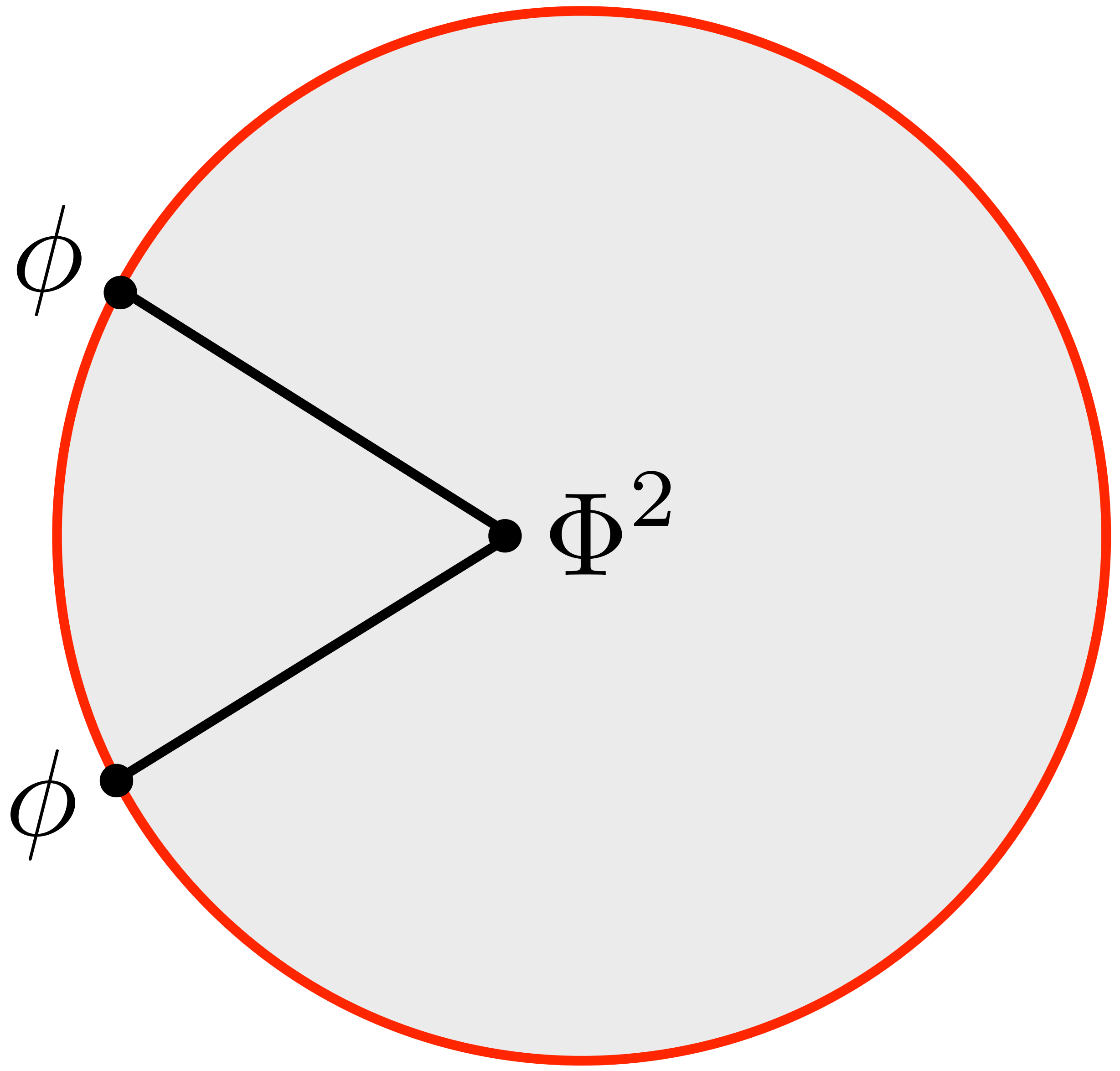
- **Swapping:** By dominated convergence
  - Poly bddness of F
  - f_n analytic and bad

Duality to GFF solution

$$\theta_m^{\tilde{\alpha}}(2\tilde{\alpha}+2n) = \delta_{mn}\ , \qquad m,n \geq 1$$



Free scalar Φ in AdS



$$\langle \Phi^2 \,|\, \phi\,\phi \rangle \sim \sum_{n=0}^{\infty} c_n\, G_{2\tilde{\alpha}+2n}(z) \qquad \tilde{\alpha} = \Delta_\phi$$

