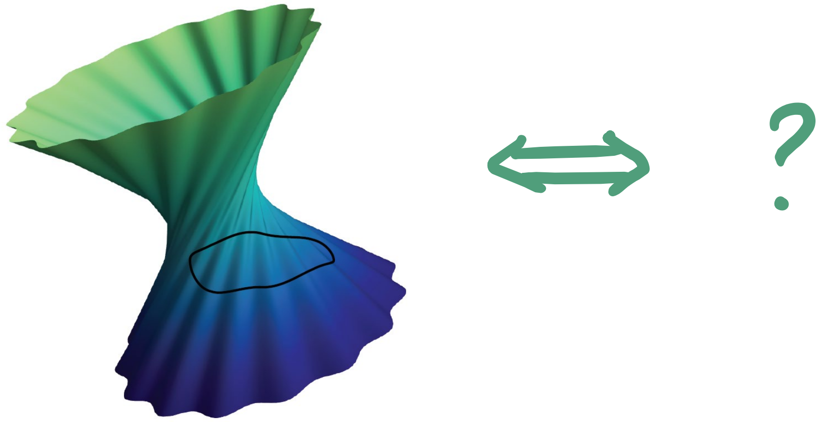
Integrable deformations of SYM



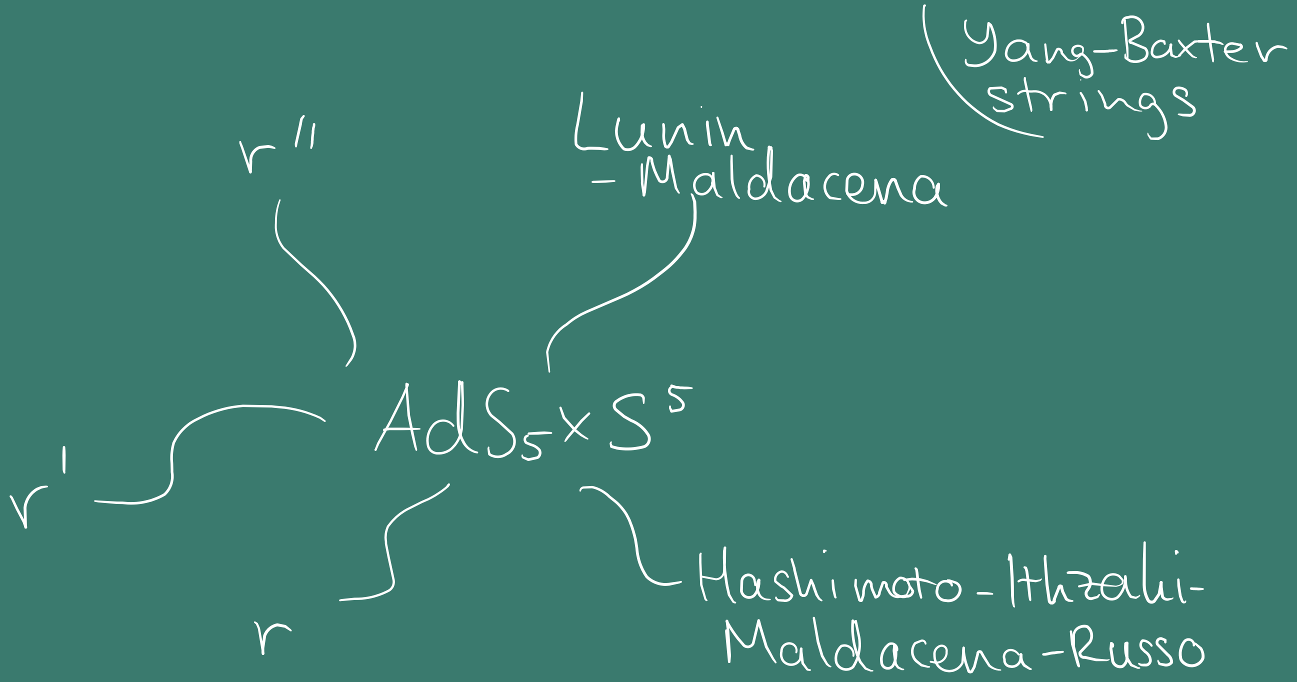
Stijn J. van Tongeren

IGST 23 Zürich

based on 2301.08757 & 2305.15470 w/ Tim Meier

Motivation

- IGST is great!
- How far can we push integrable AdS/CFT?
  - → range of observables in a theory
  - → range of theories
    - discrete steps: AdS_5/CFT_4, AdS_4/CFT_3, ...
    - deformations: Lunin-Maldacena, ...
- Many different interesting defs of AdS strings w/ less (su)sy
  ⇝ AdS/CFT? ⟿ integrable defs of $\mathcal{N}=4$ SYM

