Wilson Loop at large N
and quantum M2-brane

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work with Simone Giombi   arXiv:2303.15207

[and related work with Matteo Beccaria]

AdS/CFT: 25 years

enormous progress but still limited set of precise results

basic controlled examples:

$\mathcal{N} = 4,\ d = 4$ SYM $\leftrightarrow$ $AdS_5 \times S^5$ string

$\mathcal{N} = 6,\ d = 3$ ABJM $\leftrightarrow$ $AdS_4 \times CP^3$ string

- quantitative understanding of duality in planar limit based on integrability

integrability: allows to

- solve classical (genus 0) string theory:

spectrum of energies

- solve $N = \infty$ gauge theory:

anomalous dims, but only few correlators

- beyond planar limit?

finite N: string loops?

non-planar gauge theory corrections? hard...

other methods (for any N and $\lambda = g^2_{\text{YM}} N$):

(i) localization

[only for few special susy observables]

(ii) bootstrap

[symmetries and general principles, implicit]

• some progress by combining methods

[integrated corr $\to$ string scatt from AdS, etc]

Aims:

• use localization to check AdS/CFT
at non-planar level
on special example of $\frac{1}{2}$ BPS Wilson loop
$\rightarrow$ learn about structure of string loop corrs

• match quantum M2-brane correction
and ABJM theory localization result for WL
1-loop M2: sum of ∞ set of string loop terms

• existence of quantum supermembrane theory open question: formally non-renormalizable but semiclassical 1-loop computations are ok: no log UV div at 1-loop in 3d

[Duff, Inami, Pope, Sezgin, Stelle 88; Bergshoeff, Sezgin, Townsend 88; Fosrte 99;

Drukker, Giombi, Zhou, AT 2020]

• evidence that semiclassical quantization of M2 brane is under control and gives non-trivial check of AdS_4/CFT_3 beyond planar limit

Plan:

- localization results for WL in SYM and ABJM
- matching leading order string theory results
- higher genus strong coupling terms $\sum_n c_n (\frac{g_s^2}{T})^n$:
  $\exp(c_1 \frac{g_s^2}{T})$ in SYM and $(\sin \frac{2\pi}{k})^{-1}$ in ABJM
- $(\sin \frac{2\pi}{k})^{-1}$ as 1-loop M2 brane contribution
- generalizations

$\frac{1}{2}$ BPS circular WL in SYM and ABJM

• $\mathcal{N}=4$ $SU(N)$ SYM: $\mathcal{W} = \mathrm{Tr}\, P e^{\int (iA+\Phi)}$

Localization → gaussian matrix model: any N, g^2_{YM}

[Erickson, Semenoff, Zarembo 00; Drukker, Gross 01; Pestun 07]

$$\langle \mathcal{W} \rangle = e^{\frac{N-1}{8N} g^2_{\mathrm{YM}}} L^1_{N-1}(-\tfrac{1}{4} g^2_{\mathrm{YM}})$$

$$L^1_n(x) \equiv \frac{1}{n!} \frac{d^n}{dx^n} (x^n e^{-x})$$

Large N, fixed $\lambda = N g^2_{\mathrm{YM}}$:

$$\langle \mathcal{W} \rangle = N \Big[\frac{2}{\sqrt{\lambda}} I_1(\sqrt{\lambda}) + \frac{\lambda}{48 N^2} I_2(\sqrt{\lambda}) + ... \Big]$$

$$\lambda \gg 1: \qquad \langle \mathcal{W} \rangle = \frac{N}{\lambda^{3/4}} \sqrt{\tfrac{2}{\pi}}\, e^{\sqrt{\lambda}} + ...$$

ABJM:

3d $\mathcal{N} = 6\ U(N)_k \times U(N)_{-k}$ CS + bi-fund

[Aharony, Bergman, Jafferis, Maldacena 08]

low-energy limit of N M2's on $\mathbb{C}^4/\mathbb{Z}_k$

$z_i \to e^{\frac{2\pi i}{k}} z_i, \quad i = 1, 2, 3, 4$

large N: dual to M-theory on $\mathrm{AdS}_4 \times S^7/\mathbb{Z}_k$

large k with $\lambda = \frac{N}{k}$=fixed:

type IIA string on $\mathrm{AdS}_4 \times \mathrm{CP}^3$

can define analogous $\frac{1}{2}$ BPS circular WL

localization $\rightarrow$ matrix model: for any N, $k > 2$

[Drukker, Marino, Putrov 10; Klemm, Marino, Schiereck, Sarouush 12]