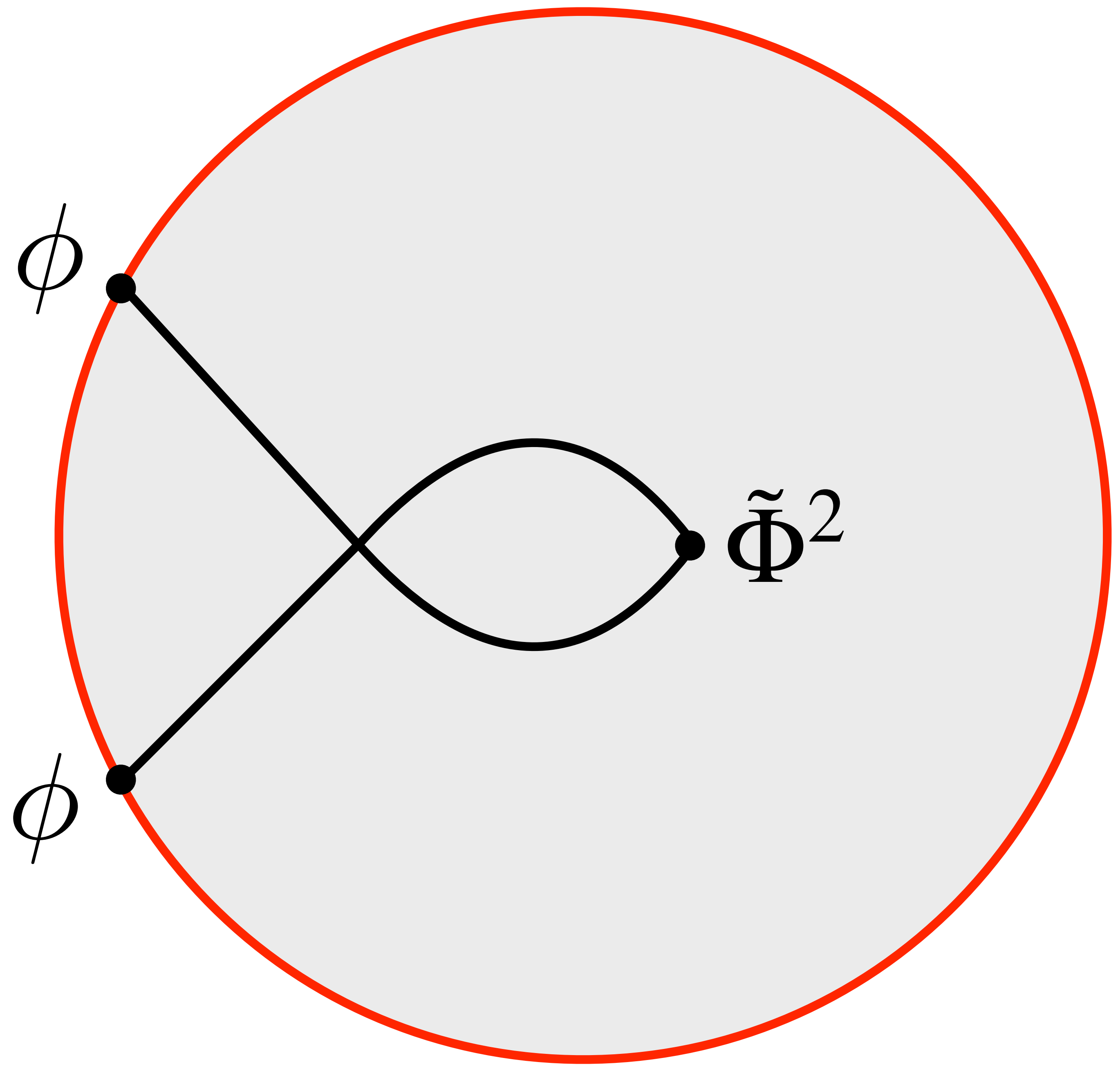
$$\theta_n^{\tilde{\alpha}}: \qquad c_0\,\theta_n^{\tilde{\alpha}}(2\tilde{\alpha}) + c_n = 0$$

$$\frac{c_n}{c_0} = -\,\theta_n^{\tilde{\alpha}}(2\tilde{\alpha})$$

Applications

(i) Perturbation theory

$\Phi, \tilde{\Phi}$ free scalars in AdS
$+ \ \Phi^2\tilde{\Phi}^2$ interaction

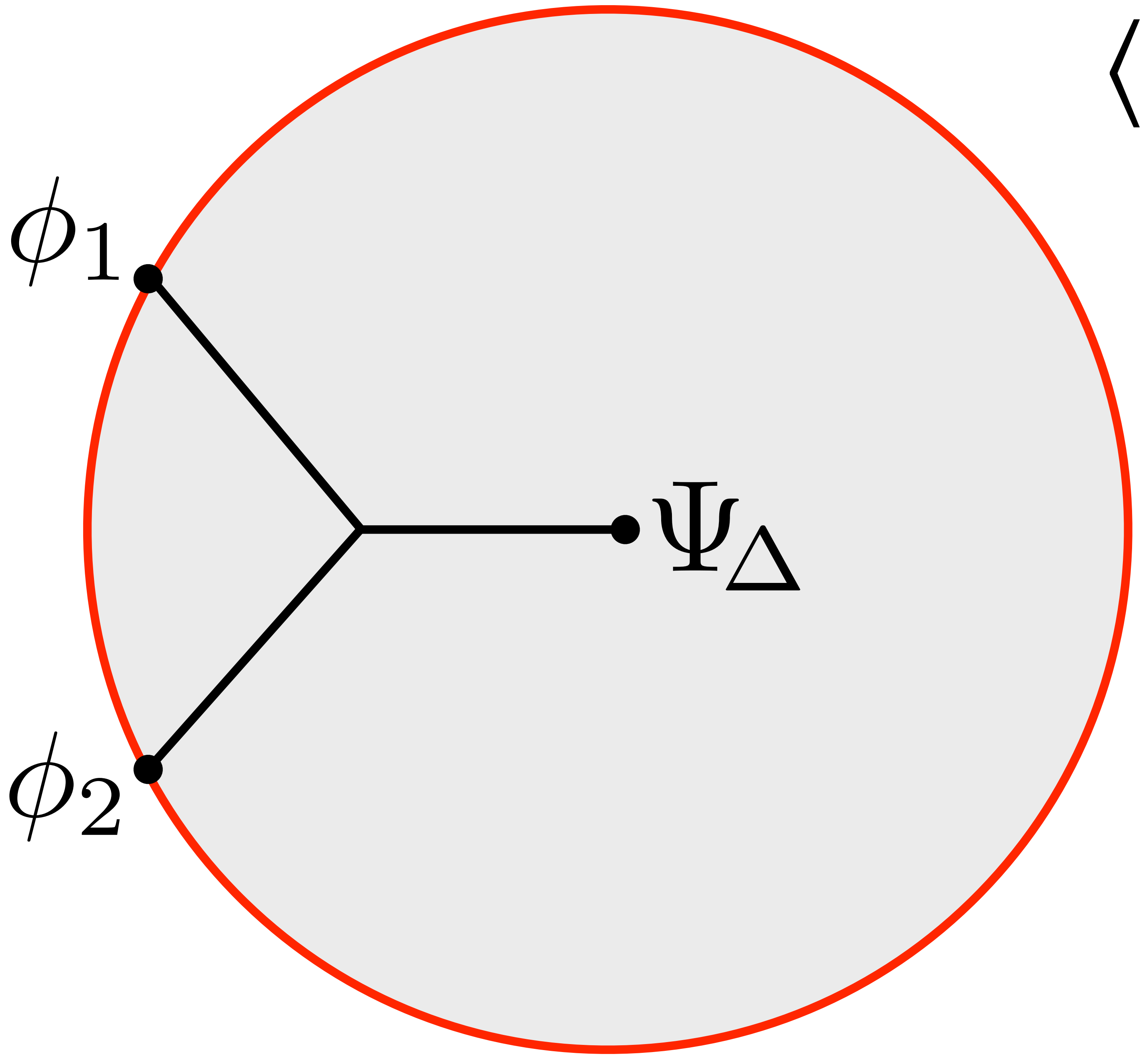


$$\langle \tilde{\Phi}^2 | \phi \, \phi \rangle \sim \sum_{n=0}^{\infty} \left(c_n \, G_{2\Delta+2n} + d_n \, G_{2\tilde{\Delta}+2n} \right)$$