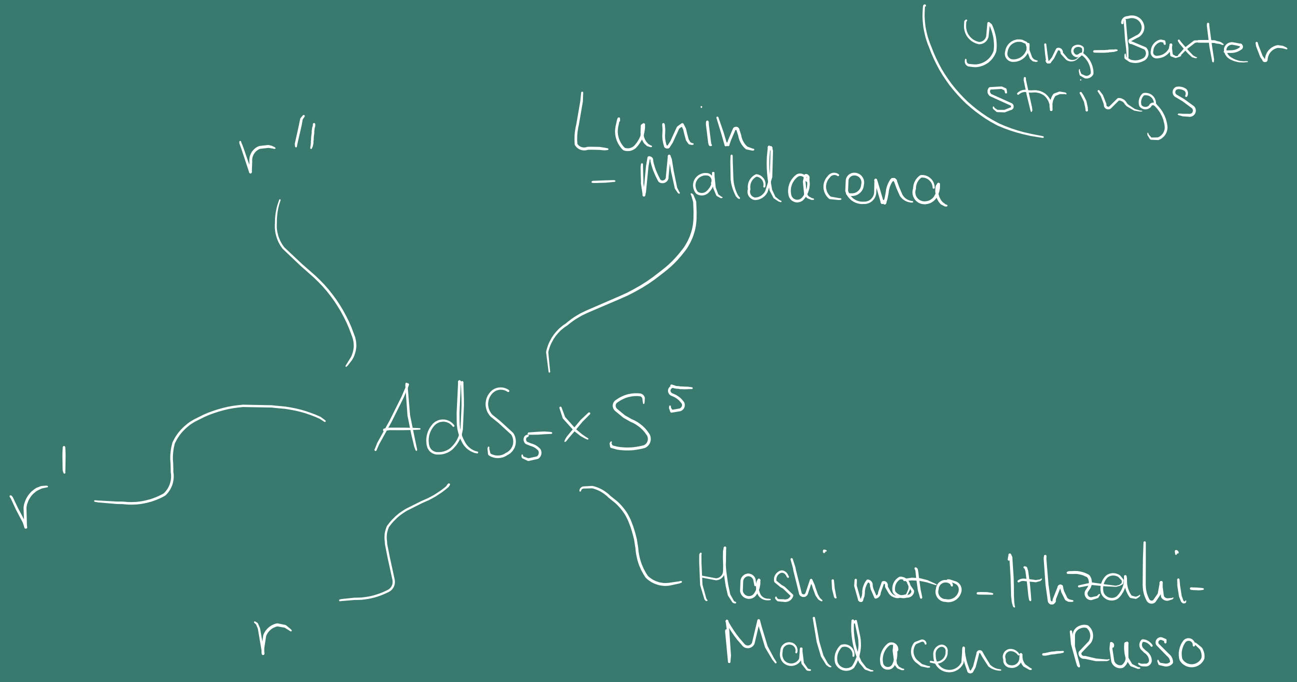


[Klimcik '02, Delduc, Magro, Vicedo '13, Kawaguchi, Matsumoto, Yoshida '14, ...]

[Riccardo's talk]

r matrix: $r \in \mathfrak{g} \otimes \mathfrak{g}$ $\quad (\mathfrak{g} = psu(2,2|4))$

- $r^t = -r$, i.e. $r = r^{ij}\, T_i \wedge T_j$
- $[r_{12}, r_{13}] + [r_{12}, r_{23}] + [r_{13}, r_{23}] = 0$ (homog. CYBE)



