SKA COSMOLOGY SWG MEETING 2023

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Quantifying and mitigating the effect of snapshot intervals in the light-cone EoR 21-cm simulation

by

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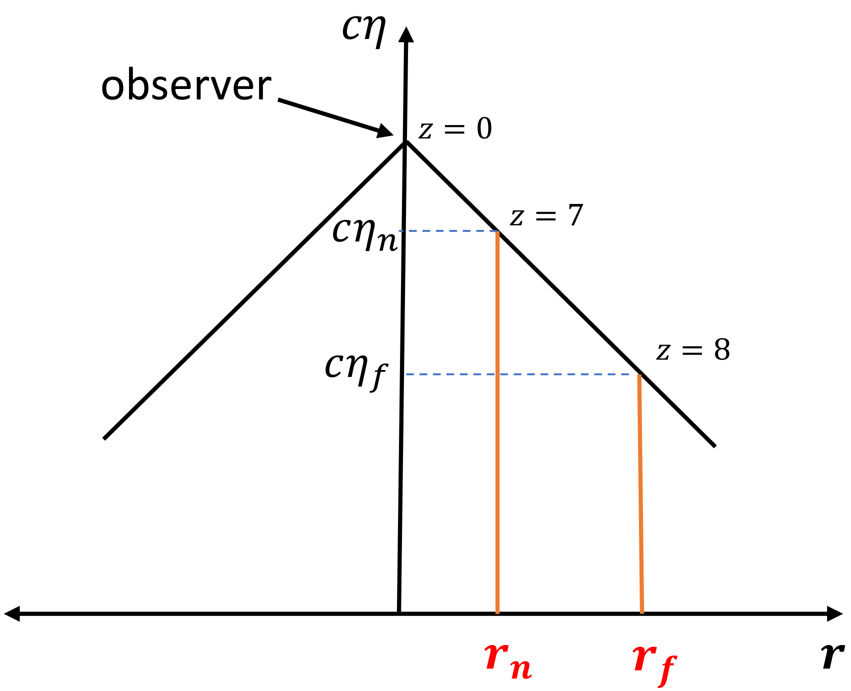
Lightcone effect



- It is the fact that our view of the universe is restricted through a backward light-cone which can be written as

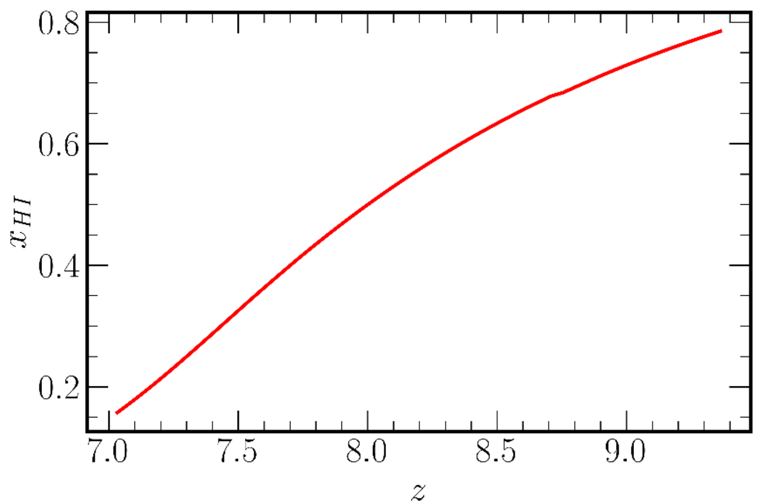
$$r = c(\eta_0 - \eta)$$

Comoving distance

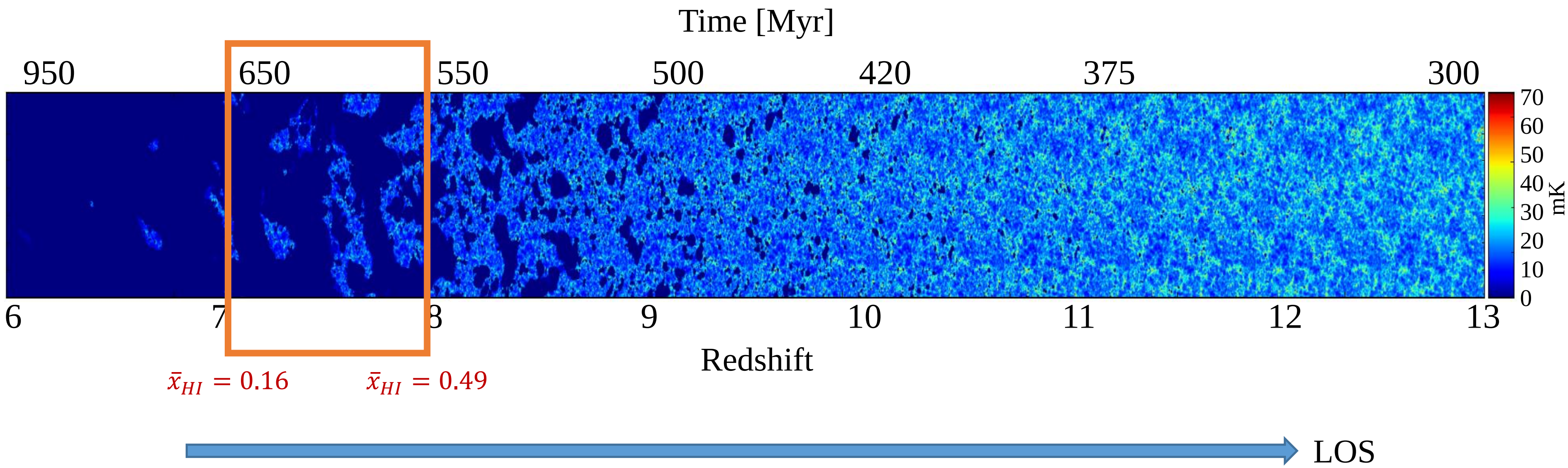
Present epoch

Epoch at the source

- Lightcone effect is significant during EoR as mean neutral fraction changes rapidly during this epoch.



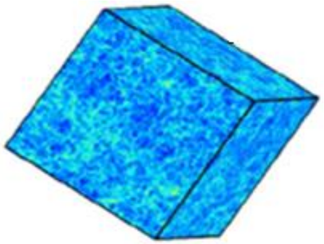
Epoch of Reionization



- A simulated box of comoving size $(286.7\ Mpc)^3$, centered around redshift $z_c = 7.46$ extends from $z = 7.03$ to 7.91 and the x_{HI} changes from 0.16 to 0.49 respectively.
- Neutral fraction, statistical properties of HI fluctuations changes substantially in the redshift range
- A simulated cube that captures redshift evolution of the signal termed as **'light cone' simulation.**

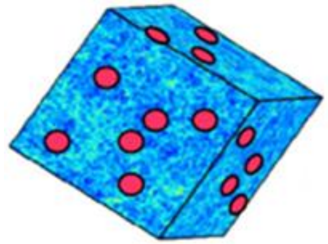
Simulating the EoR (Coeval Snapshots)

Dark matter distribution using N-body code



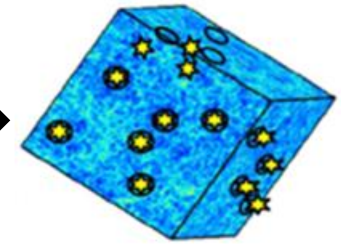
First step

Finding dark matter halos (FoF)

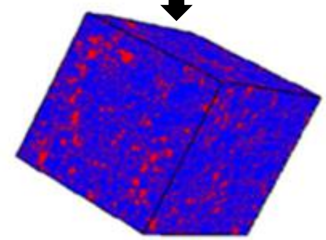


Second step (slowest)

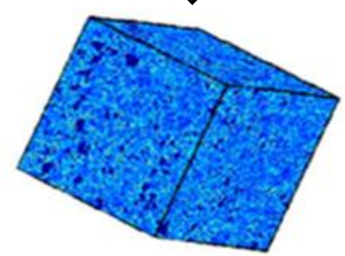
Reionization simulation (ReionYuga)