$$\langle \mathcal{W} \rangle = \frac{1}{2\,\sin\frac{2\pi}{k}}\;\frac{\mathrm{Ai}\left[(\frac{\pi^2}{2}k)^{1/3}\left(N - \frac{k}{24} - \frac{7}{3k}\right)\right]}{\mathrm{Ai}\left[(\frac{\pi^2}{2}k)^{1/3}\left(N - \frac{k}{24} - \frac{1}{3k}\right)\right]}$$

large N at fixed k ("M-theory" expansion):

$$\mathrm{Ai}(x)\Big|_{x\gg 1} \simeq \frac{e^{-\frac{2}{3}x^{3/2}}}{2\sqrt{\pi}\,x^{1/4}} \; \sum_{n=0}^{\infty} \frac{(-\frac{3}{4})^n\,\Gamma(n+\frac{5}{6})\Gamma(n+\frac{1}{6})}{2\pi\, n!\, x^{3n/2}}$$

$$\langle \mathcal{W} \rangle = \frac{1}{2\sin\frac{2\pi}{k}}\, e^{\pi\sqrt{\frac{2N}{k}}}\left[1 - \frac{\pi\left(k^2+32\right)}{24\sqrt{2}\,k^{3/2}}\frac{1}{\sqrt{N}} + \mathcal{O}(\frac{1}{N})\right]$$

't Hooft ("string") expansion: $N, k \gg 1,\, \lambda = \frac{N}{k}$

$$\langle \mathcal{W} \rangle = \frac{1}{2\sin\frac{2\pi\lambda}{N}}\, e^{\pi\sqrt{2\lambda}}\left[1 - \frac{\pi}{24\sqrt{2}}\frac{1}{\sqrt{\lambda}} + \mathcal{O}(\frac{1}{N})\right] = \frac{N}{4\pi\lambda}\, e^{\pi\sqrt{2\lambda}}\left[1+\ldots\right]$$

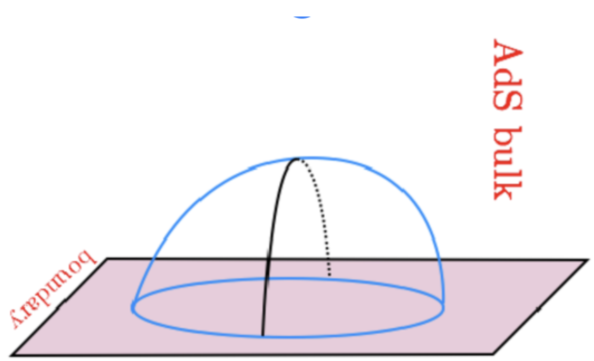
dual string in $\mathrm{AdS}_5 \times S^5$ and $\mathrm{AdS}_4 \times \mathrm{CP}^3$

parameters: g_s and effective tension $T = \frac{L_\mathrm{ads}^2}{2\pi\alpha'}$

$$\mathrm{SYM}: \quad g_\mathrm{s} = \frac{g_\mathrm{YM}^2}{4\pi} = \frac{\lambda}{4\pi N}\,, \qquad T = \frac{\sqrt{\lambda}}{2\pi}\,, \qquad \lambda = g_\mathrm{YM}^2 N$$

$$\mathrm{ABJM}: \quad g_\mathrm{s} = \frac{\sqrt{\pi}\,(2\lambda)^{5/4}}{N}\,, \qquad T = \frac{\sqrt{\lambda}}{\sqrt{2}}\,, \qquad \lambda = \frac{N}{k}$$

$\langle \mathcal{W} \rangle$ = disk partition function of string near AdS_2 minimal surface



$$ds^2 = \frac{L_{\rm ads}^2}{z^2}\left(dr^2 + r^2 d\phi^2 + dx_s dx_s + dz^2\right), \;\; r = \sqrt{1 - z^2}$$

$$\langle \mathcal{W} \rangle = Z_{\rm str} = \frac{1}{g_{\rm s}} Z_1 + \mathcal{O}(g_{\rm s}) \,, \qquad\qquad Z_1 = \int [dx] ... \, e^{-T\int d^2\sigma \, L}$$

$$\text{SYM}: \;\; \langle \mathcal{W} \rangle = \sqrt{\frac{2}{\pi}} \frac{N}{\lambda^{3/4}} \, e^{\sqrt{\lambda}} + ... \; = \frac{1}{2\pi} \, \frac{\sqrt{T}}{g_{\rm s}} \;\, e^{2\pi T} + ...$$

$$\text{ABJM}: \;\; \langle \mathcal{W} \rangle = \frac{N}{4\pi\lambda} \, e^{\pi\sqrt{2\lambda}} + ... \; = \frac{1}{\sqrt{2\pi}} \, \frac{\sqrt{T}}{g_{\rm s}} \;\, e^{2\pi T} + ...$$

universal form at strong coupling [Giombi, AT 2020]

$$\langle \mathcal{W} \rangle = c_0 \, \frac{\sqrt{T}}{g_s} \, e^{2\pi T} \Big[1 + \mathcal{O}(T^{-1}) \Big] + \mathcal{O}(g_s)$$

$$c_0 = \frac{1}{(\sqrt{2\pi})^{d-3}} \,, \qquad d = 5, \, 4$$

reason: dual string theories in $\mathrm{AdS}_n \times M^{10-n}$ have similar structure