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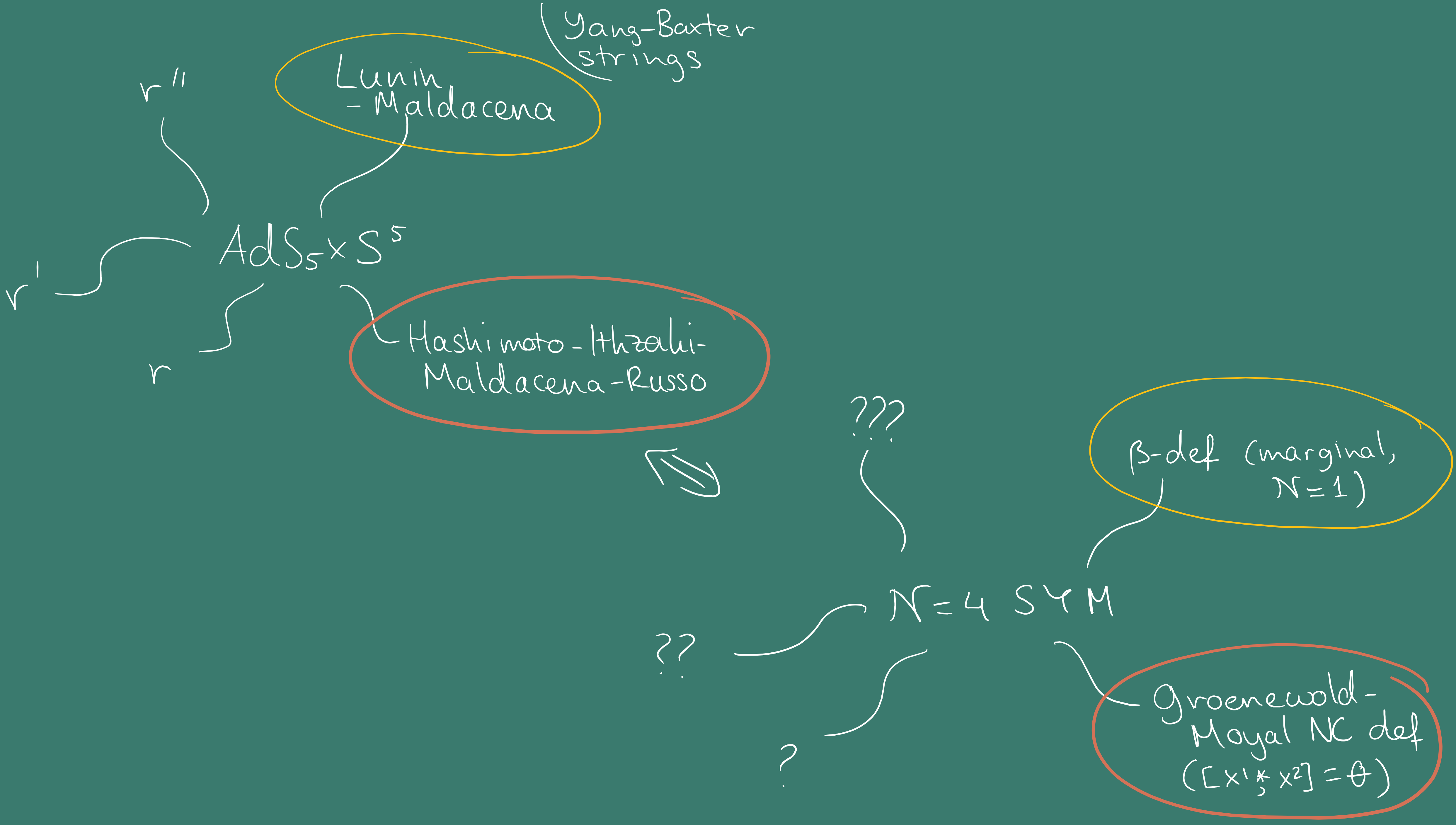
- $[r_{12}, r_{13}] + [r_{12}, r_{23}] + [r_{13}, r_{23}] = 0$ (homog. CYBE)
- $r^{ij} [T_i, T_j] = 0$ (unimodularity ⇝ Weyl inv.) [Borsato, Wulff '16]

→ many options!



Yang-Baxter strings
Lunin-Maldacena
r''
r'
r
AdS5×S5
Hashimoto-Itzhaki-Maldacena-Russo
???
β-def (marginal, N=1)
??
N=4 SYM
?
Groenewold-Moyal NC def
([x1 *, x2] = θ)

Lunin-Maldacena

- $AdS_5 \times S^5$ superstring w/

  $$r = \hat{\beta}\, Q_1 \wedge Q_2 ,$$

  Q_1, Q_2 suitable Cartan gens of $su(4)$ (S^5)

  gives string on LM background

- AdS/CFT dual: β-def SYM

  $$\mathcal{L}_\beta = \mathcal{L}_{SYM} \text{ with prod} \to * \text{ prod}$$

  $$f * g = e^{i\beta(Q_1^f Q_2^g - Q_2^f Q_1^g)} fg , \qquad \beta = \frac{\sqrt{\lambda}}{2\pi}\hat{\beta}$$

Lunin-Maldacena

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- AdS/CFT dual: β-def SYM

$$\mathcal{L}_\beta = \mathcal{L}_{SYM} \text{ with prod} \to * \text{ prod}$$

$$f * g = \mu(\mathcal{F}^{-1}(f,g)), \quad \mathcal{F} = e^{-i\beta Q_1 \wedge Q_2} = e^{ir}\Big|_{\hat{\beta}\to\beta}$$