Illuminating dark matter halos



Comparing ionizing photon vs HI density on grid positions



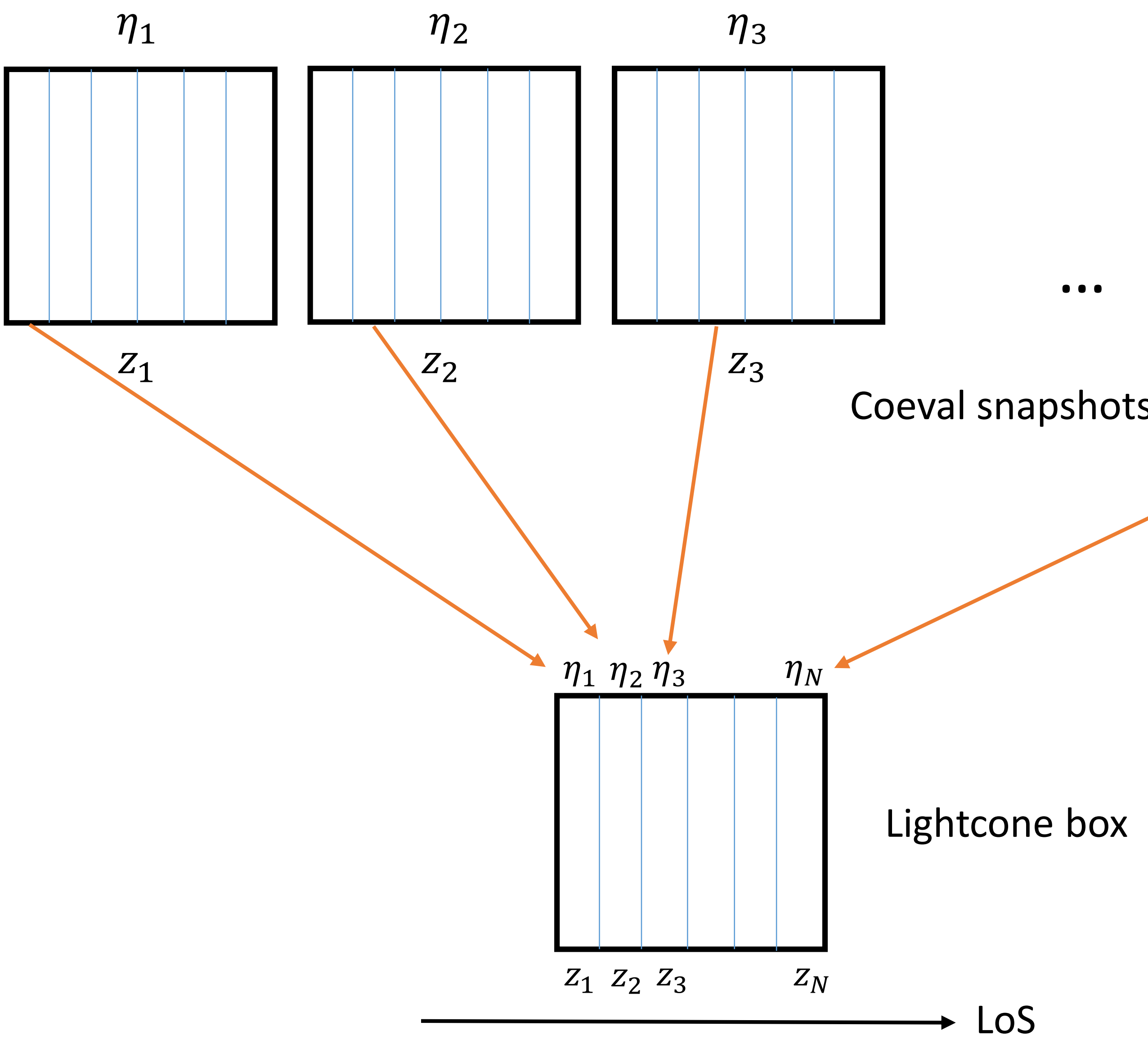
Simulating the 21-cm signal

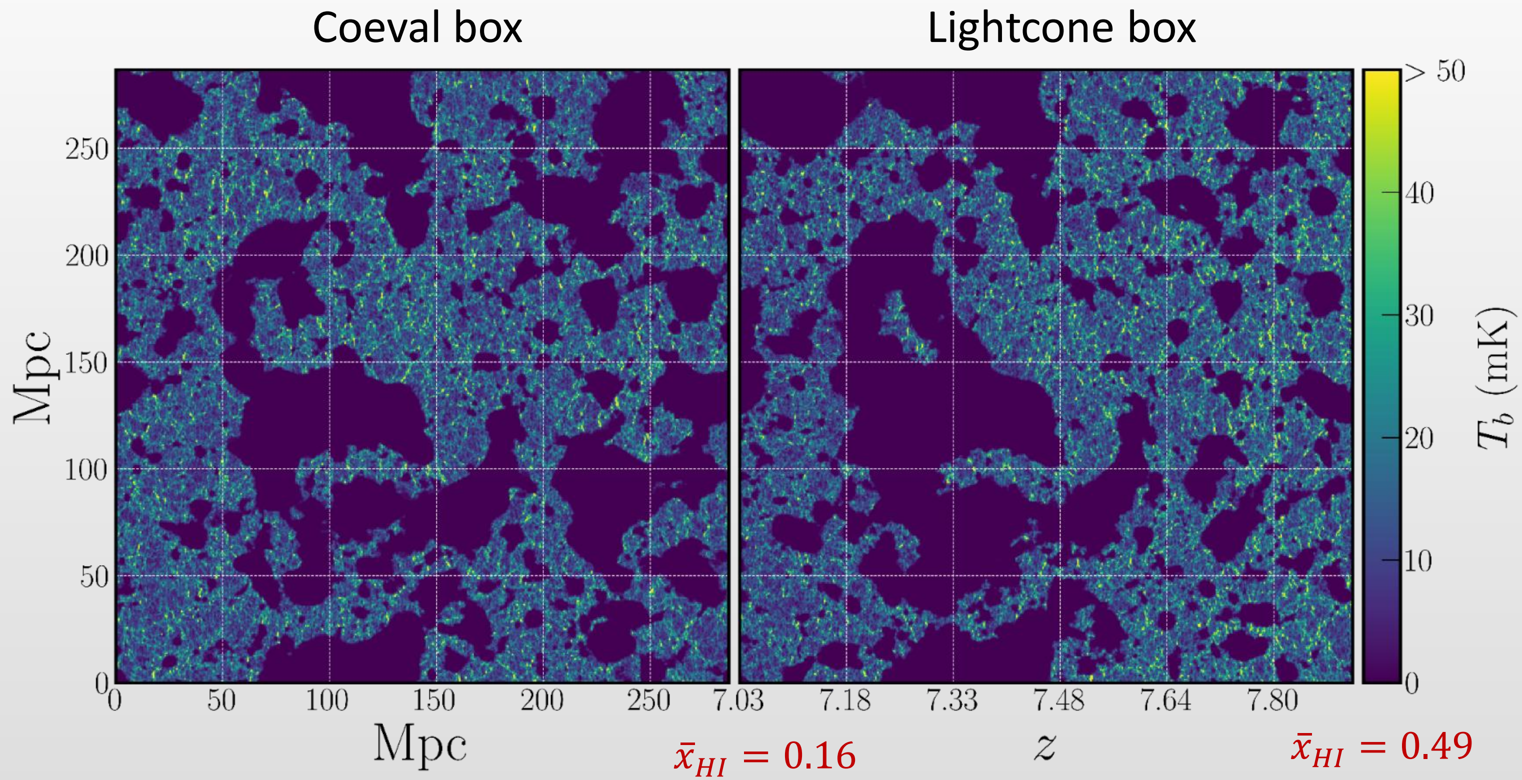
Third step

- Particle-mesh N-body (Bharadwaj & Srikant (2004))
- FoF (Mondal et al. (2015))
- ReionYuga (Choudhury et al. (2009), Majumdar et al. (2014), Mondal et al. (2016))

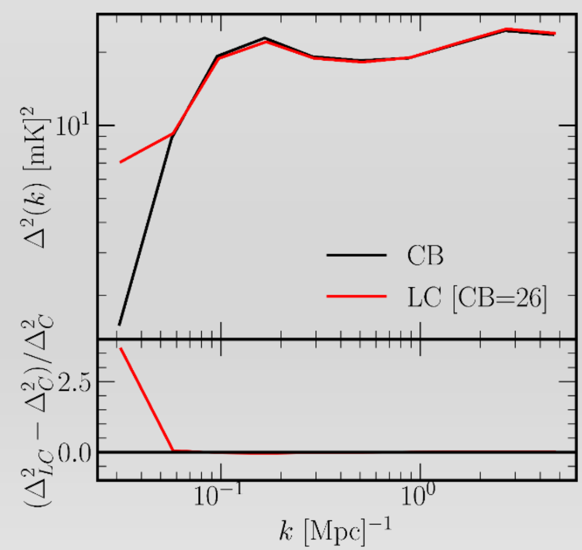
- All codes are publicly available at: https://github.com/rajeshmondal18/

Simulating the EoR lightcone



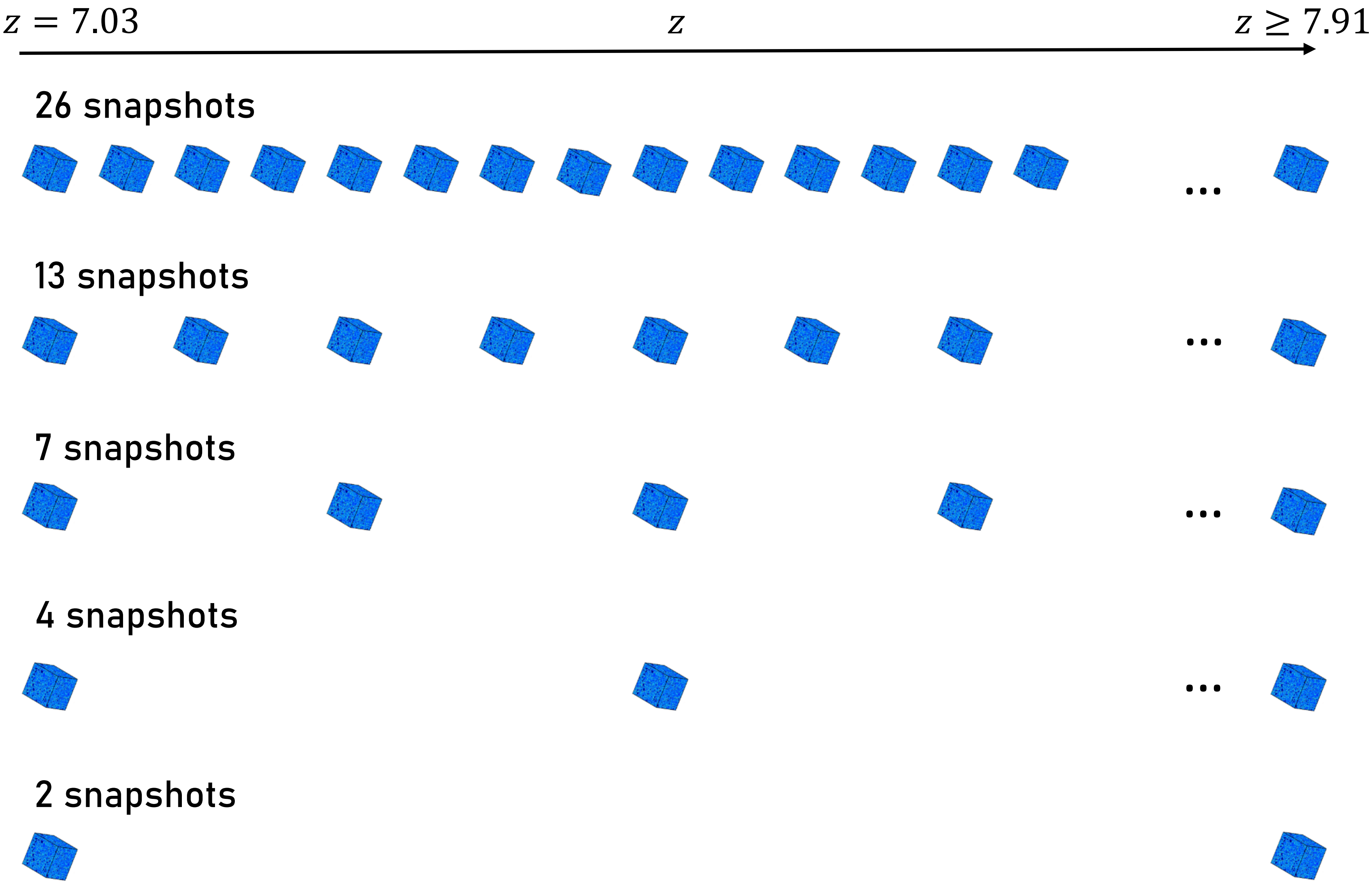


- Central redshift $z_c = 7.46$
- Central frequency $\nu_c = 167.9$ MHz
- Central commoving distance $r_c = 8986.4$ Mpc
- Bandwidth = 17.3 MHz
- Box size = $(286.7\ \mathrm{Mpc})^3$