θ_n fix one from the other

Applications

(ii) Local blocks = Exchanges

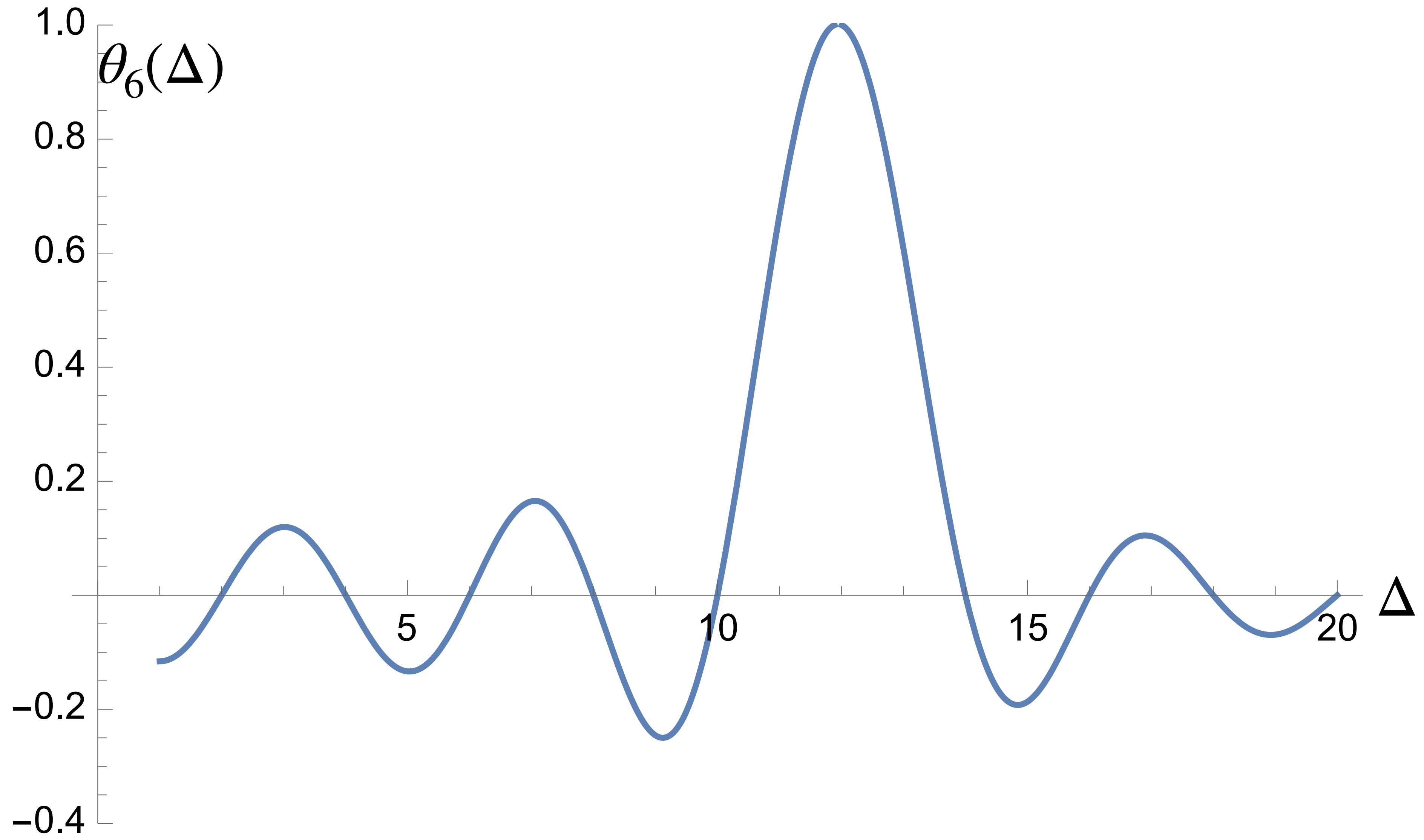


$$\langle \Psi_\Delta | \phi_1 \phi_2 \rangle = c_\Delta G_\Delta + \sum_{n \geq 1} c_n G_{\Delta_1 + \Delta_2 + 2n} = \mathscr{L}_\Delta^{\tilde{\alpha}}$$

$\tilde{\alpha} \sim \Delta_{1,2}$

$\downarrow$

$$\theta_n^{\tilde{\alpha}}(\Delta) \sim \frac{1}{(\Delta - 2\tilde{\alpha} - 2n)\, \Gamma(1 - \tilde{\alpha} - \frac{\Delta}{2})} \times (\Delta \to d - \Delta)$$