↳ normal mult: $\mu(a,b) = ab$

Groenewold-Moyal

- $AdS_5 \times S^5$ superstring w/

$$r = -\frac{1}{2}\hat{\theta}\, p_1 \wedge p_2$$

p_μ momenta of conf. alg. (Poincaré AdS)

gives string on HIMR background

- AdS/CFT dual: Groenewold-Moyal NC SYM

$$\mathcal{L}_{GM} = \mathcal{L}_{SYM} \quad \text{w/ } \star :$$

$$f \star g = e^{i\frac{\theta}{2}(\partial_1^x \partial_2^y - \partial_2^x \partial_1^y)} f(x)\, g(y)\Big|_{y \to x}$$

i.e. $[x^1 \overset{\star}{,} x^2] = i\theta$ (NC spacetime)

Groenewold-Moyal

- $AdS_5 \times S^5$ superstring w/

$$r = -\frac{1}{2}\hat{\theta}\, p_1 \wedge p_2$$

p_μ momenta of conf. alg. (Poincaré AdS)

gives string on HIMR background

- AdS/CFT dual: Groenewold-Moyal NC SYM

$$\mathcal{L}_{GM} = \mathcal{L}_{SYM} \quad \text{w/} \quad * :$$

$$f * g = \mu(F^{-1}(f,g)), \quad F = e^{ir}\Big|_{\hat{\theta}\to\theta}$$


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r-matrices
Drinfel'd twists
*
twist-NC CFT
*''
β-def (marginal, $\mathcal{N}=1$)
$\mathcal{N}=4$ SYM
*'
*
Groenewold-Moyal NC def
$([x^1 \overset{*}{,} x^2] = \theta)$

Today

SYM w/ general $\star(\mathcal{F}(r))$

Spacetime defs:

- $r = \lambda M_{01} \wedge M_{23}$ (Cartan)
- $r = \alpha M_{+1} \wedge M_{+2}$ (nilpotent)
- $r = \xi M_{+1} \wedge P_{+} + \xi P_{1} \wedge (P_{+} - P_{-})$ (nonabelian)
- ...

Today

SYM w/ general $\star$ $(\mathcal{F}(r))$

Spacetime defs, complications:

- GM $\star$ is special:
  - $p_\mu(dx^\nu) = 0$, not so for other conf. gens
  - $\partial_\mu(f \star g) = (\partial_\mu f) \star g + f \star (\partial_\mu g)$ not so for other $\star$s
- YM action not known beyond GM case
- Adjoint matter, in particular fermions...?

Today

SYM w/ general $\star$ ($F(r)$)

Spacetime defs, results:

- Action for $r \in \text{Poincaré}^{\otimes 2}$ (Hodge star), unimodular
- Twisted superconformal symmetry (✓ YB string)
- Planar equiv. Thm. for Feynman diagrams

Questions?

Outline

- r matrices, Drinfel'd twists & $\star$ prods
- twisted differential calculus
- twist-NC gauge theory
- twisted symmetry
- planar equiv. thm
- comments on AdS/CFT

Drinfel'd twists & r matrices

- $\mathcal{U}(g)$ standard Hopf alg. structure:

  $\Delta(X) = X \otimes 1 + 1 \otimes X$ $\qquad (S, \epsilon, ..)$

- $\mathcal{F} \in \mathcal{U}(g) \otimes \mathcal{U}(g)$ [Drinfel'd '83]

  - $(\mathcal{F} \otimes 1)(\Delta \otimes 1)\mathcal{F} = (1 \otimes \mathcal{F})(1 \otimes \Delta)\mathcal{F}$ (cocycle cond.)

  - $\mathcal{F} = 1 \otimes 1 + \eta \mathcal{F}^{(1)} + \ldots$ (deformation)

    (η: def param.)

- $\mathcal{F}^{(1)} - \mathcal{F}^{(1)t} = r$ solves CYBE & "vice versa"

Drinfel'd twists & * products

Given twist $\mathcal{F}$, can define * product

$$f * g = \mu(\mathcal{F}^{-1}(f,g))$$

- noncommutative...
- ... but associative (cocycl. cond)

E.g. "Lorentz" def: $\star$ w/ $\mathcal{F} = e^{i\lambda M_{01} \wedge M_{23}}$

- $[x \overset{*}{,} x] = \mathcal{O}(x^2)$, e.g. $[x^0 \overset{*}{,} x^2] = -i\lambda x^1 x^3$ (vs GM $[x \overset{*}{,} x] = \text{const.}$)
- $\partial_\mu(f*g) \neq (\partial_\mu f)*g - f*(\partial_\mu g)$

$\rightsquigarrow$ suitable formalism?