- $e^{2\pi T} = e^{-T\mathrm{vol}(\mathrm{AdS}_2)}, \qquad \mathrm{vol}(\mathrm{AdS}_2) = -2\pi$

[Berenstein, Corrado, Fischler, Maldacena 98]

- $\sqrt{T}$ from universal dependence of Z_1 on $L_{\rm ads}$
- $\mathrm{c}_0 = \frac{1}{(\sqrt{2\pi})^{d-3}} = \frac{1}{\sqrt{2\pi}}\,\bar{\mathrm{c}}_0, \quad Z_1 \sim \bar{\mathrm{c}}_0$

[Drukker, Gross, AT 00; Kruczenski, Tirziu 08; Buchbinder, AT 14; ...]

- extra $\frac{1}{\sqrt{2\pi}}$: sensitive to defn of GS

path integral measure; implicitly checked in ratio of $\frac{1}{2}$ and $\frac{1}{4}$ BPS WL's [Medina-Rincon, Zarembo, AT 18]

• will be fixed below in ABJM case
by quantum M2 brane computation

1-loop superstring partition function
in $AdS_d \times M^{10-d}$
near AdS_2 minimal surface

$$\log Z_1 = -\frac{1}{2} \log \frac{[\det(-\nabla^2 + 2)]^{d-2} \, [\det(-\nabla^2)]^{10-d}}{[\det(-\nabla^2 + \frac{1}{2})]^{2d-2} \, [\det(-\nabla^2 - \frac{1}{2})]^{10-2d}}$$

$$\log Z_1 = B_2 \log(L_{\rm ads}\,\Lambda) + \log \bar{c}_1\,, \qquad B_2 = \frac{1}{4\pi}\int d^2\sigma\,\sqrt{g}\,R^{(2)} = \chi$$

$B_2 = \zeta_{\rm tot}(0) = \chi$: universal dep. on $L_{\rm ads}$

Λ dependence should cancel

against GS string measure: $\Lambda \to \frac{1}{\sqrt{\alpha'}}$

$$Z_1 \sim (\sqrt{T})^{\chi}\,, \qquad (Z_1)_{\rm disk} \sim \sqrt{T}\,, \qquad T = \frac{L^2_{\rm ads}}{2\pi\alpha'}$$

- from dets on disk ($d = 5,\ 4$): $\bar{c}_0 = \frac{1}{(\sqrt{2\pi})^{d-4}}$

Higher genus corrections: $\chi = 1 - 2h$

• disk with h handles: $g_s^{-1} \to g_s^{\chi}, \ \sqrt{T} \to (\sqrt{T})^{\chi}$

• thus prediction on string side:

$$\langle \mathcal{W} \rangle = e^{2\pi T} \sum_{h=0}^{\infty} \mathrm{c}_h \left(\frac{g_s}{\sqrt{T}} \right)^{2h-1} \left[1 + \mathcal{O}(T^{-1}) \right]$$

remarkably, consistent with form of $\frac{1}{N}$ terms

- SYM: $N \gg 1$, then $\lambda \gg 1$

$$\langle \mathcal{W} \rangle = e^{\frac{(N-1)\lambda}{8N^2}} L^1_{N-1}(-\tfrac{\lambda}{4N}) = e^{\sqrt{\lambda}} \sum_{h=0}^{\infty} \tfrac{\sqrt{2}}{96^h \sqrt{\pi}\, h!} \frac{\lambda^{\frac{3}{4}(2h-1)}}{N^{2h-1}} \left[1 + \mathcal{O}(\tfrac{1}{\sqrt{\lambda}})\right]$$