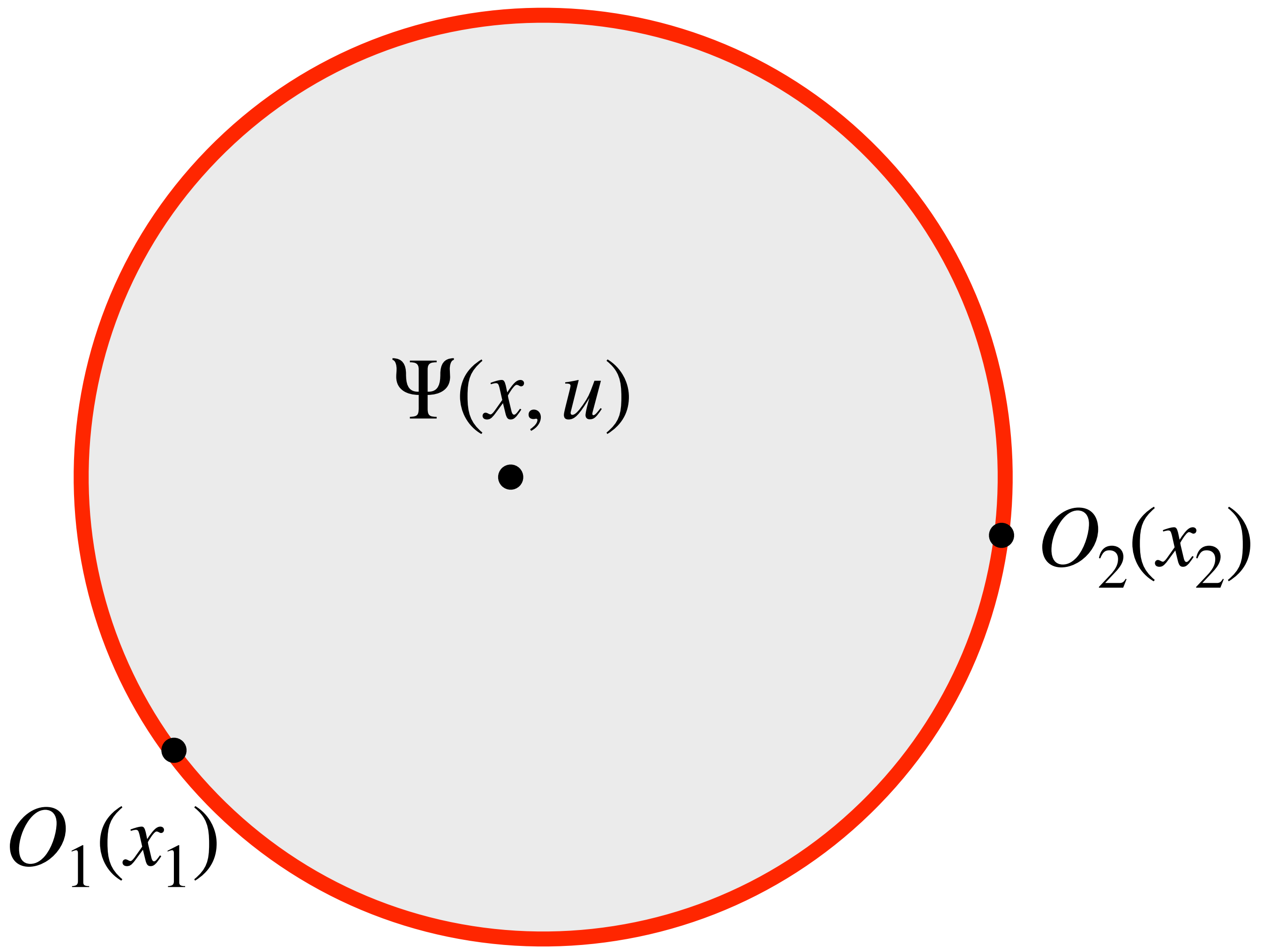


Applications

(iii) Constraints on CFT from bulk locality

Fix Ψ, vary O_1, O_2

$$\mathrm{Im}_{z\geq 1} \sum_n \mu_n \, \lambda_n^{12} \, G_{\Delta_n}^{12}(z) = 0 \qquad \forall 1,2$$



GFF example

$$\frac{\mu_n \, \lambda_n^{12}}{\mu_0 \, \lambda_0^{12}} = -\,\theta_n^{12}(\Delta_0) \qquad \forall 1,2$$

Eliminate μ !

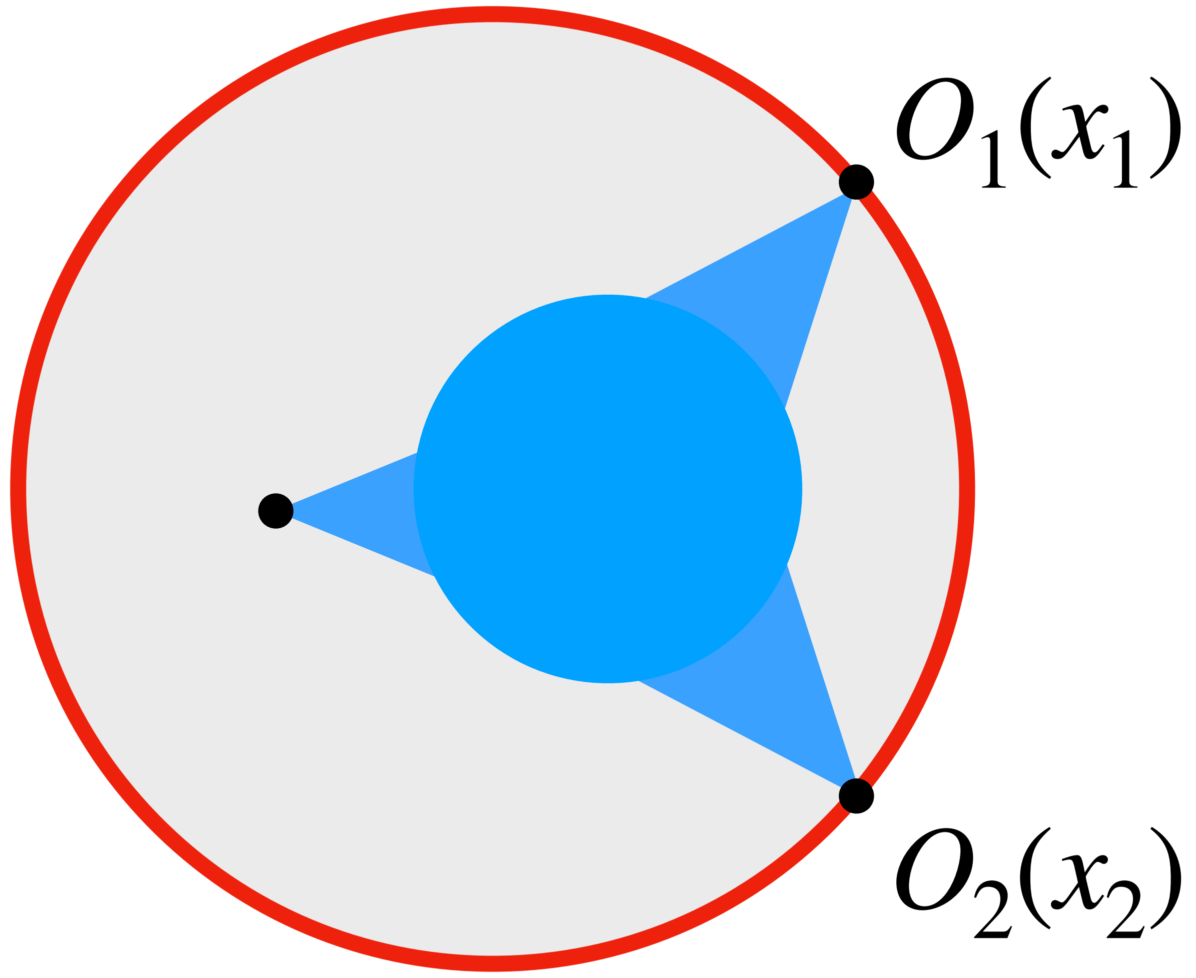
$$\frac{\lambda_n^{12}}{\lambda_0^{12}} = \frac{\theta_n^{12}(\Delta_0)}{\theta_n^{\widehat{12}}(\Delta_0)} \, \frac{\lambda_n^{\widehat{12}}}{\lambda_0^{\widehat{12}}}$$