Twisted diff. calculus [e.g. Aschieri et al. '09]

- Generators in $\mathcal{F}$ act via Lie der. ($\mathcal{L}$)
  → deform any product:

$$f \star g = \mu(\mathcal{F}^{-1}(f,g))$$

$$dx^\mu \wedge_\star dx^\nu = \wedge(\mathcal{F}^{-1}(dx^\mu, dx^\nu))$$

etc.

e.g. for Lorentz def: $f \star dx^0 = ch(\frac{\lambda}{2} M_{23})(f)\, dx^0 + sh(\frac{\lambda}{2} M_{23})(f)\, dx^1$

- Since $[d, \mathcal{L}_X] = 0$

$$d(f \star g) = df \star g + f \star dg$$

Twist-NC gauge Theory

Gauge symm is local, hence $\star$ affects it

⇝ star gauge symmetry [e.g. Szabo '01]

- $D\Phi = d\Phi + iA\star\Phi$ (Φ fund)
- $\delta_\varepsilon A = d\varepsilon + i[\varepsilon \overset{\star}{,} A]$
- $G = dA - iA\wedge_\star A$, $\delta_\varepsilon G = i[\varepsilon \overset{\star}{,} G]$
- gauge alg. only closes for $U(N)$ ($[\overset{\star}{,}]$ vs $[,]$)
  (ok for AdS/CFT!)

Action:

$$S = \int \mathrm{Tr}(G \wedge_\star \circledast G)$$

Twist-NC gauge Theory

Gauge symm is local, hence $\star$ affects it
$\rightsquigarrow$ star gauge symmetry [e.g. Szabo '01]

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- gauge alg. only closes for $U(N)$ ($[\overset{\star}{,}]$ vs $[,]$)
  (ok for AdS/CFT!)

Action:

$$S = \int \mathrm{Tr}\,(G \wedge_\star \circledast G)$$

$\hookrightarrow$???

Gauge invariance of S

$$\int \mathrm{Tr}(G \wedge_* \circledast G) \text{ is gauge invt. if:}$$

1) $\delta_\varepsilon(\circledast G) \overset{!}{=} i[\varepsilon \overset{*}{,} \circledast G]$

2) $\int \mathrm{Tr}([\varepsilon \overset{*}{,} G \wedge_* \circledast G]) \overset{!}{=} 0$

i.e. if

1) $\circledast$ is $*$ linear $\quad (\circledast(f * dx) = f * (\circledast dx))$

2) $*$ product is cyclic under integral

$\circledast$ & $*$ linearity

- Under moderate assumptions, can show that $\circledast$ can be $*$ linear only for $*$/twists based on Poincaré alg. (e.g. dilatation is not ok)

- Concrete def w/ all "usual" properties?
  - define ε via

$$dx^\mu \wedge_* dx^\nu \wedge_* dx^\rho \wedge_* dx^\sigma = \varepsilon^{\mu\nu\rho\sigma}\, dx^0 \wedge_* dx^1 \wedge_* dx^2 \wedge_* dx^3$$

  - define $\circledast$ on basis

$$\circledast\, dx^{\mu_1} \wedge_* \dots \wedge_* dx^{\mu_k} = \frac{(-1)^{\#}}{(4-k)!}\, \varepsilon_{\mu_{k+1} \dots \mu_4}{}^{\mu_1 \dots \mu_k}\, dx^{\mu_4} \wedge_* \dots \wedge_* dx^{\mu_{k+1}}$$

    & extend $*$ linearly

Integral cyclicity

Want

$$\int \omega \wedge_* \chi = (-1)^{\#} \int \chi \wedge_* \omega \quad (+\text{ tot der.})$$

Leading order:

$$\int r^{ij} \mathcal{L}_{T_i}(\omega) \wedge \mathcal{L}_{T_j}(\chi) \simeq 0$$

$$= \underbrace{\int r^{ij} \mathcal{L}_{T_i}\left(\omega \wedge \mathcal{L}_{T_j}(\chi)\right)}_{\text{tot der.}} - \underbrace{\int \omega \wedge r^{ij} \mathcal{L}_{T_i} \mathcal{L}_{T_j}(\chi)}_{=0 \text{ iff } r \text{ unimodular!}}$$

twist-NC gauge th. & SYM

- have action for twist-NC YM for unimod. Poincaré twists (14 multi-par. types)

- adjoint scalars ✓ Weyl fermions $\psi_\alpha, \bar\psi_{\dot\alpha}$?