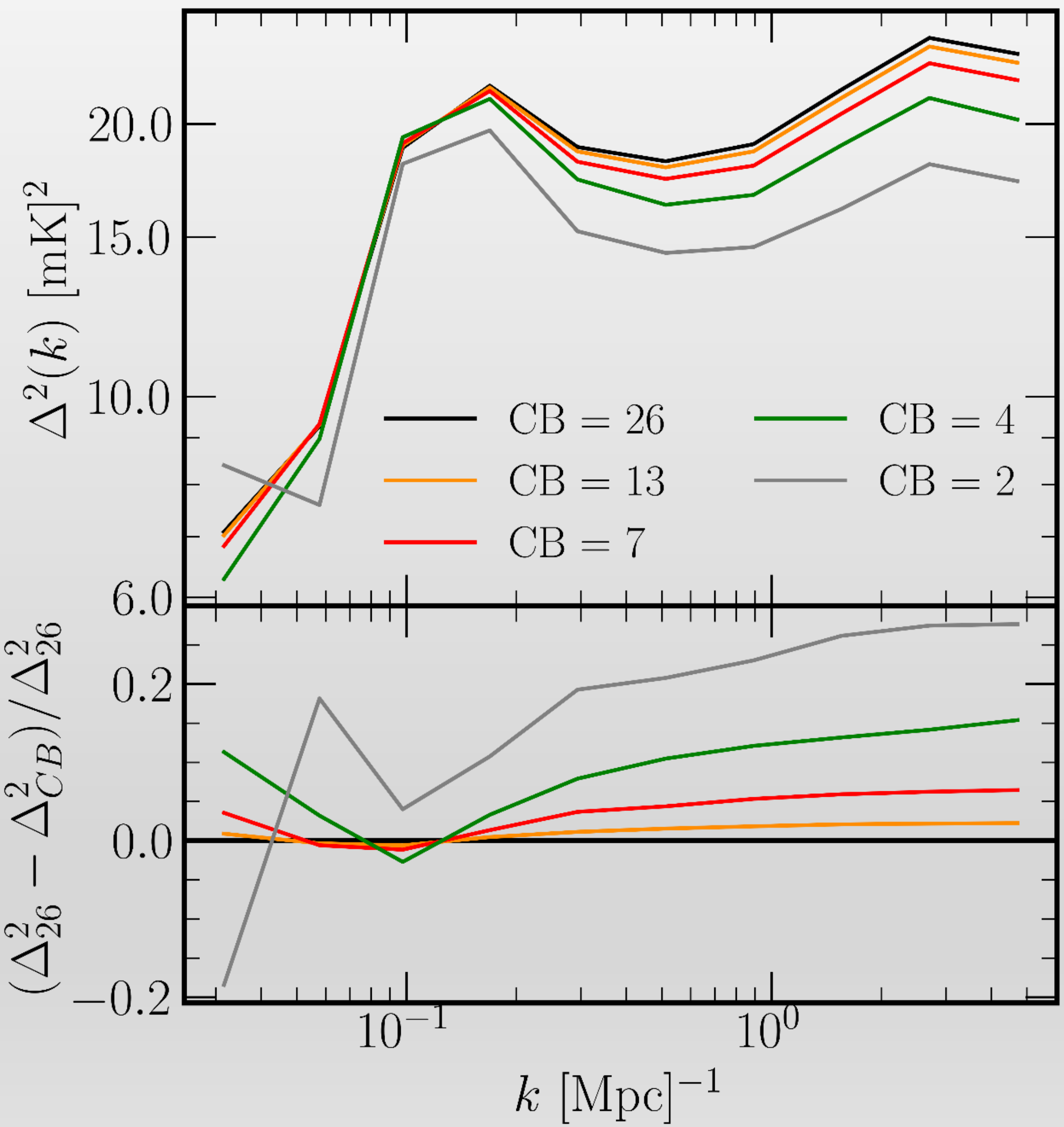
The issue is:

- The choice of snapshot intervals or the number of coeval snapshots for a particular lightcone is ad hoc.
- We want to quantify (also mitigate) the error in the simulated lightcones due to finite number of coeval snapshots used.



- All LC simulation are of same sizes $(286.7\ Mpc)^3$ and span $z_n = 7.03$ to $z_f = 7.91$

Spherically averaged power spectrum

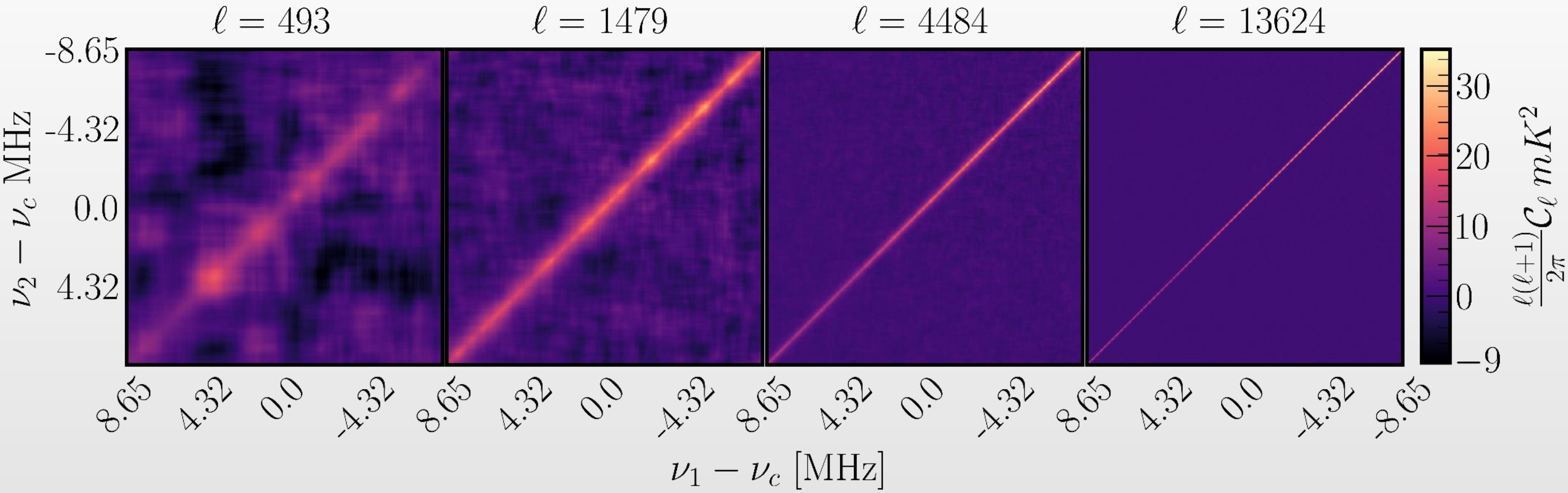


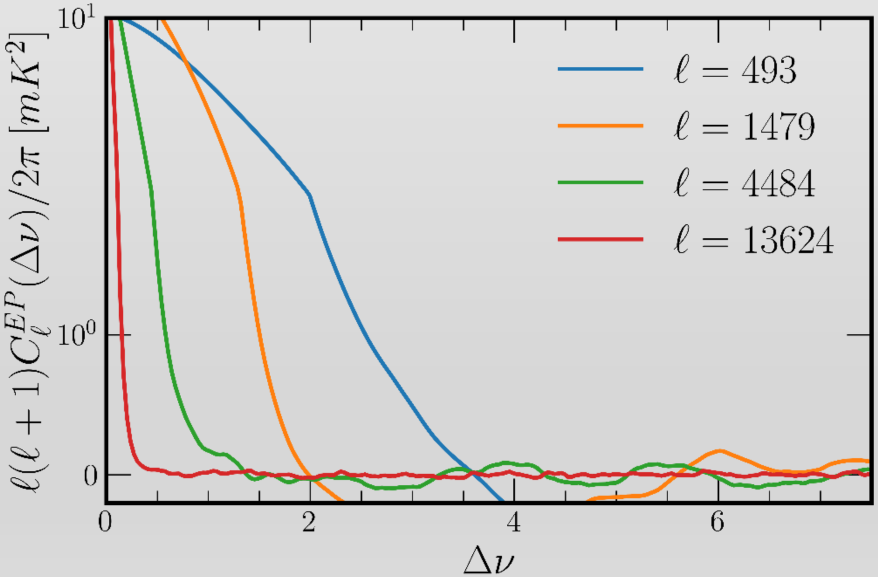
- The overall power decreases as we decrease no. of CB
- Largest scales are cosmic variance dominated
- At $k = 0.06$ the error is -0.003 for LC(CB=13) which reaches 0.18 for LC(CB=2)
- At $k = 2.7$ the errors are 0.02 and 0.27 respectively

The appropriate metric to quantify the error is Multifrequency Angular Power Spectrum (MAPS)

- Power spectrum cannot accurately quantify this error and gives biased estimate as it assumes statistical homogeneity and imposes periodicity on the signal which cannot be justified in the presence of LC effect (Trott 2016; Mondal et al. 2018).

- In contrast MAPS does not have any such intrinsic assumptions and an appropriate tool for our study.