E.g. computed $\lambda_{(\phi^2)_n \, (\phi^2)_1 \, \phi^2}$

Flat space limit

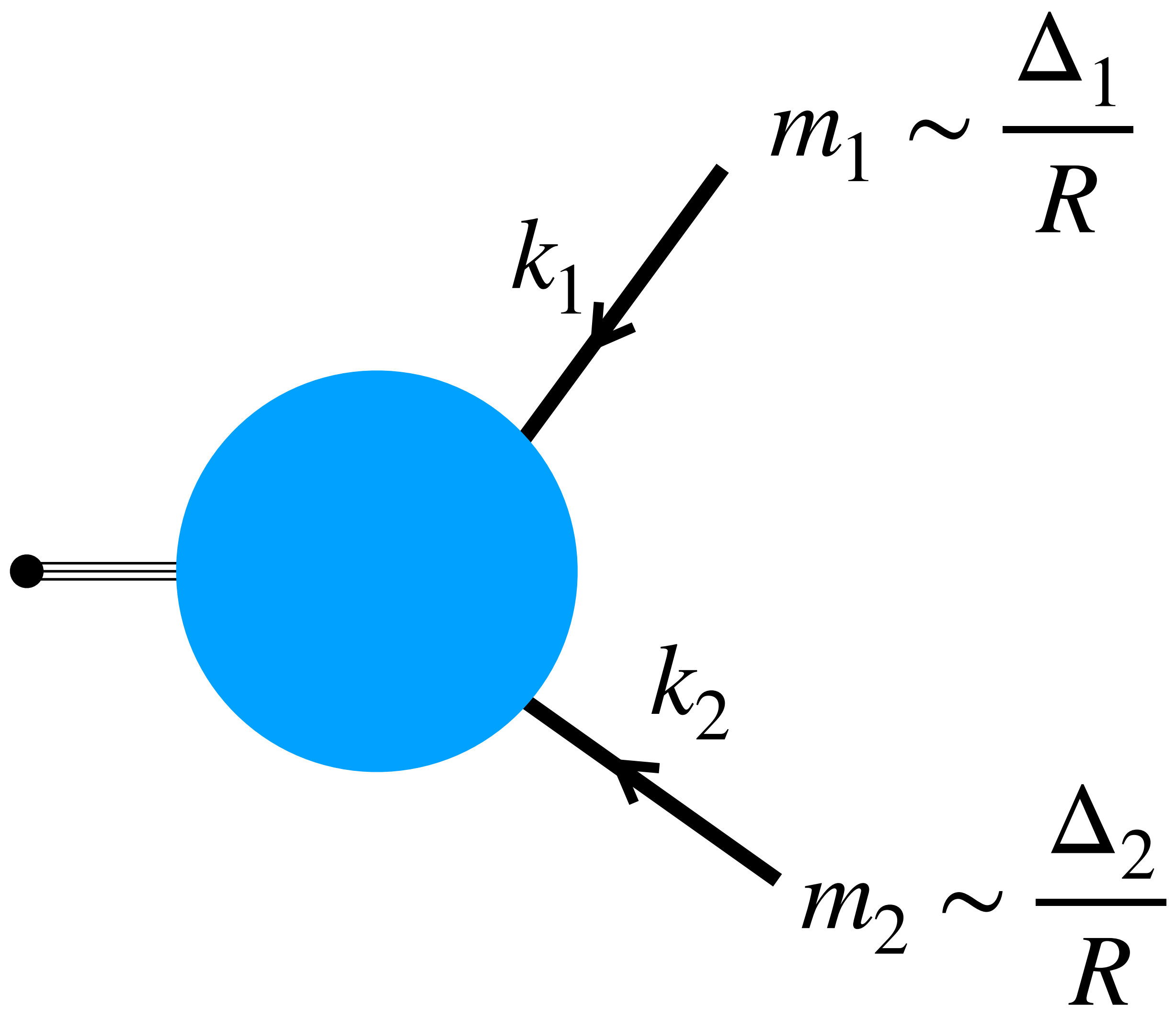
1. Rotate to Lorentz
2. $R_{\text{AdS}} \to \infty \ , \quad m^2 = \dfrac{\Delta(\Delta - d)}{R^2}$ fixed

cf. [Paulos, Penedones, Toledo, van Rees, Vieira] …
S-matrix



AdS form factor
$F(z) = \langle \Psi | O_1 O_2 \rangle$

$R_{\text{AdS}} \to \infty$



Flat space form factor
$\mathcal{F}(s) = \langle 0 | \Psi | k_1, k_2 \rangle$

Phase-shift formula

$(s = (k_1 + k_2)^2 > (m_1 + m_2)^2 + i\epsilon)$

$$\mathcal{F}(s) = \lim_{R\to\infty} \sum_{\Delta>\Delta_1+\Delta_2} 2e^{-i\frac{\pi}{2}(\Delta-\Delta_1-\Delta_2)} \frac{(\mu\lambda)_\Delta}{(\mu\lambda)^{\text{free}}_\Delta} \mathcal{N}(\Delta, s)$$

In 2d : Watson's equation

"Minimal" BOE spectrum $\{\Delta\} = \{2\Delta_\phi + 2n + \gamma_n\}$

$$\Longrightarrow \quad \mathcal{F}(s + i\epsilon) = \mathcal{F}(s - i\epsilon)\, S(s + i\epsilon)$$