  → basis spinors $s^\alpha, \bar s^{\dot\alpha}$ in spin reps of Poincaré

  - $\psi = s^\alpha \psi_\alpha, \; \bar\psi = \bar\psi_{\dot\alpha} \bar s^{\dot\alpha}$
  - $\sigma = s^\alpha \sigma_{\mu\alpha\dot\alpha} \bar s^{\dot\alpha} dx^\mu$

  $\Bigg\} \rightsquigarrow \int d^2 s \int d^2 \bar s \int \bar\psi \star \sigma \wedge_\star \circledast D\psi$

  ↑ σ uncharged w.r.t. $\star$

- … $\rightsquigarrow$ $S^\star_{SYM} = S_{SYM}\big|_{\text{prod} \to \star\text{prod}}$ (index free)

Twisted Lorentz symm for GM ⋆

Recall undeformed:

- Lorentz gens act via Lie ders on fields

  e.g. scalar: $\delta_M \phi = M\phi \qquad (M = x\partial - x\partial)$

- Symmetry: $\delta_M S = \int \text{tot der} \simeq 0$

  e.g. mass term:

  $$\delta_M(\phi^2) = (M\phi)\phi + \phi(M\phi) = M(\phi^2) = \text{tot der.}$$

  ↑ prod rule

- Formalize in Hopf alg. language:

$$\delta_M(\phi^2) = \mu(\Delta(M)(\phi,\phi)) = M\,\mu(\phi,\phi) = M(\phi^2)$$

Twisted Lorentz symm for GM $\star$

Deformed [Chaichian et al. '04, Wess '04]:

- in NC setting usual Lorentz is broken

$$\delta_M(\phi \star \phi) \equiv (M\phi)\star\phi + \phi\star(M\phi) \neq M(\phi\star\phi)$$

- not gone, but twisted:

$$\delta^\star_M(\phi\star\phi) \equiv \mu\left(\mathcal{F}^{-1}\Delta_{\mathcal{F}}(M)(\phi,\phi)\right) = \text{tot. der.}$$

$$\hookrightarrow = \mathcal{F}\Delta(M)\mathcal{F}^{-1}$$

$\rightsquigarrow$ twisted symmetry (action on field dep. on prod.)

e.g. $\delta^\star_{M_{13}}(\phi\star\phi) = (M_{13}\phi)\star\phi + \phi\star(M_{13}\phi) + \theta\left(P_{[2}\phi\right)\star\left(P_{3]}\phi\right)$

twisted superconf. symm.

- S^*_{SYM} is similarly invt under twisted Poincaré symm...

- ... & full twisted $psu(2,2|4)$!

  → Matches expectations from YB strings

[Vicedo '15, SvT '15, ...]
[Ahn, Bajnok, Nepomechie '10, ...]

Planar equivalence Theorem

- matrix valued fields & traces
  ⇝ "planarity" of Feynman diagrams
- same for ⋆ w/ cyclic integration
- for GM def. scalars [Filk '96]:
  planar diagrams are undef up to external ⋆ prod

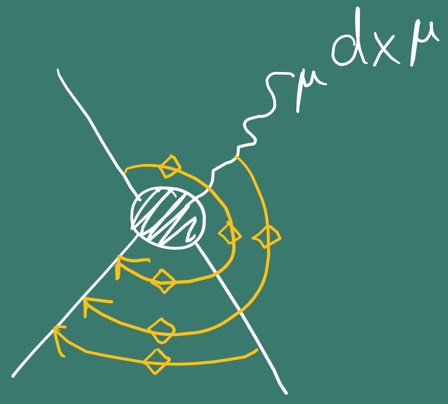
 : undef

Planar equivalence Theorem

- matrix valued fields & traces
  ⇝ "planarity" of Feynman diagrams

- same for $\star$ w/ cyclic integration

- for all our $\star$'s & any field content :
  planar diagrams are undef up to external $\star$ prod



(index free combinations)

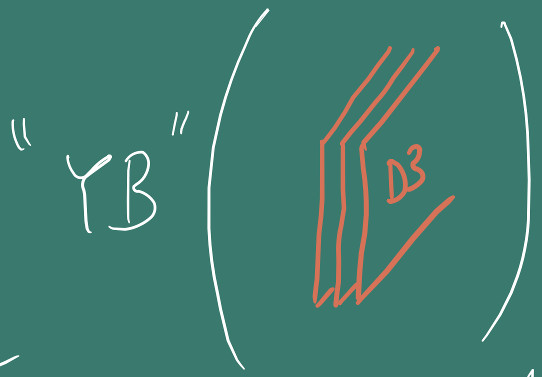
(hatched circle) : undef

$\diamond$: $\star\big|_{F^{-1} \to F^t}$ ($= \star$ for abelian twists)

Comments on AdS/CFT

Are twist-NC SYMs really dual to YB strings?

- planar symmetry matches
- brane picture:



[Seiberg, Witten '99, ...]