- $\frac{g_s}{\sqrt{T}} \sim \frac{\lambda^{\frac{3}{4}}}{N}$ appears as expansion parameter
- for gauge-theory: $c_h = \frac{1}{2\pi\, h!} \left(\frac{\pi}{12}\right)^h$, $c_0 = \frac{1}{2\pi}$

large $T = \frac{\sqrt{\lambda}}{2\pi}$ terms at each order in $g_s = \frac{\lambda}{N}$

exponentiate: [Drukker, Gross]

$$\langle \mathcal{W} \rangle = W_1 \, e^H \left[1 + \mathcal{O}(T^{-1}) \right], \qquad W_1 = \frac{1}{2\pi} \frac{\sqrt{T}}{g_s} \, e^{2\pi T}$$

$$H \equiv \frac{\pi}{12} \frac{g_s^2}{T} = \frac{1}{96\pi} \frac{\lambda^{3/2}}{N^2}$$

conjectured interpretation: "handle operator"

computing even 1-loop string term is challenge

but will derive analog of $e^{\frac{\pi}{12} \frac{g_s^2}{T}}$ in ABJM!

$1/N$ strong-coupling expansion of $\frac{1}{2}$ BPS circular WL in ABJM

• above: in both $AdS_5 \times S^5$ and $AdS_4 \times CP^3$

universal form of expansion in small g_s, large T

$$\langle \mathcal{W} \rangle = e^{2\pi T} \frac{\sqrt{T}}{g_s} \Big(c_0 + O(T^{-1}) + \frac{g_s^2}{T} \big[c_1 + O(T^{-1}) \big] + \big(\frac{g_s^2}{T} \big)^2 \big[c_2 + ... \big] + ... \Big)$$

• ABJM: $\frac{g_s^2}{T} \sim \frac{\lambda^2}{N^2} = \frac{1}{k^2}$, corrections $T^{-1} \sim \frac{\sqrt{k}}{\sqrt{N}}$

• exponentiation of leading terms? no

localization: they summed by $\frac{1}{\sin\frac{2\pi}{k}}$: [Beccaria, AT 20]

$$\langle \mathcal{W} \rangle = \frac{1}{2\sin\frac{2\pi}{k}}\, e^{\pi\sqrt{\frac{2N}{k}}}\Big[1+\mathcal{O}(\tfrac{1}{\sqrt{N}})\Big]$$

$$= \frac{1}{2\sin\Big(\sqrt{\frac{\pi}{2}}\,\frac{g_s}{\sqrt{T}}\Big)}\, e^{2\pi\, T}\Big[1+O(T^{-1})\Big], \qquad \sqrt{\frac{\pi}{2}}\,\frac{g_s}{\sqrt{T}} = 2\pi\,\frac{\lambda}{N} = \frac{2\pi}{k}$$

$$\frac{1}{2\sin\Big(\sqrt{\frac{\pi}{2}}\,\frac{g_s}{\sqrt{T}}\Big)} = \frac{\sqrt{T}}{\sqrt{2\pi}g_s}\,\Big[1+\frac{\pi}{12}\frac{g_s^2}{T}+\frac{7\pi^2}{1440}\Big(\frac{g_s^2}{T}\Big)^2+\ldots\Big]$$

Main claim: $\frac{1}{\sin\frac{2\pi}{k}}$ comes from

1-loop M2 contribution in M-theory [Giombi, AT 2023]

- large N, fixed k:

$\frac{1}{2}$ BPS WL described by M2-brane on $\mathrm{AdS}_2 \times S^1$

$e^{-S_{\rm M2}} = e^{\pi\sqrt{\frac{2N}{k}}}$ from classical M2 action

- 1-loop M2 correction $\to Z_1 = \frac{1}{\sin\frac{2\pi}{k}}$

• leading quantum M2 correction in $AdS_4 \times S^7/\mathbb{Z}_k$
describes large T terms at all orders in g_s
in type IIA string theory on $AdS_4 \times CP^3$, i.e.
gives all c_h coeffs in string genus expansion

• highly non-trivial check of
AdS_4/CFT_3 duality at all orders in $1/N$

Review of basic relations

$$S_{11} = \frac{1}{2\kappa_{11}^2} \int d^{11}x \sqrt{-G} \left(R - \frac{1}{2 \cdot 4!} F_{mnk\ell} F^{mnk\ell} + \cdots \right)$$

M2 action in 11d background [Bergshoeff,Sezgin,Townsend 87]

$$S_{\rm M2} = T_2 \int d^3\sigma \left[\sqrt{-\det g_{mn}} + \hat{C}_3 \right]$$

$$g_{mn} = G_{MN}(x)\, \Pi_m^M \Pi_n^N + \ldots , \;\; \hat{C}_3 = \tfrac{1}{6} \epsilon^{mnk}\, C_{MNK}(x)\, \Pi_m^M \Pi_n^N \Pi_k^K$$

$$\Pi_m^M = \partial_m x^M - i\bar{\theta}\Gamma^M \partial_m \theta \, , \qquad x^M = x^M(\sigma)$$

$$2\kappa_{11}^2 = (2\pi)^8\,\ell_P^9\,, \qquad\qquad T_2 = (\frac{2\pi^2}{\kappa_{11}^2})^{1/3} = \frac{1}{(2\pi)^2\ell_P^3}$$

$$ds_{11}^2 = e^{-\frac{2}{3}\phi}\,ds_{10}^2 + e^{\frac{4}{3}\phi}(dx_{11} + e^{-\phi}A)^2, \qquad x_{11} \sim x_{11} + 2\pi\bar{R}_{11}$$

$$g_{\rm s} = e^{\phi}; \quad R_{11} = g_{\rm s}^{2/3}\bar{R}_{11}; \quad 2\kappa_{10}^2 = (2\pi)^7 g_{\rm s}^2\alpha'^4$$

- M2 wrapped on $x_{11} \to$ string [Duff, Howe, Inami, Stelle 87]

$$T_2 = \frac{1}{(2\pi)^2\, \ell_P^3}, \qquad T_1 = 2\pi\, \bar{R}_{11}\; T_2 = \frac{1}{2\pi\alpha'}$$

- 11d M2-brane solution [Duff, Stelle 90] $\to \mathrm{AdS}_4 \times S^7$

$$ds_{11}^2 = \mathrm{L}^2 \Big(\frac{1}{4} ds^2_{\mathrm{AdS}_4} + ds^2_{S^7} \Big), \quad F_4 = dC_3 \sim \hat{\mathrm{N}}\, \epsilon_4, \quad \big(\frac{\mathrm{L}}{\ell_P} \big)^6 = 32\pi^2 N$$

- M2 on orbifold: $\mathrm{AdS}_4 \times S^7/\mathbb{Z}_k, \qquad N \to Nk$

- S^7 as S^1 fibration over CP^3 and $\mathbb{Z}_k$ quotient

$$ds^2_{S^7} = ds^2_{\mathrm{CP}^3} + \frac{1}{k^2}(d\varphi + kA)^2\,, \qquad \varphi \equiv \varphi + 2\pi$$

$$ds^2_{\mathrm{CP}^3} = \tfrac{dw^s d\bar{w}^s}{1+|w|^2} - \tfrac{w_r \bar{w}_s}{(1+|w|^2)^2} dw^s d\bar{w}^r\,, \qquad dA = i[\tfrac{\delta_{sr}}{1+|w|^2} - \tfrac{w_s \bar{w}_r}{(1+|w|^2)^2}] dw^r \wedge d\bar{w}$$

$$R_{11} = g_{\mathrm{s}}^{2/3}\,\bar{R}_{11} = \frac{\mathrm{L}}{k}\,, \qquad \frac{\mathrm{L}}{\ell_P} = (2^5 \pi^2 N k)^{1/6}$$

$$ds^2_{10} = L^2\Big(\frac{1}{4} ds^2_{AdS_4} + ds^2_{\mathrm{CP}^3}\Big), \qquad L = g_{\mathrm{s}}^{1/3}\,\mathrm{L}$$

$$g_s = \left(\frac{\mathrm{L}}{k\,\ell_P}\right)^{3/2} = \frac{\sqrt{\pi}\,(2\lambda)^{5/4}}{N}\,, \qquad \lambda = \frac{N}{k}$$

$$T = \frac{L^2_{\text{ads}}}{2\pi\alpha'} = g_s^{2/3}\frac{\mathrm{L}^2}{8\pi\alpha'} = \frac{\sqrt{\lambda}}{\sqrt{2}}\,, \qquad \frac{g_s^2}{8\pi\,T} = \frac{\lambda^2}{N^2} = \frac{1}{k^2}$$

- M-theory expansion: $\frac{\mathrm{L}}{\ell_P} \gg 1$ or $T_2\mathrm{L}^3 \gg 1$
  or large N for fixed $k = 1, 2, ...$

• $\frac{1}{2}$ BPS WL: probe M2 brane intersecting AdS_4 boundary (multiple M2's) over line or circle

• compute M2 partition function for $\mathrm{L}^3 T_2 \gg 1$ compare to large N, fixed k expansion of $\langle \mathcal{W} \rangle$

• $\mathrm{AdS}_2 \times S^1$ M2 solution dual $\frac{1}{2}$-BPS Wilson loop: wrapping S^1_φ of $S^7/\mathbb{Z}_k$ and AdS_2 of AdS_4

$$S_{\mathrm{M2}} = \frac{1}{4}\mathrm{L}^3 T_2 \mathrm{vol}(\mathrm{AdS}_2)\, \frac{2\pi}{k} = -\pi\sqrt{\frac{2N}{k}}$$

$e^{-S_{\mathrm{M2}}}$ matches leading factor in $\langle \mathcal{W} \rangle$

1-loop M2 brane partition function

• expand M2 action near $AdS_2 \times S^1$ solution

fix 3d diff and κ- gauges $\rightarrow$ 8+8 3d fluctuations

• spectrum of fluctuations [Sakaguchi, Shin, Yoshida 2010]

static gauge: M2 coordinates

σ_1, σ_2 = AdS_2 directions; σ_3 = 11d φ

• KK expansion of 3d fields in $\sigma_3 = (0, 2\pi)$:

tower $(n = 0, \pm 1, ...)$ of B+F 2d fields on AdS_2

• bosonic fluctuations in 2 $\perp$ AdS_4 directions:
tower of complex scalars η_n

$$m^2_{\eta_n} = \tfrac{1}{4}(kn-2)(kn-4)\,, \qquad\qquad n \in \mathbb{Z}$$

• fluctuations of CP^3 directions:
tower of 3 complex ζ^s_n ($s = 1, 2, 3$)

$$m^2_{\zeta^s_n} = \tfrac{1}{4}kn(kn+2)\,, \qquad n \in \mathbb{Z}$$

• fermions: tower of 8 two-component spinors

$$m_{\vartheta^a_n} = \tfrac{1}{2}kn \pm 1 \ \ \text{(3+3 modes)}\,, \qquad m_{\vartheta^i_n} = \tfrac{1}{2}kn \ \ \text{(2 modes)}\,, \qquad n \in \mathbb{Z}$$

• string limit $k \to \infty$: $n \neq 0$ modes decouple
$n = 0$: same as 2d fluctuations around AdS_2
in IIA superstring on $AdS_4 \times CP^3$:
B: 2 of $m^2 = 2$;   6 of $m^2 = 0$;
F: 3+3 of $m = \pm 1$ and 2 of $m = 0$
• spectrum consistent with 2d susy:
combination of AdS_2 $\mathcal{N} = 1$ multiplets
scalar + Majorana fermion $m_B^2 = m_F(m_F - 1)$

- 1-loop M2 partition function on $\mathrm{AdS}_2 \times S^1$

$$Z_{\rm M2} = Z_1\, e^{-S_{\rm M2}} \Big[1 + \mathcal{O}\big(\frac{1}{\mathrm{L}^3 T_2}\big)\Big]\,, \qquad S_{M2} = -\frac{\pi}{k}\mathrm{L}^3 T_2$$

$$Z_1 = \prod_{n=-\infty}^{\infty} \mathcal{Z}_n\,, \qquad \mathcal{Z}_0 = \mathrm{AdS}_4 \times \mathrm{CP}^3 \text{ string on } \mathrm{AdS}_2$$

$$\mathcal{Z}_n = \frac{\Big[\det\Big(-\nabla^2 - \tfrac{1}{2} + (\tfrac{kn}{2}+1)^2\Big)\Big]^{\frac{3}{2}} \Big[\det\Big(-\nabla^2 - \tfrac{1}{2} + (\tfrac{kn}{2}-1)^2\Big)\Big]^{\frac{3}{2}} \det\Big(-\nabla^2 - \tfrac{1}{2} + (\tfrac{kn}{2}\Big)^2)}{\det\Big(-\nabla^2 + \tfrac{1}{4}(kn-2)(kn-4)\Big) \Big[\det\Big(-\nabla^2 + \tfrac{1}{4}kn(kn+2)\Big)\Big]^3}$$

compute dets by spectral zeta-function in AdS_2

[Drukker, Gross, AT 00; Buchbinder, AT 14]

$$\Gamma_1 = \tfrac{1}{2}\log\det(-\nabla^2 + m^2) = -\tfrac{1}{2}\zeta(0;m^2)\log\Lambda^2 - \tfrac{1}{2}\zeta'(0;m^2)$$

$$\zeta_B(0;m_B^2) = \frac{m_B^2}{2} + \frac{1}{6}\,, \quad \zeta'_B(0;m_B^2) = -\tfrac{1}{12}(1+\log 2) - \int_0^{m_B^2+\frac{1}{4}} dx\,\psi(\sqrt{x}+\tfrac{1}{2})$$

$$\zeta_F(0;m_F) = -\frac{m_F^2}{2} + \frac{1}{12}\,, \quad \zeta'_F(0;m_F) = -\tfrac{1}{6} + 2\log A + |m_F| + \int_0^{m_F^2} dx\,\psi(\sqrt{x})$$

• cancellation of log UV ∞ in $\Gamma_1 = -\log Z_1$:

$$\zeta_{\rm tot}(0) = \tfrac{1}{2} \sum_{n\in\mathbb{Z}} \big(-2+4\big) = \sum_{n\in\mathbb{Z}} 1 = 1 + 2\zeta_R(0) = 0$$

contribution of all $n \neq 0$ massive KK modes
cancels log UV div of $AdS_4 \times CP^3$ string ($n = 0$)
• cancellation was to be expected:
no 1-loop log UV div in 3d theory

- Z_1 is thus finite:

$$\Gamma_1 = -\log Z_1 = -\tfrac{1}{2}\zeta'_{\text{tot}}(0)\,, \qquad \zeta'_{\text{tot}}(0) = \sum_{n=-\infty}^{\infty} \zeta'_{\text{tot}}(0;n)$$

$$\begin{aligned}\zeta'_{\text{tot}}(0;n) = {} & 2\zeta'_{B}\big(0;\tfrac{1}{4}(kn-2)(kn-4)\big) + 6\zeta'_{B}\big(0;\tfrac{1}{4}kn(kn+2)\big) \\ & + 3\zeta'_{F}(0;\tfrac{kn}{2}+1) + 3\zeta'_{F}(0;\tfrac{kn}{2}-1) + 2\zeta'_{F}(0;\tfrac{kn}{2})\end{aligned}$$

- combining B and F: remarkable simplifications

- $n=0$ string contribution =0 [Giombi, AT 2020]; $n>0$:

$$\zeta'_{\rm tot}(0;n)+\zeta'_{\rm tot}(0;-n)=-2\log(\tfrac{k^2n^2}{4}-1)\,,\ kn>2$$

$$=\log\pi^2\,,\ kn=2;\ -\log\tfrac{9}{4}\,,\ kn=1$$

$$\Gamma_{1,k>2}=\sum_{n=1}^{\infty}\log\big(\tfrac{k^2n^2}{4}-1\big)=2\sum_{n=1}^{\infty}\log\tfrac{kn}{2}+\sum_{n=1}^{\infty}\log\big(1-\tfrac{4}{k^2n^2}\big)$$

$\zeta_R(0)=-\frac{1}{2}$, $\zeta'_R(0)=-\frac{\log(2\pi)}{2}$:
$2\sum_{n=1}^{\infty}\log\frac{kn}{2}=-\log\frac{k}{4\pi}$

- Euler's relation: $\sin \pi x = \pi x \prod_{n=1}^{\infty}(1 - \frac{x^2}{n^2})$

$$\sum_{n=1}^{\infty} \log\left(1 - \tfrac{4}{k^2 n^2}\right) = \log\Big[\prod_{n=1}^{\infty}\left(1 - \tfrac{4}{k^2 n^2}\right)\Big] = \log\left(\frac{k}{2\pi}\sin\frac{2\pi}{k}\right)$$

- final result for $k > 2$

$$Z_1 = e^{-\Gamma_1} = \frac{1}{2\sin\frac{2\pi}{k}}$$

- precise agreement with localization

- cases of $k = 1, 2$ require a separate treatment

$$\Gamma_1^{k=1} = \frac{1}{2}\log\frac{9}{4} - \frac{1}{2}\log\pi^2 + \sum_{n=3}^{\infty}\log\left(\frac{n^2}{4} - 1\right) = \log 4\,, \quad Z_1^{k=1} = \frac{1}{4}$$

$$\Gamma_1^{k=2} = -\frac{1}{2}\log\pi^2 + \sum_{n=2}^{\infty}\log\left(n^2 - 1\right) = 0\,, \qquad Z_1^{k=2} = 1$$

localization result for $\langle \mathcal{W} \rangle$ singular for $k = 1, 2$
may need reconsideration (susy $\mathcal{N} = 6 \to 8$)

Generalizations and open problems

- $\frac{1}{\sqrt{N}}$ corrections: from higher M2 loops

dimensionless expansion parameter: inverse of effective M2 brane tension $\frac{1}{\mathrm{L}^3 T_2} = \frac{\pi}{\sqrt{2k}} \frac{1}{\sqrt{N}}$

$$\langle \mathcal{W} \rangle = \frac{1}{2\sin\frac{2\pi}{k}} e^{\pi\sqrt{\frac{2N}{k}}} \Big[1 - \frac{\pi(k^2+32)}{24\sqrt{2}\,k^{3/2}} \frac{1}{\sqrt{N}} + \mathcal{O}(\frac{1}{N})\Big]$$

$$= \frac{1}{2\sin\frac{2\pi}{k}} e^{\frac{\pi^2}{k}\mathrm{L}^3 T_2} \Big[1 - \frac{k^2+32}{24k} \frac{1}{\mathrm{L}^3 T_2} + \mathcal{O}(\frac{1}{(\mathrm{L}^3 T_2)^2})\Big]$$

• $\frac{1}{\sqrt{N}} \sim$ 2-loop M2 contribution

UV finite despite apparent non-renormalizability?