Interacting functionals

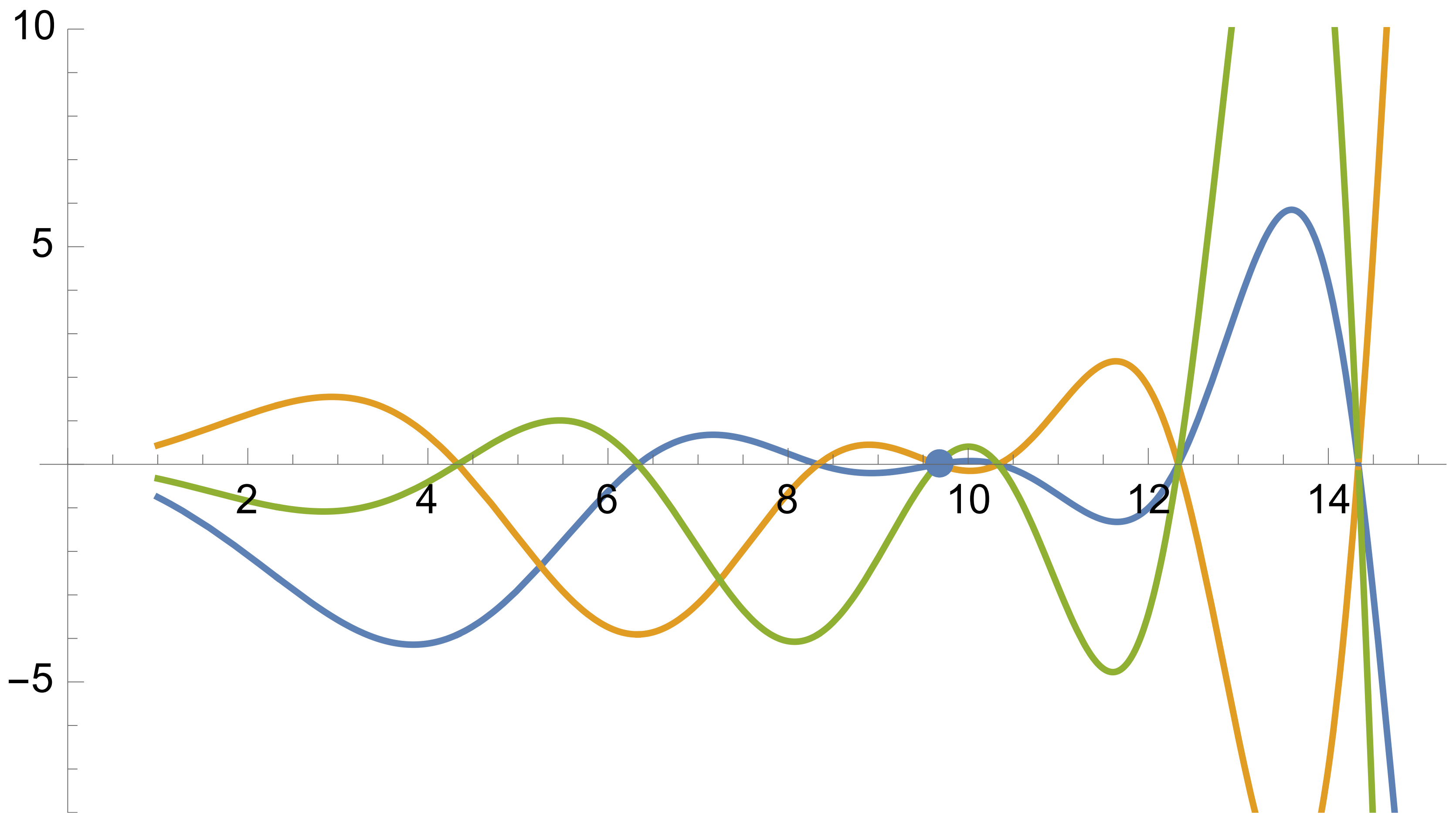
[Part II 23xx.xxxxx NL, Paulos]

Towards integrability in AdS

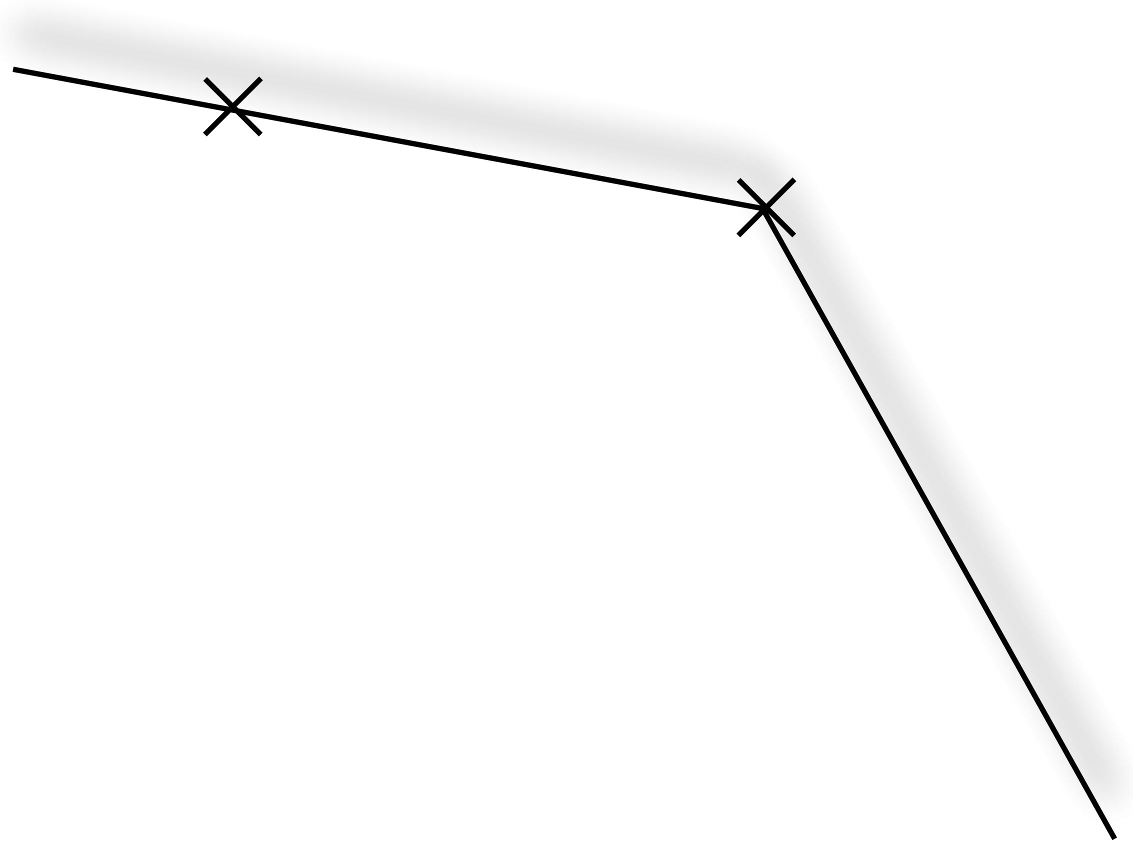
Functionals $\xleftrightarrow{\text{dual}}$ GFF solution

cf. 1d crossing [Mazac, Paulos]

We can build ***interacting functionals*** dual to any "minimal" spectrum !



$$\Delta_n = 2\Delta_\phi + 2n + \gamma_n \, , \qquad \gamma_n \overset{n\to\infty}{\sim} n^{-\epsilon}$$



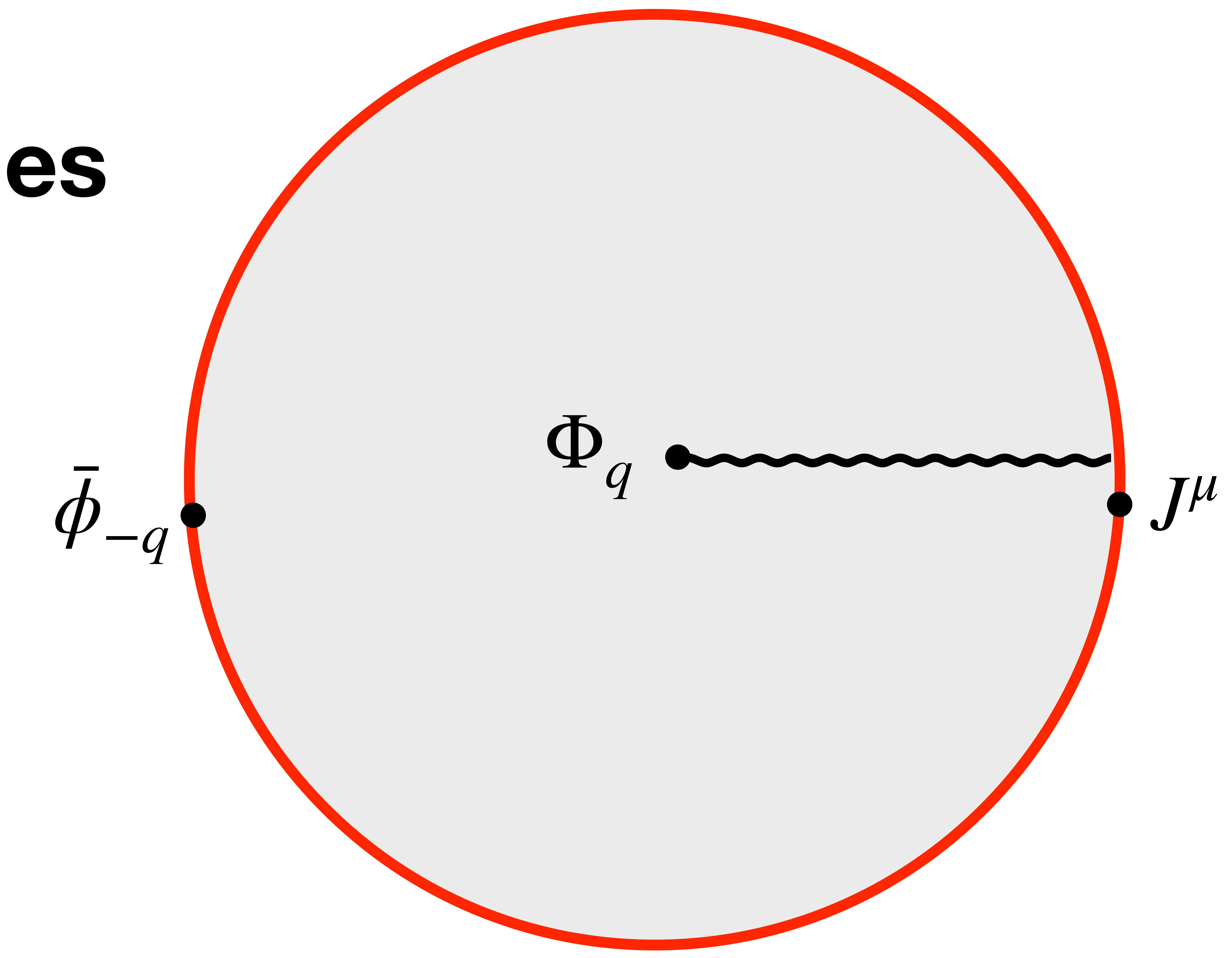
Dream: interacting functionals for 1d crossing

→ Integrable theories on AdS_2 ??

Future directions

- **Part III: Gauge/gravitational theories**

  Specific non-locality from dressing



- Bulk locality : extra constraint on CFT

  cf. [Behan, Di Pietro, Lauria, van Rees] for bulk free field

  Flat space : “strong locality” constraint on S-matrix ?

Future directions

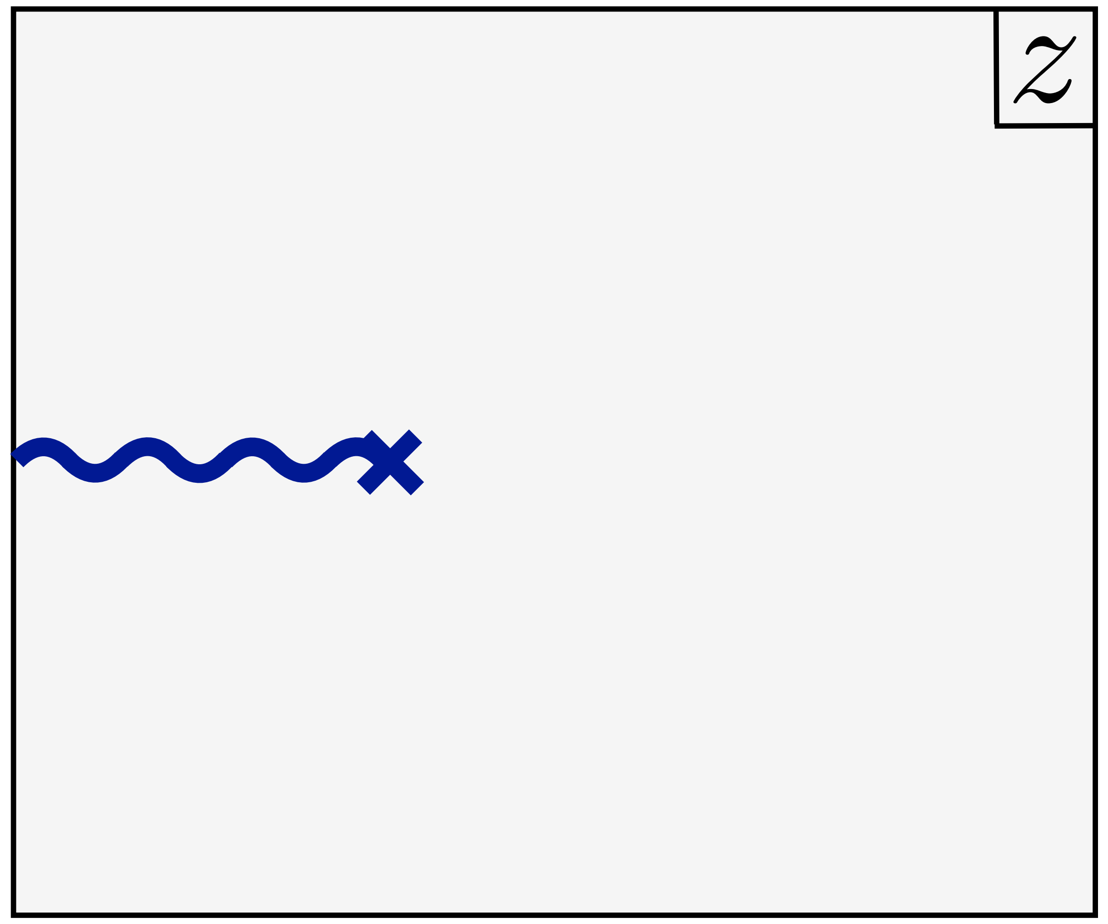
- Combine with crossing in semi-definite way?

  BCFT context?

  Higher point functions

cf. [Meineri Penedones Spirig] for stress tensor

- Constrain structure constants in 1d WL defect in $\mathcal{N} = 4$?



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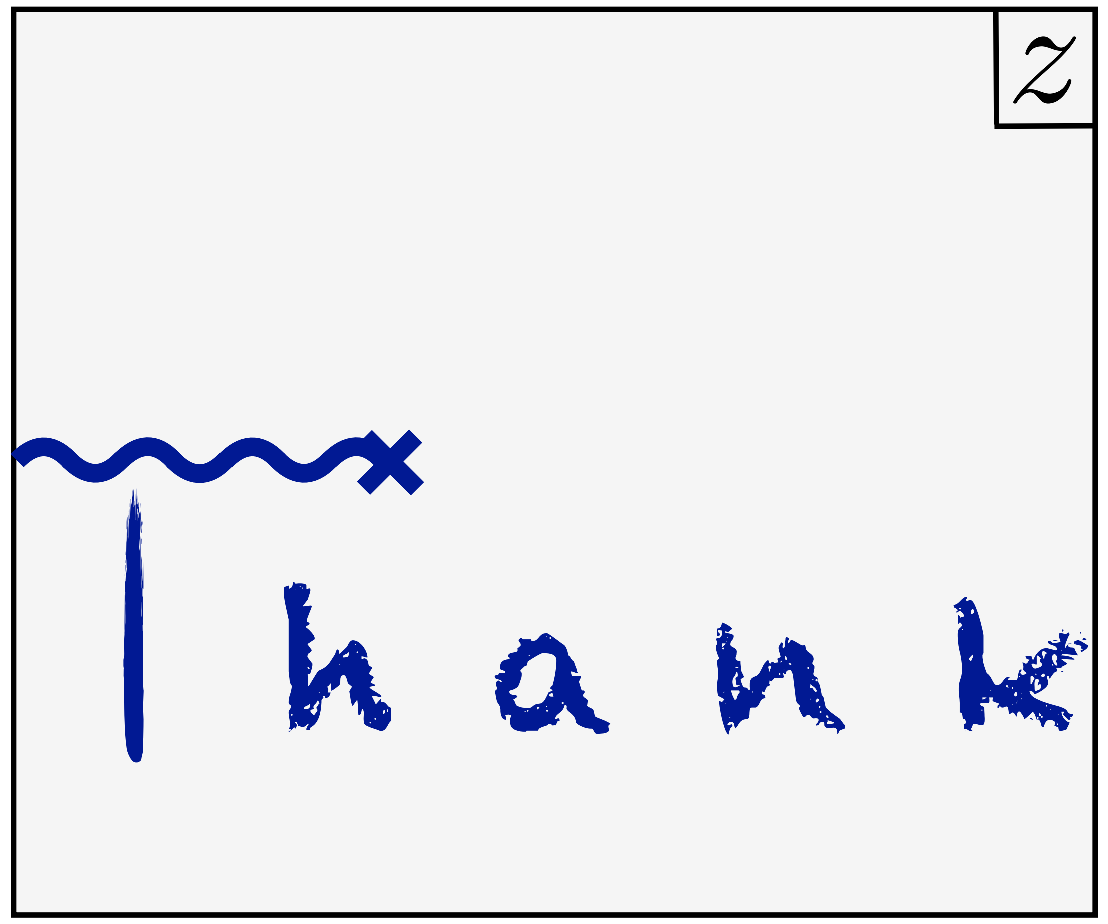

Thank you!