YB def of $AdS_5 \times S^5$ $\Longleftrightarrow$ NC SYM w/ $[x^\mu \overset{*}{,} x^\nu] = r^{\mu\nu} + \dots$

"✓"

subtleties:
- limit not ok for timelike $r^{\mu\nu}$
- cases w/ no susy may be unstable
- YB strings can have unbounded dilaton

Summary & outlook

- Constructed twist-NC versions of SYM
  ~ dual to YB AdS_5 strings
  - twisted symmetry
  - planar equiv. Thm
  - some certainly dual, others less clear (only planar?)

- For now limited to Poincaré-based defs
  - extend to conformal?
  - mix w/ R-symmetry "✓" (dipole)
  - include supercharges? ⇝ Jordanian? (cf. Riccardo's talk!)

→ goal: integrable YB deformed AdS/CFT

Integrable outlook

Integrability for planar twist-NC SYM:

- Yangian inv. of action in spirit of
  [Beisert, Garus, Rosso '18], [Garus '17]
- Twisted spin chain for 2pt fns.
  - in progress for Lorentz def ($\rightsquigarrow$ [Beisert, Roiban '05])
  - open question for other defs (also on string side!)
    - break D/ AdS global τ symmetry
      $\rightsquigarrow$ which spectrum? "exact" S matrix?
    - nilpotent / nonabelian twists (Bethe A. / Q-system)
      [Guica, Levk.-Maslyuk, Zarembo '17]

      $\rightsquigarrow$ CSC [Borsato, Driezen, Nieto-G, Wyss '22]

Thank you!

Example: light-cone Lorentz def

- $r = \alpha\, M_{+1} \wedge M_{+2}$ (nilpotent gens)
- nontrivial quadratic NC def w/

$$[x^1 \overset{*}{,} x^2] = i\alpha\, x^{-2} + \mathcal{O}(\alpha^2)$$

$$[x^+ \overset{*}{,} x^{1/2}] = \pm i\alpha\, x^{2/1}\, x^- + \mathcal{O}(\alpha^2)$$

- Hodge $\circledast$ uses ε w/ deformed
  $\varepsilon^{++++-} = -2i\alpha \quad \varepsilon^{+1+1} = \varepsilon^{+2+2} = i\alpha$
  (+ graded cyclic)
- preserves 16 supercharges & D
- admits nonabelian extension
  w/ 8 supercharges
- brane geom. has good limit

- YB dual:

$$g + B = \frac{1}{g^{-1}_{AdS} + r}$$

$$F^{(n)} = \ldots$$

$$e^{2\phi} = \frac{z^4}{z^4 + \hat{\alpha}^2 (x^-)^4} \quad \text{(bounded)}$$

$$S^\star_{\rm SYM} = -\frac{1}{2}{\rm tr}\int {\rm D}\phi^m \wedge_\star *{\rm D}\phi_m - \frac{1}{4g^2_{\rm YM}}{\rm tr}\int G\wedge_\star *G - \frac{g^2_{\rm YM}}{4}{\rm tr}\int {\rm d}^4x \;\; [\phi^m \overset{\star}{,} \phi^n]\star[\phi_m \overset{\star}{,} \phi_n]$$
$$+\,{\rm tr}\int {\rm d}^2s{\rm d}^2\bar{s}\int \bar{\psi}^I \star \sigma \wedge_\star *{\rm D}\psi_I$$
$$-\,\frac{ig_{\rm YM}}{2}\;{\rm tr}\int {\rm d}^2s\int {\rm d}^4x\;\sigma^{IJ}_m \psi_I \star [\phi^m \overset{\star}{,} \psi_J] - \frac{ig_{\rm YM}}{2}\;{\rm tr}\int {\rm d}^2\bar{s}\int {\rm d}^4x\;\sigma^m_{IJ}\bar{\psi}^I\star[\phi_m \overset{\star}{,} \bar{\psi}^J]\,,$$

$$\mathcal{L} = {\rm tr}\left(-\frac{1}{2}D_\mu\phi^m \star D^\mu\phi_m - \frac{1}{4g^2_{\rm YM}}G_{\mu\nu}\star G^{\mu\nu} + \bar{\psi}_{\dot\alpha}{}^I\sigma^{\mu\dot\alpha\alpha}\star D_\mu\psi_{\alpha I}\right)$$
$$-\,\frac{g^2_{\rm YM}}{4}\;{\rm tr}\left([\phi^m \overset{\star}{,} \phi^n]\star[\phi_m \overset{\star}{,} \phi_n]\right)$$
$$-\,\frac{ig_{\rm YM}}{2}\sigma^{IJ}_m\;{\rm tr}\left(\bar{F}_\alpha{}^\gamma\psi^\alpha{}_I\star\phi^m\star(F_{op})_\gamma{}^\beta\,\psi_{\beta J} - (\bar{F}_{op})_\alpha{}^\beta\,\psi^\alpha{}_I\star F_\beta{}^\gamma\psi_{\gamma J}\star\phi^m\right)$$
$$-\,\frac{ig_{\rm YM}}{2}\sigma^m_{IJ}\;{\rm tr}\left(F_{\dot\beta}{}^{\dot\alpha}\bar{\psi}_{\dot\alpha}{}^I\star\phi_m\star(\bar{F}_{op})_{\dot\gamma}{}^{\dot\beta}\,\bar{\psi}^{\dot\gamma J} - (F_{op})_{\dot\beta}{}^{\dot\alpha}\,\bar{\psi}_{\dot\alpha}{}^I\star\bar{F}_{\dot\gamma}{}^{\dot\beta}\bar{\psi}^{\dot\gamma J}\star\phi_m\right)$$