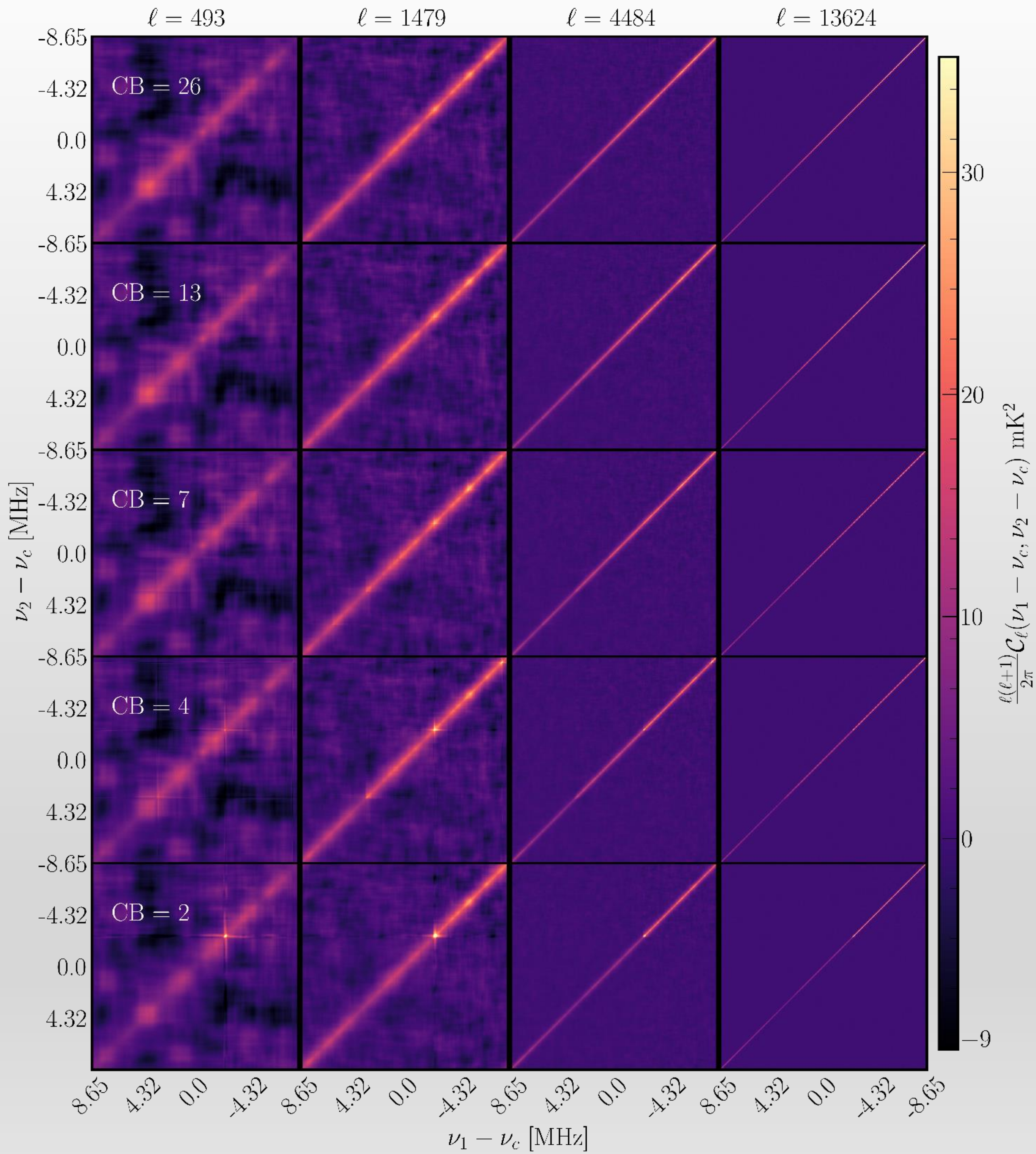




This shows dimensionless
MAPS
$l(l+1)C_l(\nu_1, \nu_2)/2\pi$
For lightcone simulation
(CB=26) at central frequency
$\nu_c = 167.9$ MHz
correspondind to a central
redshift $z = 7.46$

MAPS $[C_\ell(\nu_1, \nu_2)]$

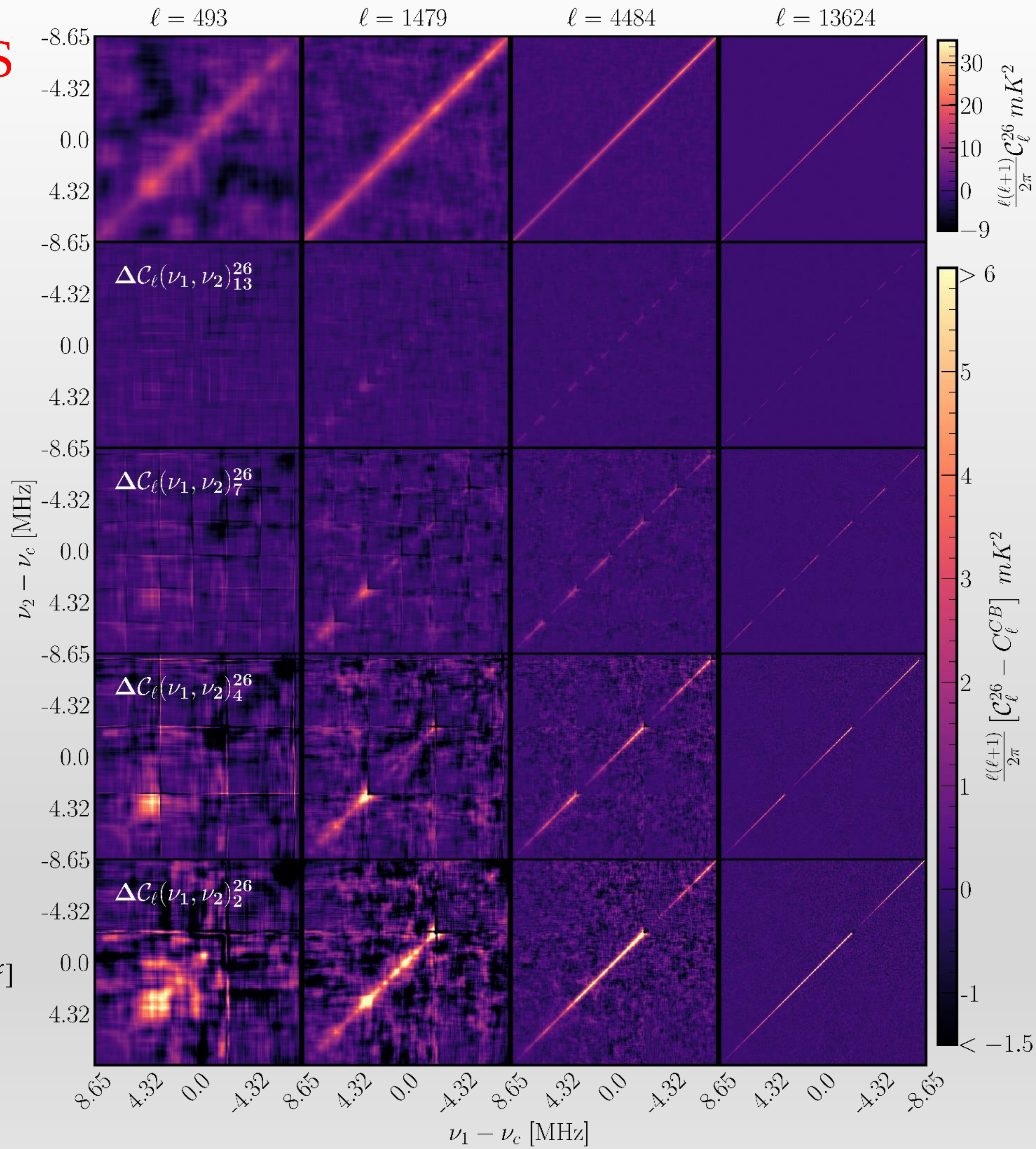
- The differences are not very clearly visible here.

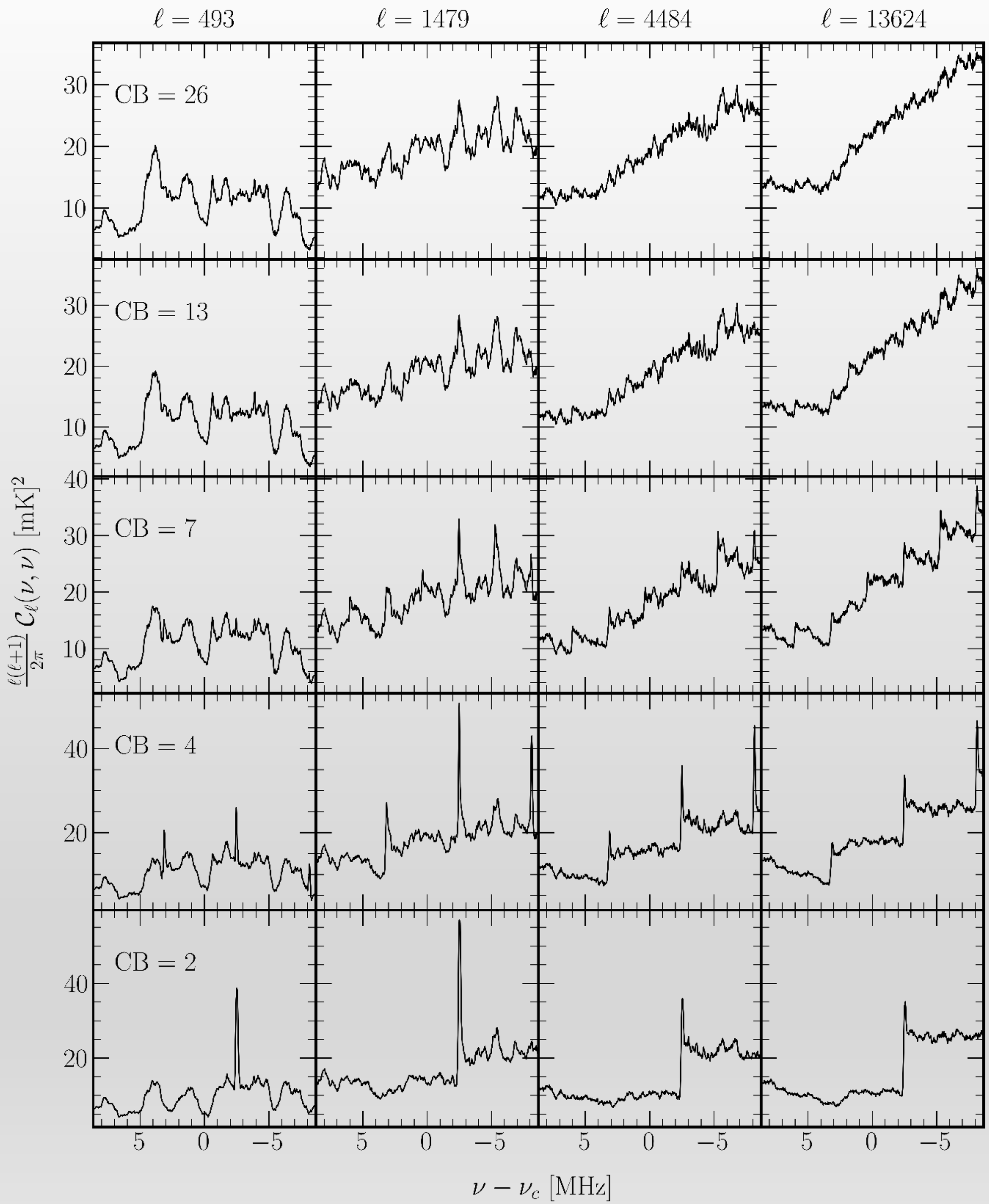


Differences of MAPS

- The differences are shown for other LC simulations w.r.t LC($CB = 26$).
- Patterns can be seen at the stitching boundaries of CBs.
- Differences are significant at all angular length scales

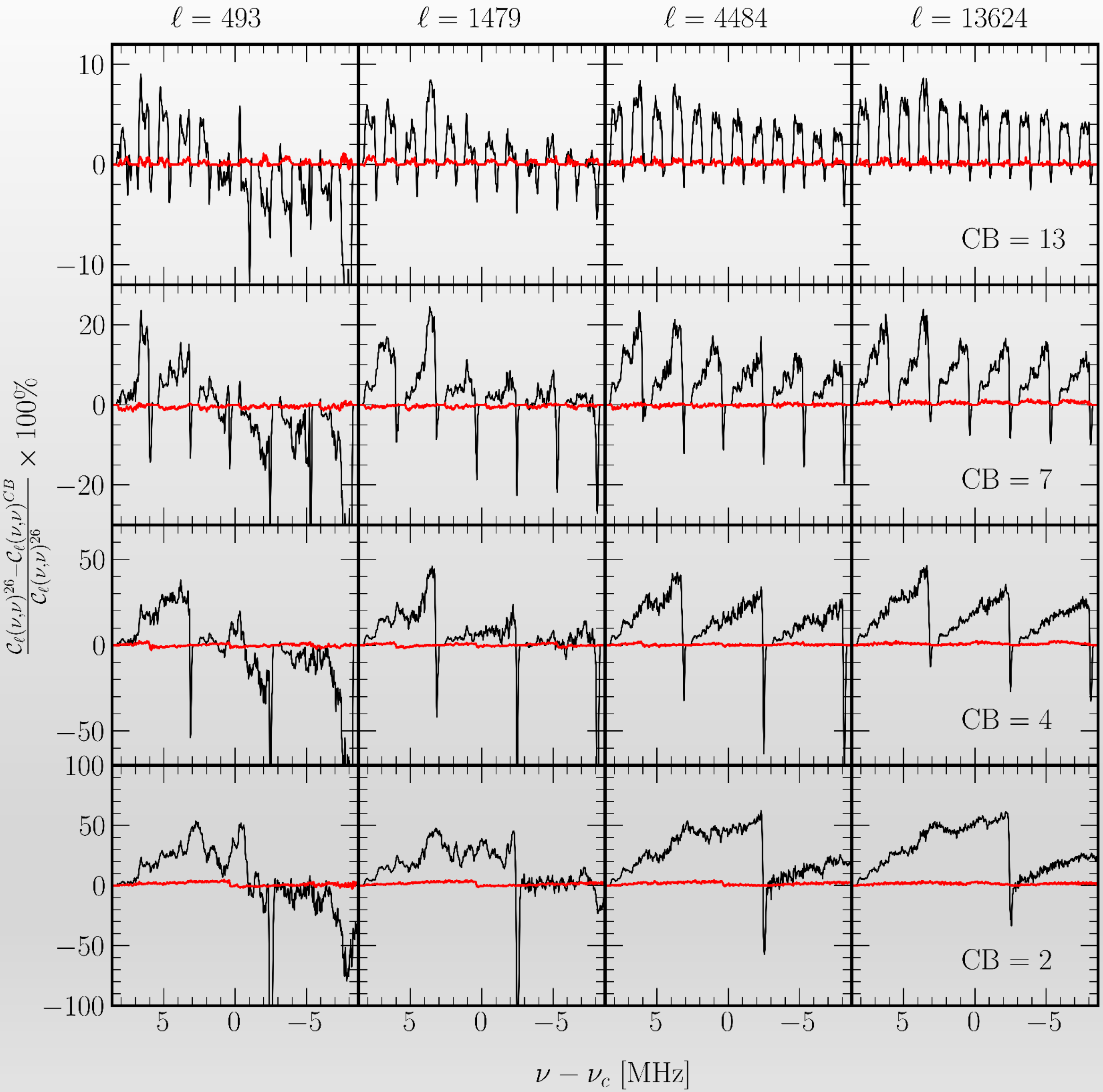
$$\Delta C_\ell(\nu_1, \nu_2)^{26}_x = \frac{\ell(\ell+1)}{2\pi}\left[C_\ell(\nu_1,\nu_2)^{26} - C_\ell(\nu_1,\nu_2)^x\right]$$





Diagonal element of MAPS $[C_\ell(\nu,\nu)]$

- Discontinuities are seen at the stitching boundaries of CB slices.
- This discontinuities are arising due to abrupt changes in average neutral fraction $(\bar{x}_{HI})$ at the stitching boundaries.
- The changes are smooth when number of CB is high.



$\ell = 493$
$\ell = 1479$
$\ell = 4484$
$\ell = 13624$
CB = 13
CB = 7
CB = 4
CB = 2
$\frac{C_\ell(\nu,\nu)^{26} - C_\ell(\nu,\nu)^{CB}}{C_\ell(\nu,\nu)^{26}} \times 100\%$
$\nu - \nu_c$ [MHz]

- One should use significantly large number of coeval snapshots to reduce the error.

- But, that demands huge computational resources in terms of RAM, storage, processing power and time.

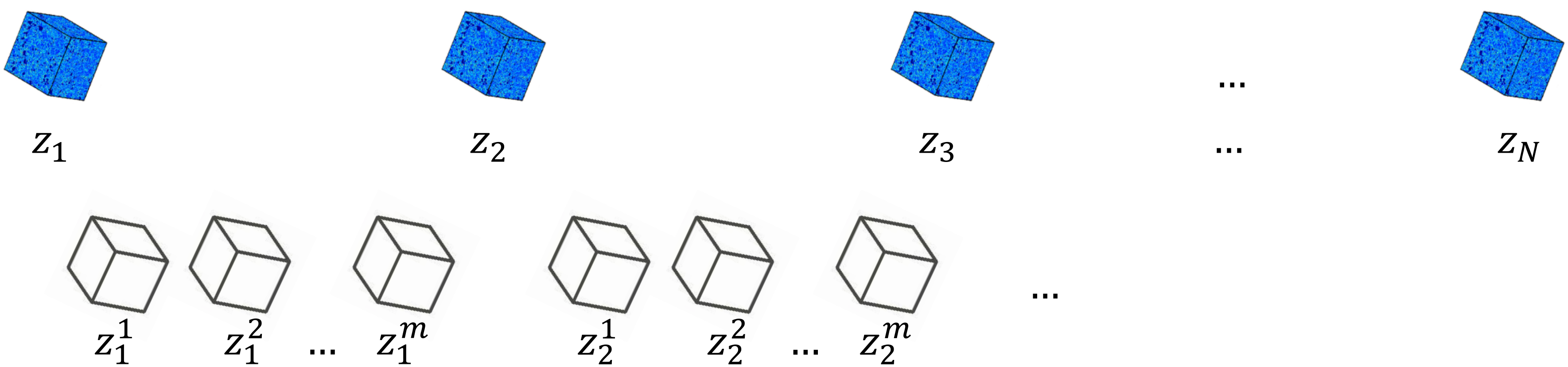
Introducing a new technique to mitigate the error

- The idea is to generate 'in-between' CB simulation using available CBs.
- Here we skip the 1st and 2nd steps of reionization simulation, which are the most time taking $(\sim\mathbf{97.5}\%)$ steps.
- 1st step is computing dark matter density field using N-body code (Writing the data on a typical disk takes 22.5% of the total time and for one z 201 GB space).
- 2nd step is identifying dark matter halos (75% of total time).
- Here we directly do the 3rd step and find the reionization map.

Introducing a new technique to mitigate the error

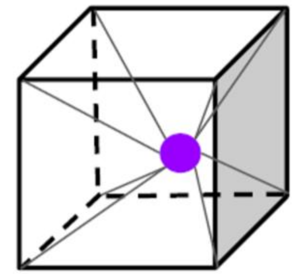
- Let us assume we have N number of available CB simulations within the bandwidth of the observation, at different z_i where i = 1, 2…, N

- Let us also assume we want to generate m number of CB simulations in-between every z_i and z_{i+1}.

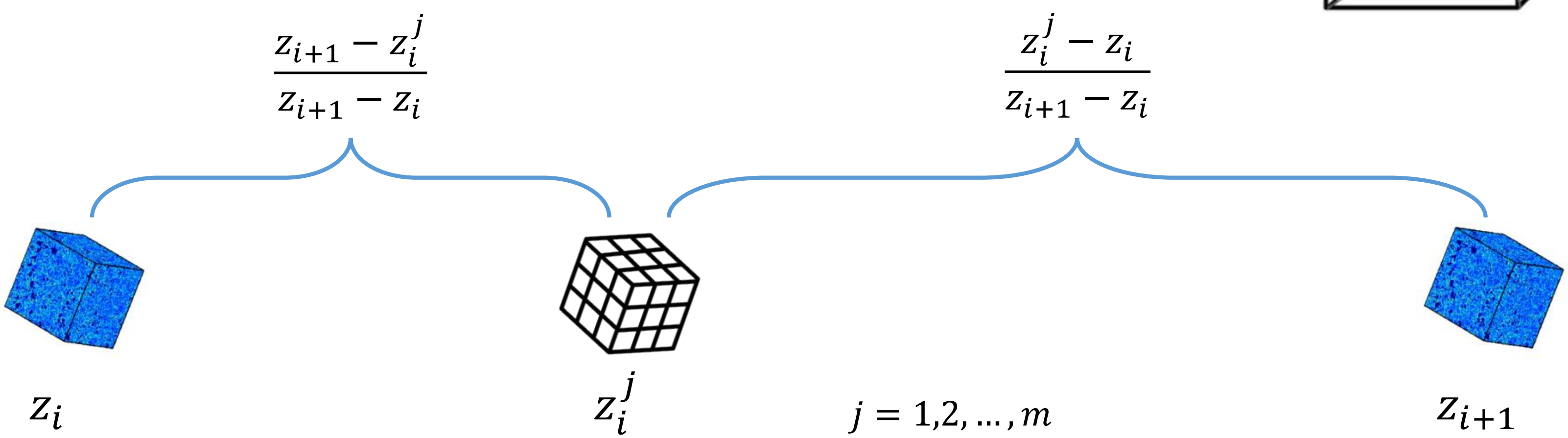


- Here we assume a linear cosmological evolution.

Introducing a new technique to mitigate the error

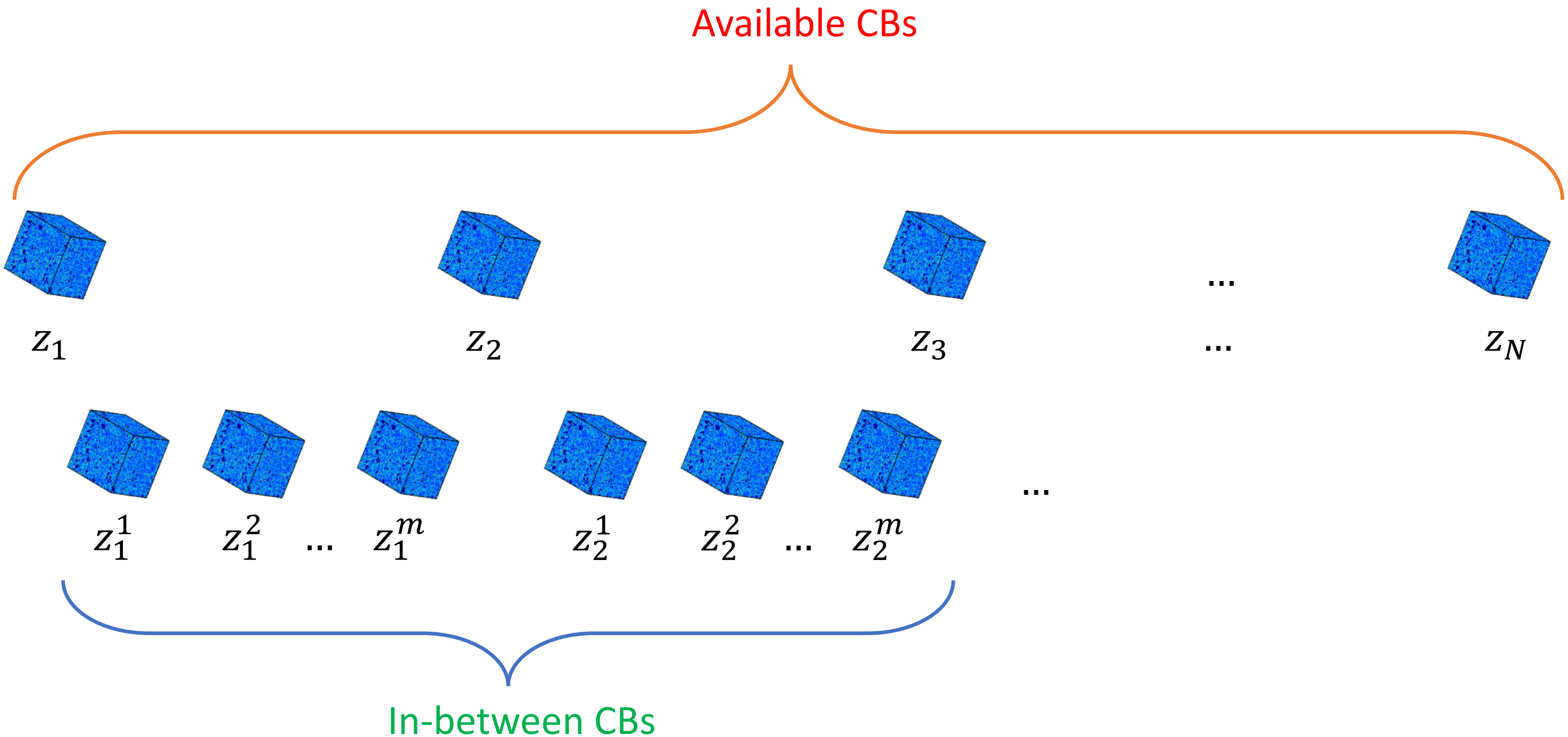


- We take the dark matter distributions and halo catalogues at z_i and z_{i+1} and calculate their contribution on grid positions (**griding**).
- We then calculate the weighted average of these dark matter distribution and halo catalogue at the desired in-between redshifts.

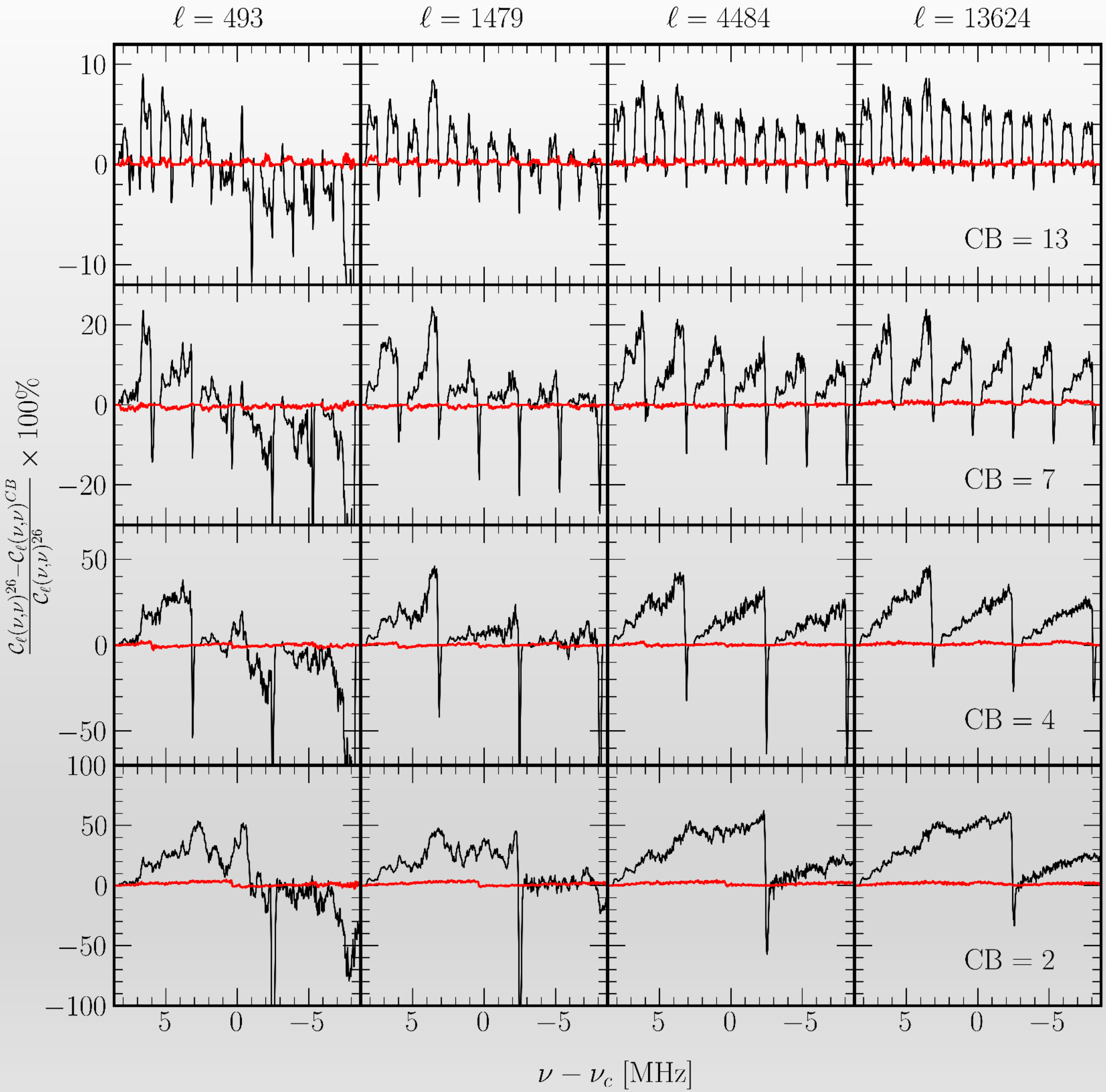


- The weight is taken in proportion to the redshift interval.
- These averaged DM density field and halo catalog are then pass in the third step of the reionization simulation to compute the final H I 21-cm brightness temperature maps.
- Particle positions and frequencies are directly taken from the CB at z_i. However, we have checked the same with CB at z_{i+1} which gives similar results.

Finally we have



This process increases the number of CB from N to $[N + m \times (N - 1)]$



$\ell = 493$
$\ell = 1479$
$\ell = 4484$
$\ell = 13624$
CB = 13
CB = 7
CB = 4
CB = 2
$\frac{\mathcal{C}_\ell(\nu,\nu)^{26} - \mathcal{C}_\ell(\nu,\nu)^{CB}}{\mathcal{C}_\ell(\nu,\nu)^{26}} \times 100\%$
$\nu - \nu_c$ [MHz]

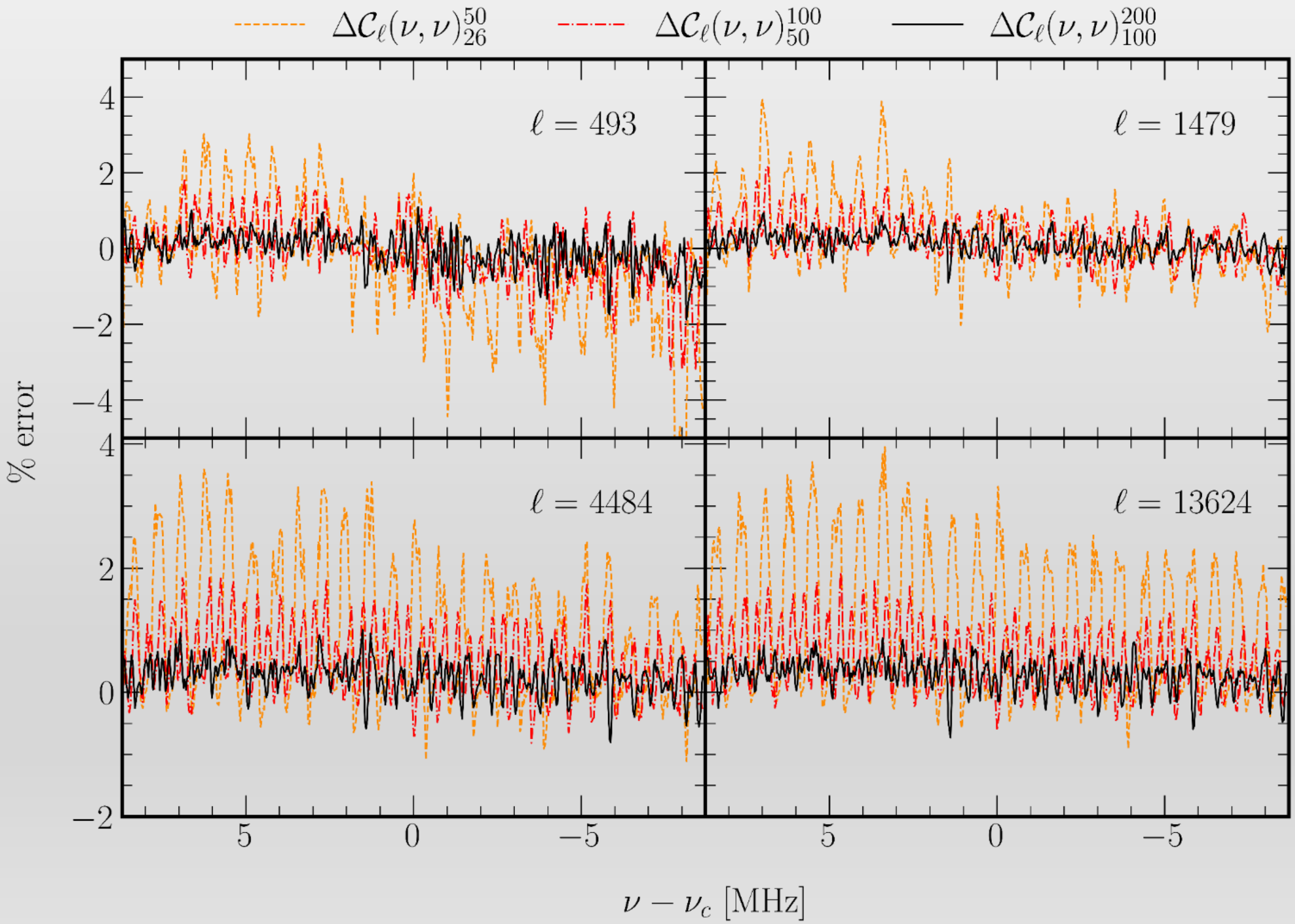
Average percentage errors in the $C_\ell(\nu, \nu)$ before and after mitigation.

	CB | ℓ	493	1479	4484	13624
Before mitigation	13	2.39	1.56	2.23	2.56
	7	6.79	4.73	6.78	7.44
	4	16.18	10.16	15.45	16.71
	2	22.77	18.09	26.51	28.78
After mitigation	13	0.18	0.19	0.15	0.11
	7	0.45	0.32	0.21	0.47
	4	0.65	0.52	0.40	0.81
	2	1.45	1.56	1.60	1.55

Pramanick et al. in prep.

- We use 26 available CBs and generate 1, 3 and 7 in-between CBs within every z_i and z_{i+1}

- This allow us to simulate lightcones using 50, 100 and 200 CBs respectively



Successive error reduces as we increase the number of CB

- This method is very effective to reduce the error in simulated lightcones.

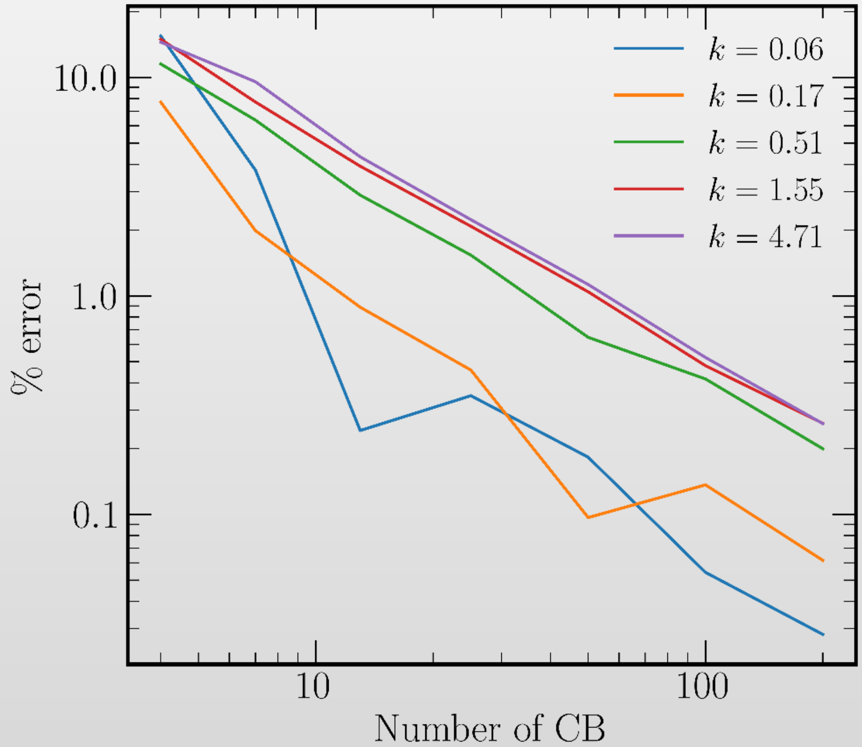
- For example, in traditional way (doing all three steps), one in-between CB simulation takes around 18 hrs time and around 240 GB space.

- 175 numbers of in-between CB simulation then would take $175 \times 18 = 3150\ hrs \approx$ 131 days and $240 \times 175 = 42000\ GB \approx 41\ TB$ space.

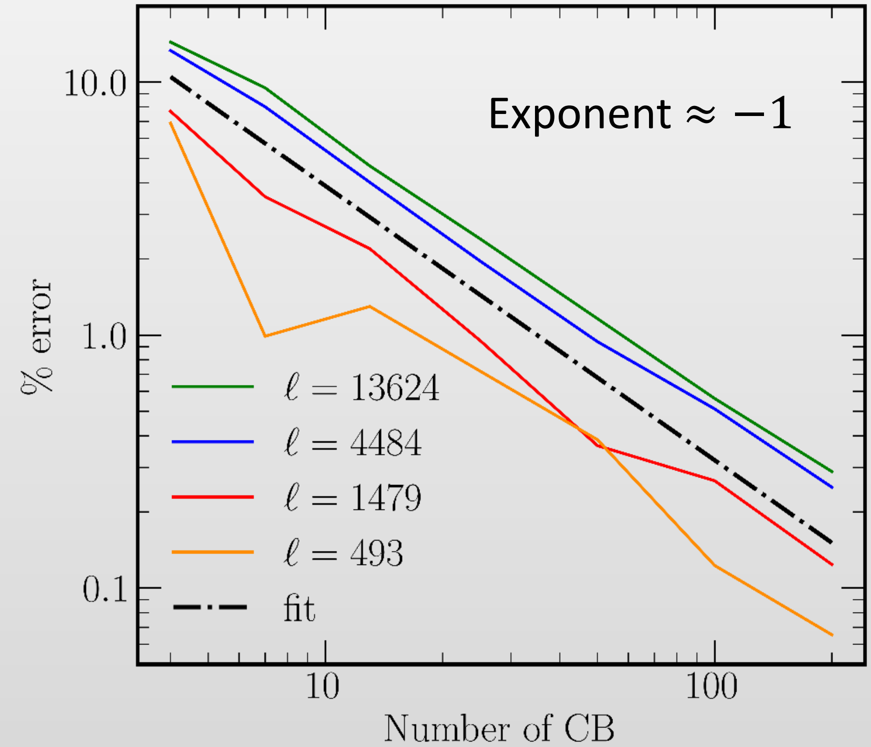
- This method generates 175 in-between CBs within < 2 days! And takes $\sim 3\ TB$ of space!

Average percentage error

For 3D power spectrum



For $C_\ell(\nu, \nu)$



$$\textbf{Average \% error in LC} = \frac{\mathbf{47.22}}{\textbf{No. of CB}}$$

Summary and Conclusion

- The error in the LC simulation is inversely proportional to the number of CB used.
- The error can be calculated using the relation:
  Average % error in $\mathrm{LC} = \frac{47.22}{\mathrm{No.of\ CB}}$
- The error can be reduced to a great extent assuming linear cosmological evolution to generate in-between CB simulations.
- This technique can be used to fasten the LC simulation pipeline which is highly effective to generate multiple realizations.

Thank You

Photo Credit: Ruobing Dong

Appendix

Simulating the Light-cone 21-cm signal from EoR

- For the 21-cm radiation originated from the point $\boldsymbol{n}r$, the cosmological expansion and the radial component of HI peculiar velocity $\boldsymbol{n}\,.\,\boldsymbol{v}\,(\boldsymbol{n}r,\eta)$ together determine the frequency ν at which the signal is observed, and we have

$$\nu = a(\eta)\left[1 - \boldsymbol{n}\,.\,\frac{\boldsymbol{v}(\boldsymbol{n}r,\eta)}{c}\right] \times \nu_e$$

- Assuming that spin temperature is much greater than the background CMB temperature, i.e $T_s \gg T_\gamma$, the HI 21-cm brightness temperature can be expressed as

$$T_b(\boldsymbol{n},\nu) = T_0\,\frac{\rho_{HI}}{\rho_H}\left(\frac{H_0\nu_e}{c}\right)\left|\frac{\partial r}{\partial \nu}\right|$$

Where, $T_0 = 4.0\;mK\,\left(\frac{\Omega_b h^2}{0.02}\right)\left(\frac{0.7}{h}\right)$

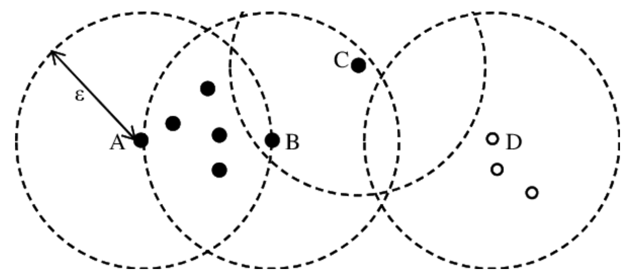
- The comoving HI density can be obtained by assigning the HI mass in the particles to a uniform rectangular grid in comoving space $\rho_{HI} = (\Delta r)^{-3}\sum_m [M_{HI}]_m$ where $(\Delta r)^3$ is the volume of each grid cell
- Here, we use a uniform grid in solid angle $(\Delta\Omega)$ and frequency $(\Delta\nu)$ to define a modified density

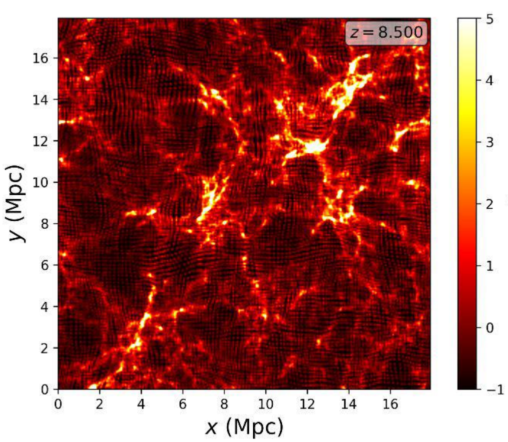
$$\rho'_{HI} = (\Delta\Omega\;\Delta\nu)^{-1}\left(\frac{H_0\nu_e}{c}\right)\sum_m \frac{[M_{HI}]_m}{r_m^2}$$

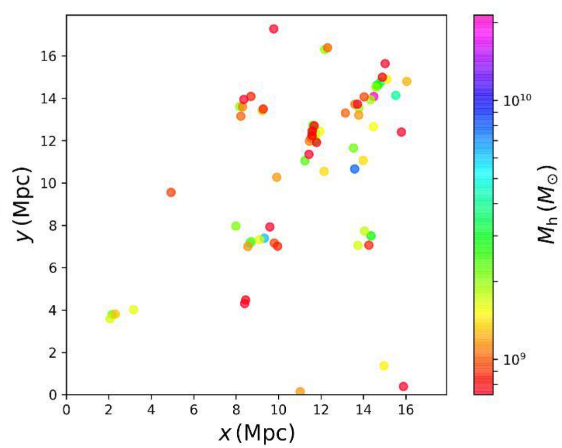
- Then we can calculate the brightness temperature using $T_b(\boldsymbol{n},\;\nu) = T_0\,\frac{\rho'_{HI}}{\rho_H}$

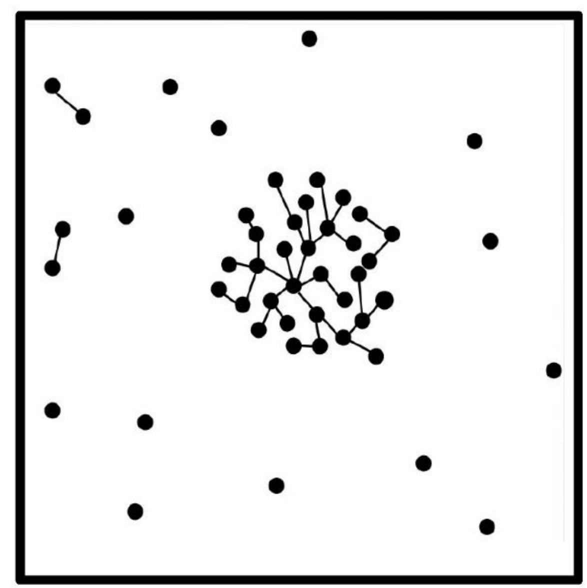
Steps of FoF halo finder

- DM are represented by discrete particles of mass = $1.09 \times 10^8 M_{sun}$
- Group of DM particles (gravitationally bound within close vicinity) forms halo
- We look for such group of DM particles within *N*-body outputs using Friends-of-Friends (FoF) algorithm
- We call a particle friend of another particle if they are within a certain distance which is **Fixed linking length** $L_{fof} = 0.2 \times$ Grid separation = 14 Kpc
- We call a grop halo if it contains more than a minimum number of DM particles M_{min} (Which is a parameter of the EoR model)





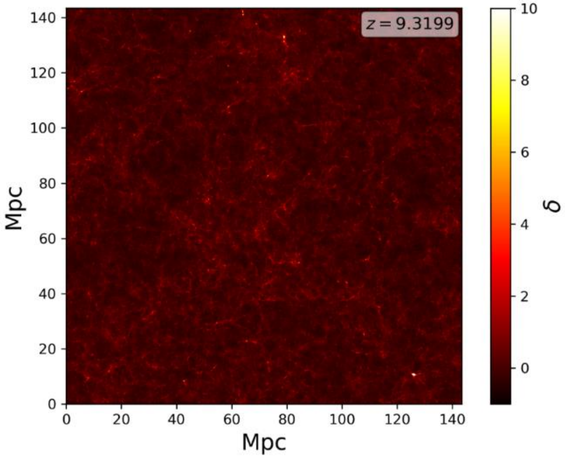


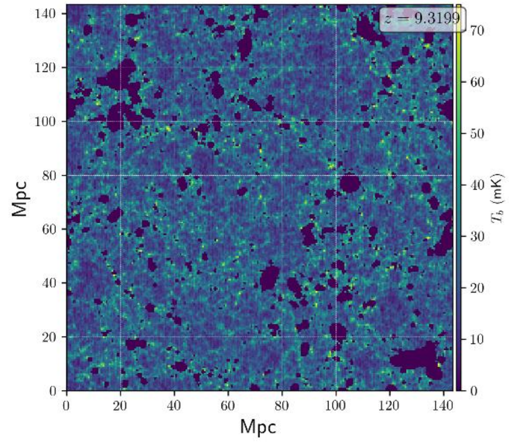


Generating EoR 21-cm signal (final step)

- From the N-body simulation we have DM density field, we assume that baryon will follow DM with some bias, these creates the neutral hydrogen (HI) field.
- From FoF we have dark matter halo locations and their masses.
- We illuminate those DM halo in proportion to their mass.
- So in the DM halo locations we have High photon number density (N_γ), then we smooth them using convolution with spherical tophat function.
- The smoothing radius vary from $R_{min} = grid\ size = 0.7$ Mpc to R_{mfp}
- R_{mfp} is the mean free path of the ionizing photon through IGM.
- We then compare the photon number (N_γ) and neutral hydrogen number (N_H) on a grid.
- A grid is fully ionized ($x_{HII} = 1$) is $N_\gamma > N_H$
- If $N_\gamma < N_H$ then $x_{HII} = \frac{N_\gamma}{N_H}$
- Main observable of EoR is the differential brightness temperature, which can be calculated using (Bharadwaj & Ali 2005)

$$\delta T_b(\boldsymbol{x}) = 27\, x_{HI}(z,\boldsymbol{x})[1+\delta_B(z,\boldsymbol{x})]\left(\frac{H}{\frac{dv_r}{dr}+H}\right)\left(\frac{\Omega_B h^2}{0.023}\right)\left(\frac{0.15}{\Omega_m h^2}\frac{1+z}{10}\right)^{\frac{1}{2}}\left[1-\frac{T_\gamma(z)}{T_S(z,\boldsymbol{x})}\right] mK$$





Redshift Space Distortion

- In the sky we can only observe the angular position ($\boldsymbol{\theta}$) of a source.
- If the source emits a known emission line (like 21-cm line from neutral H) then from the spectral-shift of that line one can calculate its redshift ($z = \frac{\nu_{em}}{\nu_{obs}} - 1$).
- From z we calculate the commoving distance of the source using best available cosmological model

$$r_z = c\int_a^1 \frac{da}{aH(a)} \text{ where, } a = \frac{1}{1+z}$$

- If the source have some local velocity (peculiar velocity), apart from the Hubble flow then that will add-up in the redshift measurement
- Redshift-space distance $\boldsymbol{s}$ can be calculated and comes out as

$$\boldsymbol{s} = \boldsymbol{r} + \frac{\boldsymbol{v}_p.\boldsymbol{n}}{aH(a)}$$

Where, $\boldsymbol{v}_p$ is the peculiar velocity and $\boldsymbol{n}$ is the line of sight direction, a is the scale factor and $H(a)$ is the Hubble parameter

Multifrequency Angular Power Spectrum (MAPS)

- Here we decompose brightness temperature fluctuations $\delta T_b(\hat{\boldsymbol{n}},\ \nu)$ in terms of spherical harmonics $Y_\ell^m(\hat{\boldsymbol{n}})$ using

$$\delta T_b(\hat{\boldsymbol{n}}, \nu) = \sum_{\ell,m} a_{\ell m}(\nu)\, Y_\ell^m(\hat{\boldsymbol{n}})$$

- And define MAPS as

$$C_\ell(\nu_1, \nu_2) =< a_{\ell m}(\nu_1) a^*_{\ell m}(\nu_2) >$$

- In this work, it suffices to adopt the flat-sky approximation where we decompose the θ dependence of $\delta T_b(\theta, \nu)$ into 2D Fourier modes $\tilde{T}_{b2}(\boldsymbol{U}, \nu)$. Here, $\boldsymbol{U}$ is the Fourier conjugate of θ, and we define the MAPS using

$$C_\ell(\nu_1, \nu_2) = C_{2\pi U}(\nu_1, \nu_2) = \Omega^{-1} < \tilde{T}_{b2}(\boldsymbol{U}, \nu)\, \tilde{T}_{b2}(-\boldsymbol{U}, \nu) >$$

Where Ω is the solid angle subtended by the simulation at the observer.