• analogy: GS string in $AdS_5 \times S^5$ is formally non-renormalizable but 2-loop $\frac{1}{\sqrt{\lambda}}$ correction to cusp anom dim is finite: $E - S = f(\lambda) \log S$

$f(\lambda) = a_0\sqrt{\lambda} + a_1 + \frac{a_2}{\sqrt{\lambda}} + ...$

[Roiban, AT 07; Giombi, Ricci, Roiban, Vergu, AT 10]

matches $f(\lambda)$ in SYM [Basso, Korchemsky, Kotanski 07]

• similar 2-loop result in $\mathrm{AdS}_4 \times \mathrm{CP}^3$ string case also UV finite [Bianchi, Bianchi, Bres, Forini, Vescovi 14]

[alternative? e.g. UV cutoff $\Lambda \sim {\ell_P}^{-1} \sim T_2^{-1/3}$ then log div term $\log(\mathrm{L}\Lambda) = \frac{1}{6}\log(Nk) + ...$ but no $\log N$ term in localization result for $\langle \mathcal{W} \rangle$]

• conjecture: div's cancel also at higher M2 loops

GS in $\mathrm{AdS}_5 \times S^5$ or $\mathrm{AdS}_4 \times \mathrm{CP}^3$ constrained by integrability; hidden symm also in M2 theory?

• compare to IIA string regime:

't Hooft expansion $N, k \gg 1$, $\lambda = \frac{N}{k}$=fixed

$$g_\mathrm{s} = \frac{\sqrt{\pi}\,(2\lambda)^{5/4}}{N}\,, \quad T = \frac{\sqrt{\lambda}}{\sqrt{2}}\,, \qquad \frac{1}{k} = \frac{1}{\sqrt{8\pi}}\,\frac{g_\mathrm{s}}{\sqrt{T}}$$

$\frac{1}{2\sin\frac{2\pi}{k}}$ sums all leading $\frac{g_\mathrm{s}}{\sqrt{T}}$ corrections

$\frac{1}{\mathrm{L}^3 T_2} = \frac{\sqrt{\pi}}{\sqrt{32}}\,\frac{g_\mathrm{s}}{T^{3/2}}$ terms give subleading

large tension corrections at each order in g_s

• generalization to $\frac{1}{6}$-BPS Wilson loop:
from localization [Klemmm et al 12]

$$\langle W_{\frac{1}{6}} \rangle = \frac{i}{2\sin\frac{2\pi}{k}} \sqrt{\frac{2N}{k}}\, e^{\pi\sqrt{\frac{2N}{k}}}(1+\dots)$$

origin of $\sqrt{\frac{2N}{k}}$ as in string case [Drukker, Plefka, Young 08]
[solution smeared over CP^1 in CP^3:
two 0-modes $(\sqrt{T})^2 \sim \sqrt{\lambda}$]
M2: should also be smeared: $\mathrm{L}^3 T_2 \sim \sqrt{N}$

• problem: study fluctuations to get prefactor

• explore defect CFT defined by $\frac{1}{2}$-BPS WL
(as in $\mathrm{AdS}_5 \times S^5$ or SYM case [Giombi, Roiban, AT 2017])
already studied in type IIA string regime

[Bianchi, Bliard, Forini, Griguolo, Seminara 20]

• M2-brane regime of large N, fixed k limit:
defect CFT interpretation of higher KK modes?
their boundary correlators?

• Lesson: take quantum M2 brane seriously
use it to derive strong coupling corrections to
non-BPS observables not known
from localization or integrability

• Example: cusp anom dim in ABJM
at strong coupling beyond planar limit [Giombi, AT]

$$f(\lambda, N) = \sqrt{2\lambda} - \frac{5}{2\pi}\log 2 + q_1(k) + \mathcal{O}(\frac{1}{\sqrt{\lambda}})$$

$$q_1 = \frac{2\pi}{3k^2} + \frac{2\pi^3}{45k^4} + ... = \frac{2\pi\lambda^2}{3N^2} + \frac{2\pi^3\lambda^4}{45N^4} + ...$$

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