$$G_{\mu\nu} = \partial_{[\mu}A_{\nu]} - i\bar{F}_\rho{}^\kappa{}_\sigma{}^\tau\bar{F}_{[\nu|}{}^\sigma(A_\kappa)\star\bar{F}^\rho{}_{|\mu]}(A_\tau), \qquad {\rm D}\Phi = {\rm D}^\star_\mu\Phi\star{\rm d}x^\mu = (\partial^\star_\mu\Phi + iA^\star_\nu\star\bar{R}_\mu{}^\nu\Phi)\star{\rm d}x^\mu.$$

Twisted symmetry, details

Twisted Poincaré ($\ni X$) relies on:

- $[\mathcal{L}_X, d] = 0$ (autom.)
- $[\mathcal{L}_X, \circledast] = 0$

Twisted conf. relies on:

- $[\mathcal{L}_D, \circledast] =$ usual
- $[\mathcal{L}_k, \circledast] =$ usual

Twisted susy:

- index free formulation: $Q = S^\alpha Q_\alpha$ etc.
- cancellations ($\circledast(\sigma \wedge_\star \circledast d\phi) = \sigma^\mu \partial_\mu \phi$, Fierz id.)

$$
\begin{aligned}
Q^K(\phi_m) &= \sigma_m^{IJ}\psi_J \\
\bar{Q}_K(\phi^m) &= \sigma_{IJ}^m\bar{\psi}^J \\
Q^J(s'^{\,\alpha}\,\psi_{\alpha I}) &= \frac{1}{2}\delta_I^J\int \mathrm{d}^2\bar{s}\;\ast(\sigma\wedge_\star\sigma'\wedge_\star\ast F)+\frac{ig}{2}\;s^\alpha\star s'_\alpha\star[\phi^m\stackrel{\star}{,}\phi_n]\,\sigma_m^{JK}\sigma_{KI}^n \\
\bar{Q}_I(s^\alpha\psi_{\alpha J}) &= \ast\frac{1}{2}\sigma_m^{IJ}\,(\sigma\wedge_\star\ast D\phi^m) \\
Q^I(\bar{\psi}^J{}_{\dot\alpha}\bar{s}^{\dot\alpha}) &= \ast\frac{1}{2}\sigma_m^{IJ}\,(\sigma\wedge_\star\ast\mathrm{D}\phi^m) \\
\bar{Q}_J(\bar{\psi}^I_{\dot\alpha}\bar{s}'^{\dot\alpha}) &= \frac{1}{2}\delta_J^I\int \mathrm{d}^2s\ast(\sigma\wedge_\star\sigma'\wedge_\star\ast F)+\frac{ig}{2}\bar{s}^{\dot\alpha}\star\bar{s}'_{\dot\alpha}\star[\phi^m\stackrel{\star}{,}\phi_n]\,\sigma_m^{IK}\sigma_{KJ}^n \\
Q^I(A) &= ig\;\int \mathrm{d}^2\bar{s}\;\sigma\star\bar{\psi}^I \\
\bar{Q}_I(A) &= ig\;\int \mathrm{d}^2s\;\psi_I\star\sigma.
\end{aligned}
$$

$$
\ast\,(\sigma\wedge_\star\ast\sigma')=s^\alpha s'_\alpha\bar{s}_{\dot\alpha}\;\bar{s}'^{\,\dot\alpha}
$$

Planar equiv. Thm; details

relies on:

- propagators undeformed
  (integr. cycl. + $\circledast d \circledast d = \partial^\mu \partial_\mu$)

- Poincaré inv. of (index-free)
  propagators:
  $$(X_x - X_y)\Delta(x-y) = (X_x + S(X_y))\Delta(x-y) = 0$$

- Hopf alg. rels + $\mu(S \otimes 1\, \mathcal{F}^{-1}) = 1$ ($\Leftarrow$ integr. cycl.)

E.g. cyclicity property: