

An introduction to the Tully-Fisher relation

Federico Lelli

INAF - Arcetri Astrophysical Observatory
Florence, Italy



Why studying the Tully-Fisher relation?

1. Empirical tool to measure galaxy distances

→ Measure H_0 and peculiar velocities (galaxy flows)

Need: Large galaxy samples (low-resolution HI surveys)

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→ Test galaxy formation models in Λ CDM & alternatives

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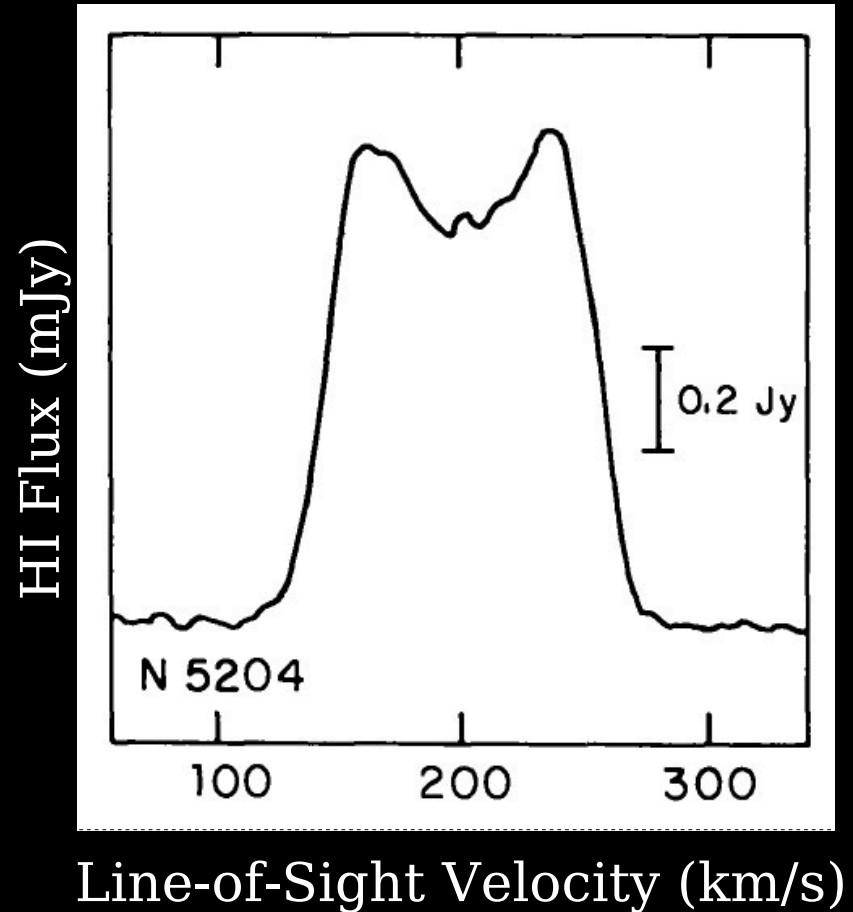
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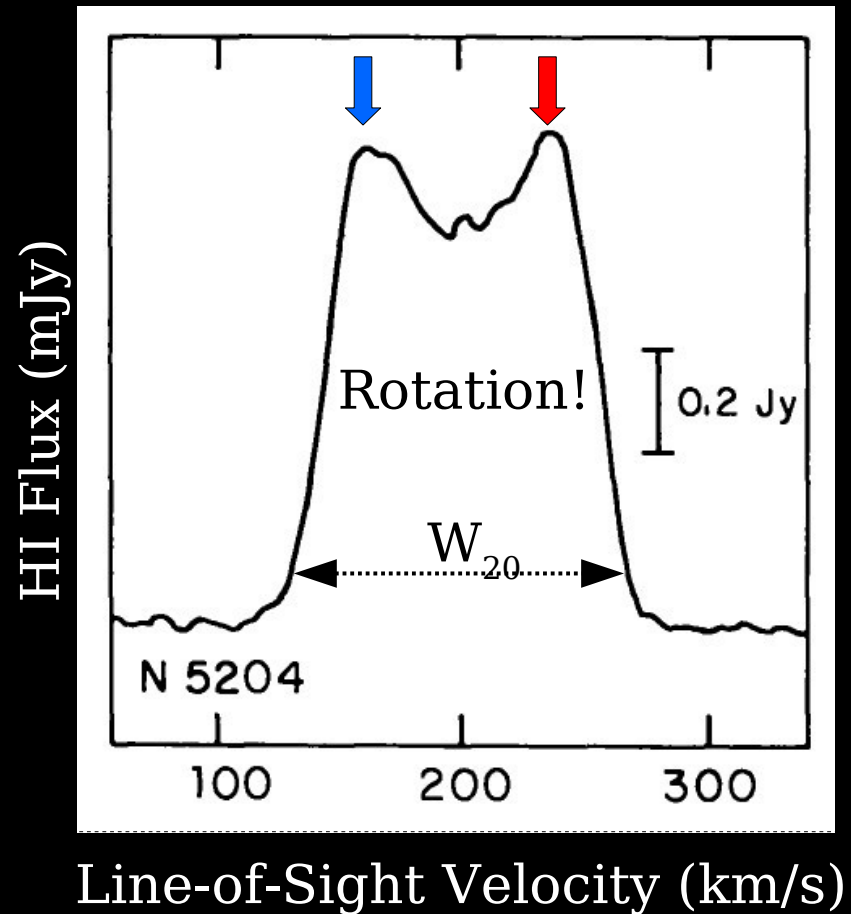
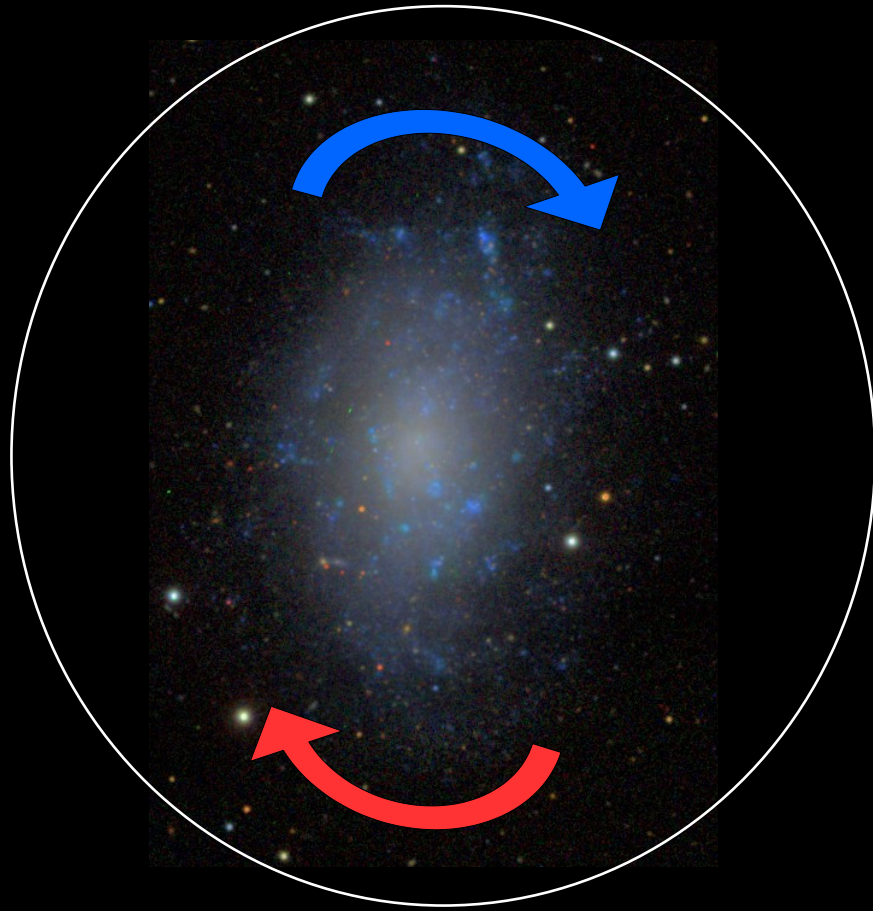
→ Test galaxy formation models in Λ CDM & alternatives

Need: High-resolution HI surveys (small galaxy samples)

Spatially Integrated HI Observations

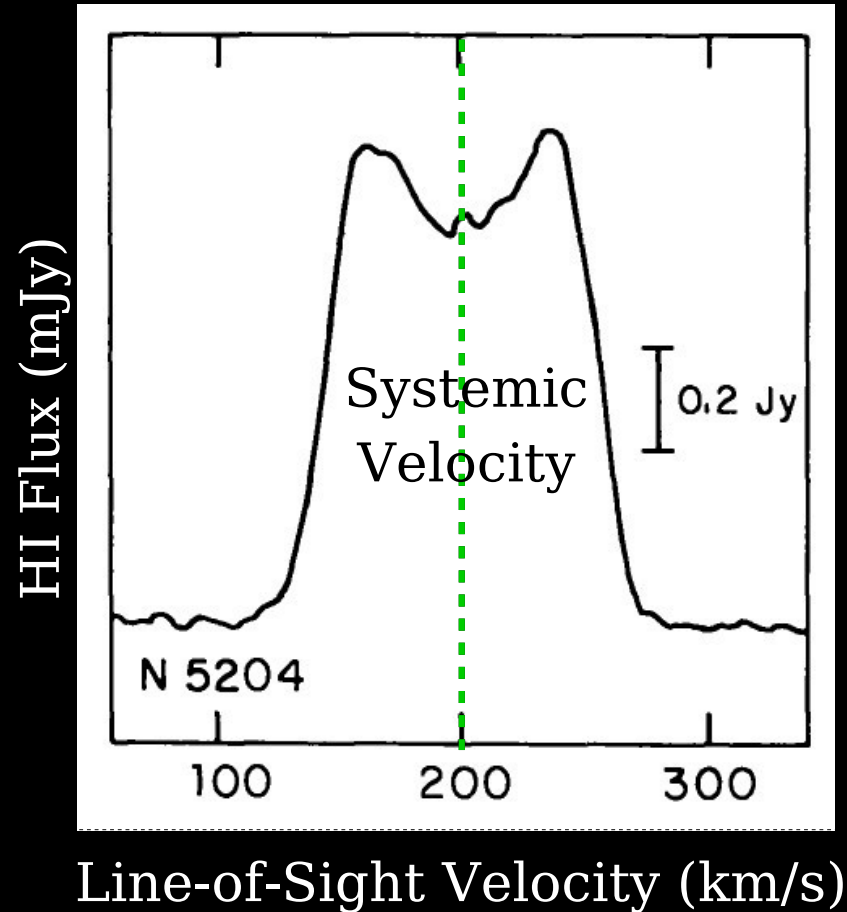


Spatially Integrated HI Observations



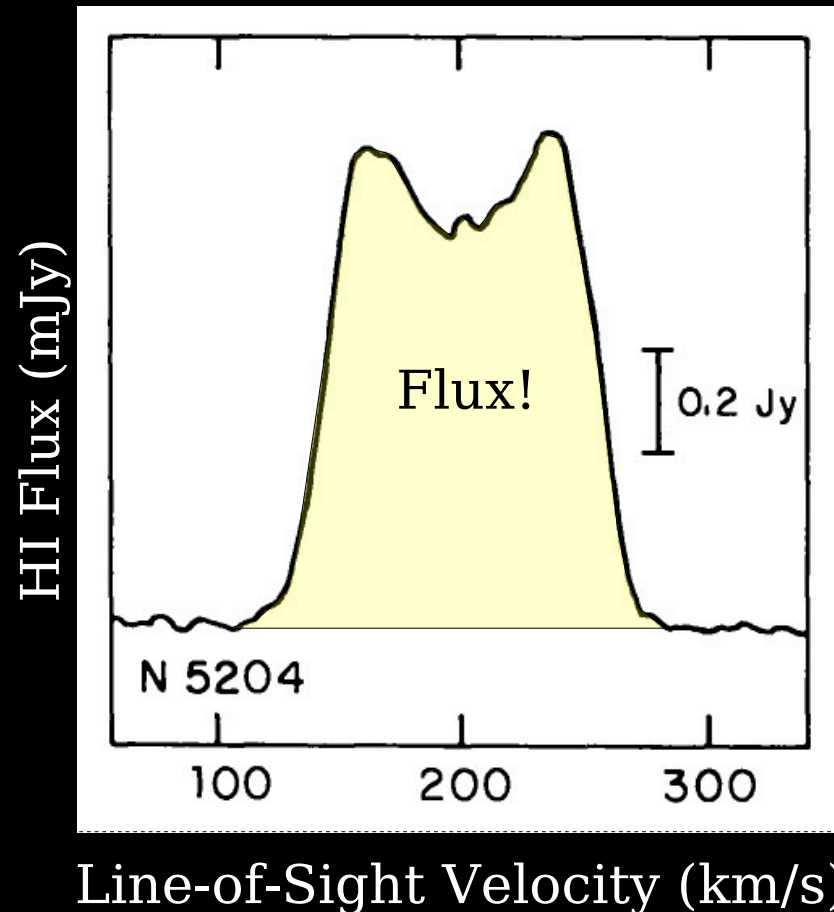
- HI Line-Width: W_{20} (20% of peak flux) $\approx 2 \langle V_{\text{rot}} \rangle \sin(i)$

Spatially Integrated HI Observations



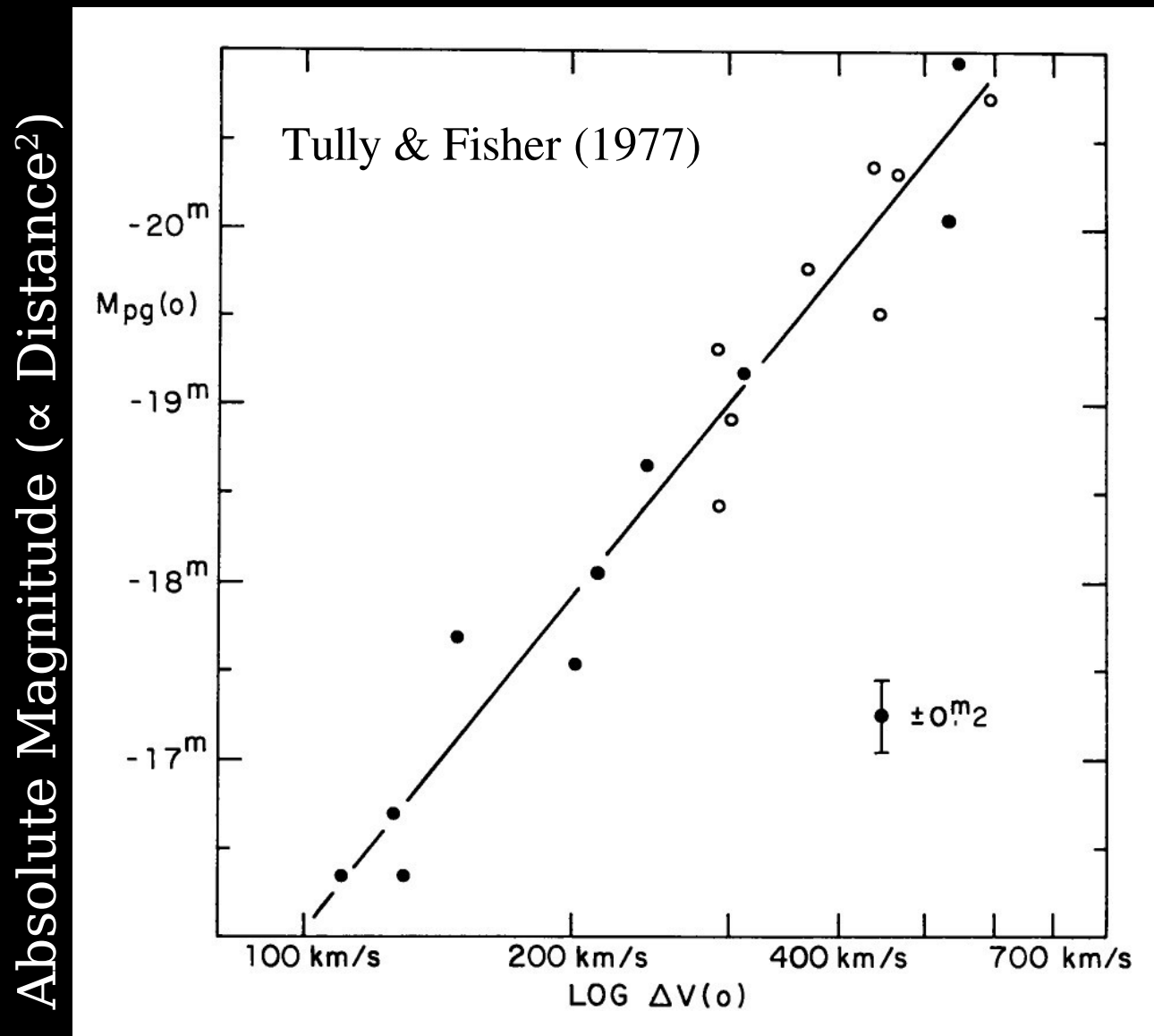
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- Systemic Velocity / Redshift: $z \approx V_{\text{sys}} / c$ for low z
- Total HI flux / HI mass: $M_{\text{HI}} = 236 d_L^2 [\text{Mpc}] S_{\text{HI}} [\text{mJy km/s}]$

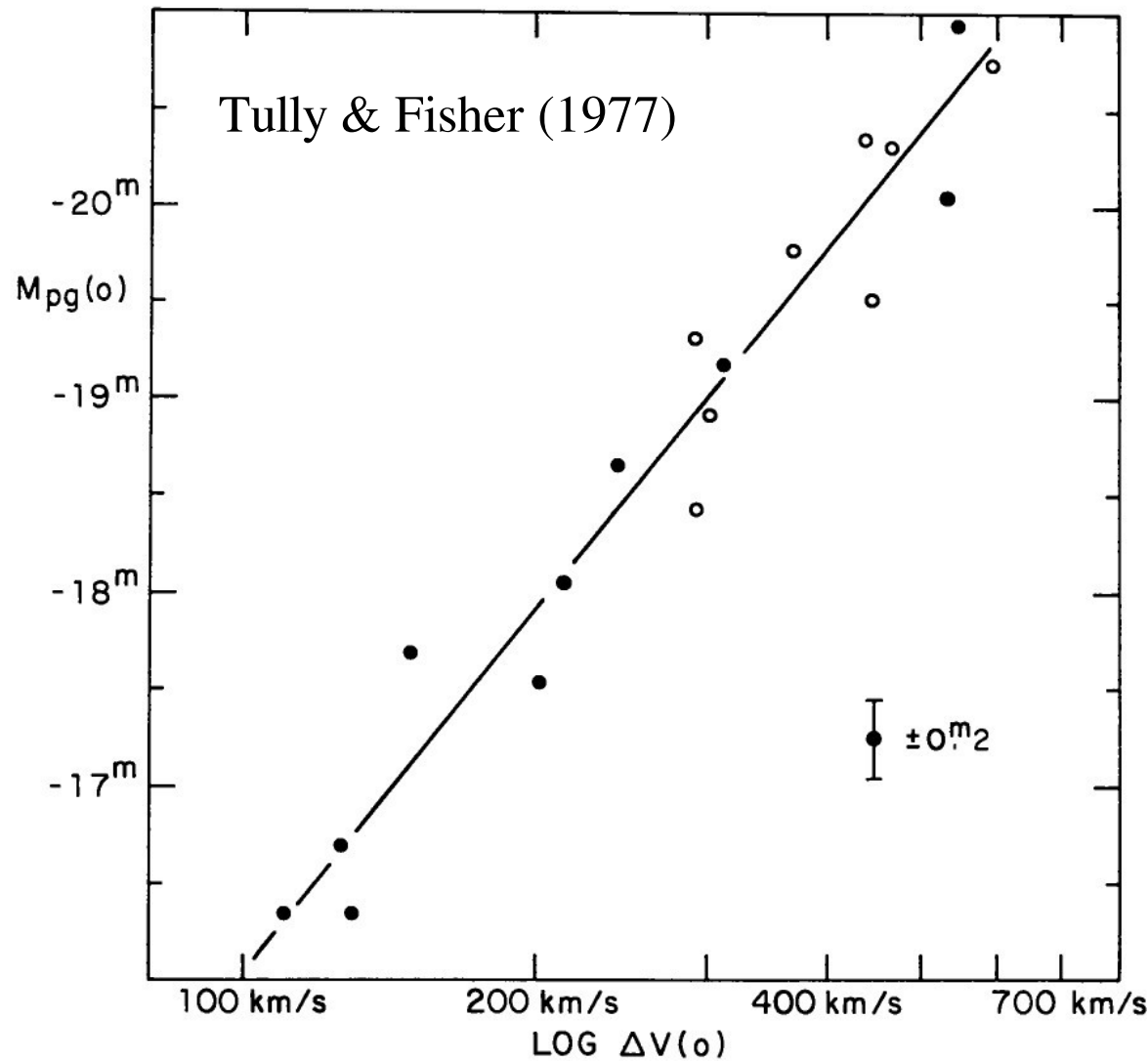
The Classic Tully-Fisher (TF) Relation



HI Line-Width (Distance Independent)

The Classic Tully-Fisher (TF) Relation

Absolute Magnitude (\propto Distance²)



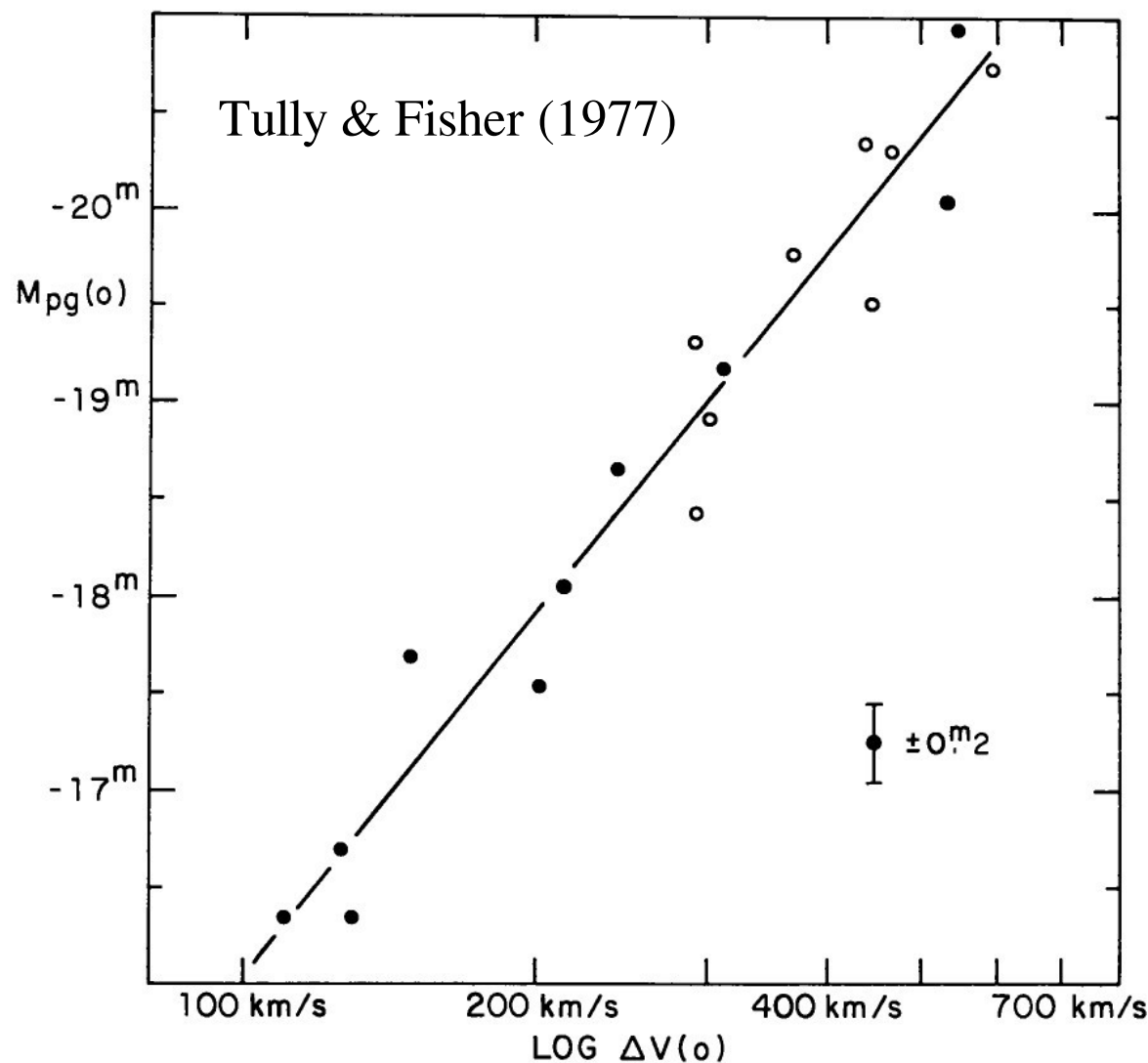
HI Line-Width (Distance Independent)

STEP 1:

Calibrate TF relation using galaxies with known distance (from Cepheids, TRGB, etc.)

The Classic Tully-Fisher (TF) Relation

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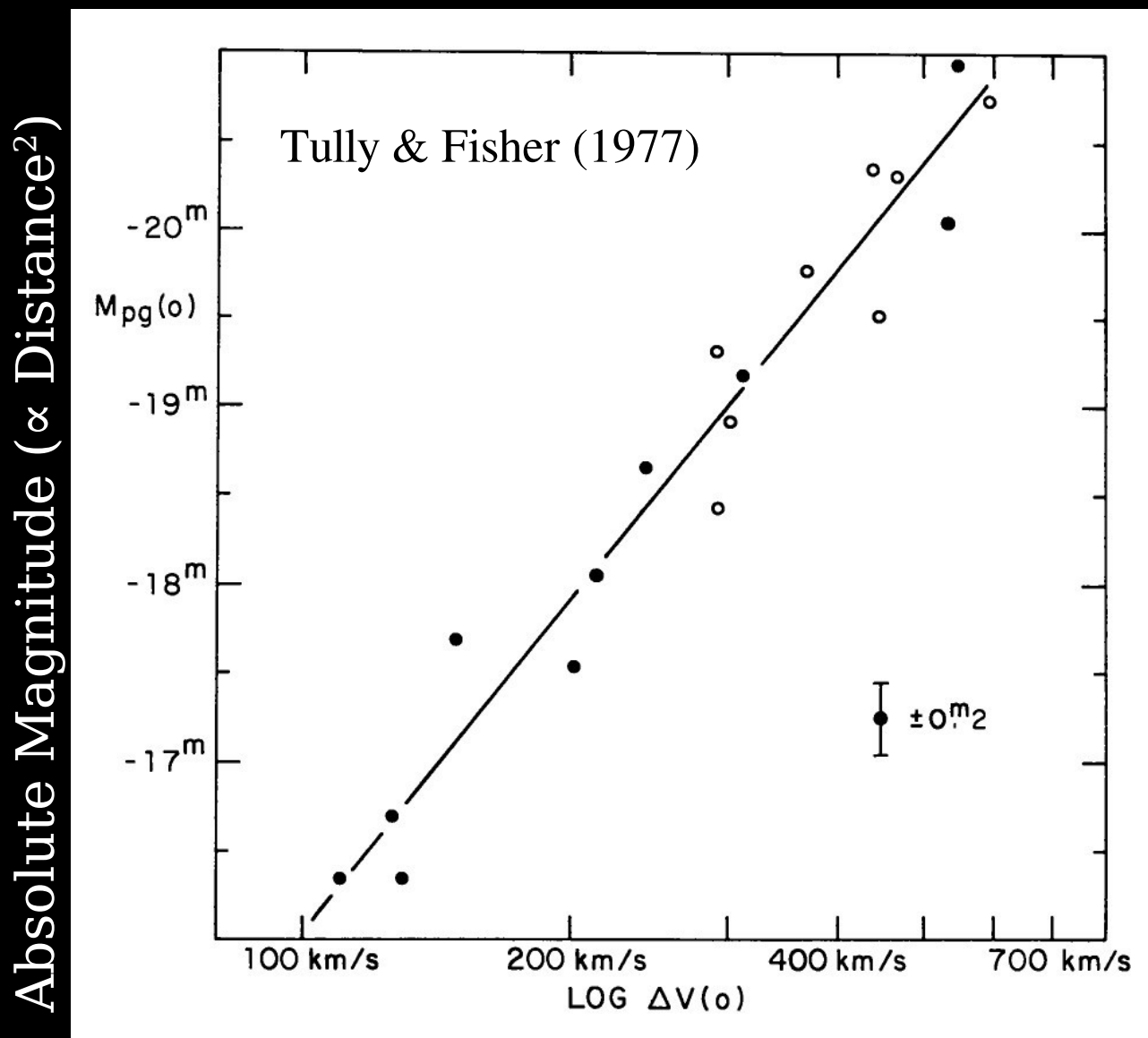


HI Line-Width (Distance Independent)

STEP 1:
Calibrate TF relation using galaxies with known distance (from Cepheids, TRGB, etc.)

STEP 2:
Measure HI line-widths & apparent mags from radio & optical/IR surveys

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STEP 3:

Infer distances for large galaxy samples (< 300 Mpc)

$\rightarrow H_0 = 80 \text{ km/s/Mpc}$

(Tully & Fisher 1977)

Classic Applications of the TF relation

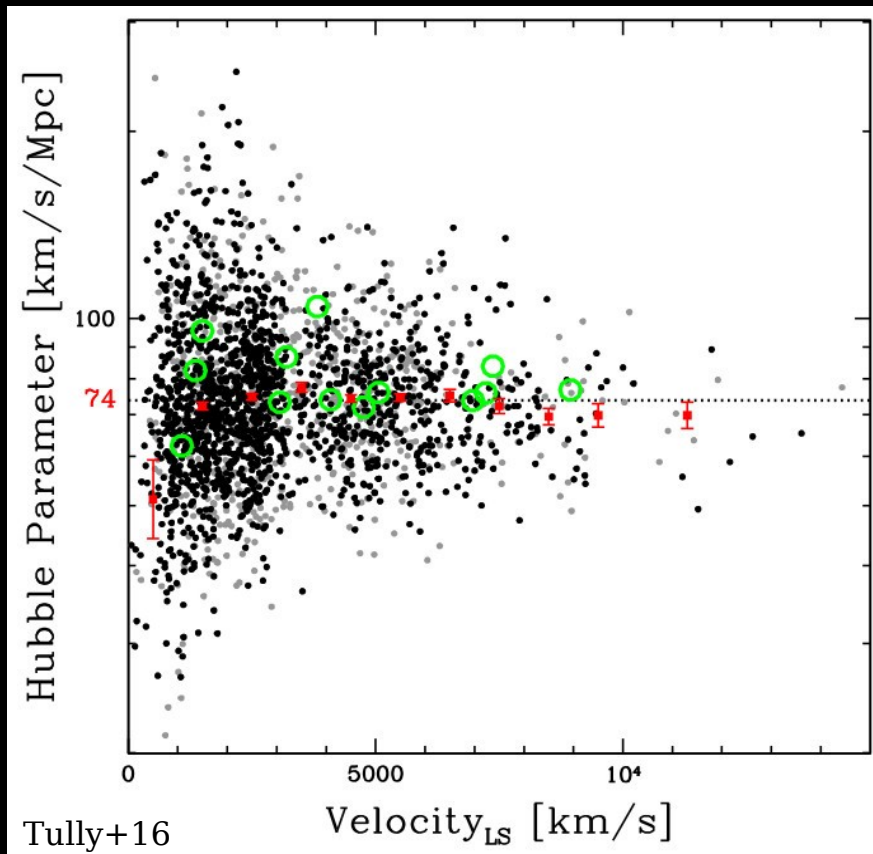
1 - Hubble constant

$$V_{\text{sys}} \simeq H_0 d_L + V_{\text{pec}} \text{ at low } z$$

$$H_0 = 75.0 \pm 2.0 \text{ km/s/Mpc (Tully+16)}$$

$$H_0 = 75.1 \pm 2.3 \text{ km/s/Mpc (Schombert+19)}$$

$$H_0 = 75.5 \pm 2.5 \text{ km/s/Mpc (Kourkchi+22)}$$



Classic Applications of the TF relation

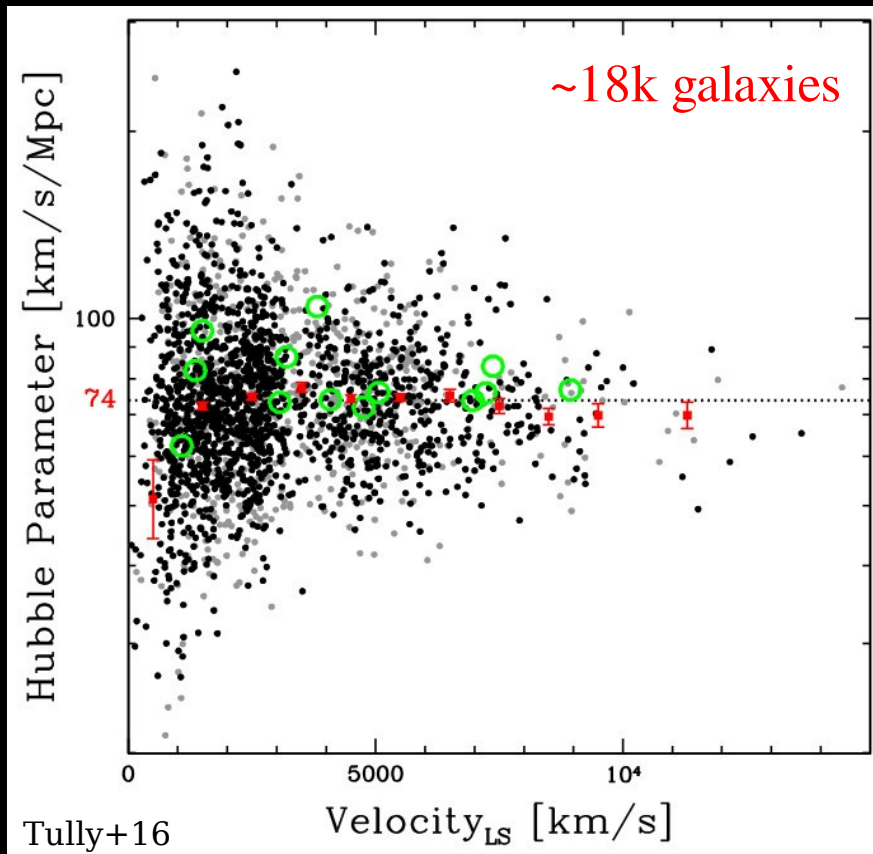
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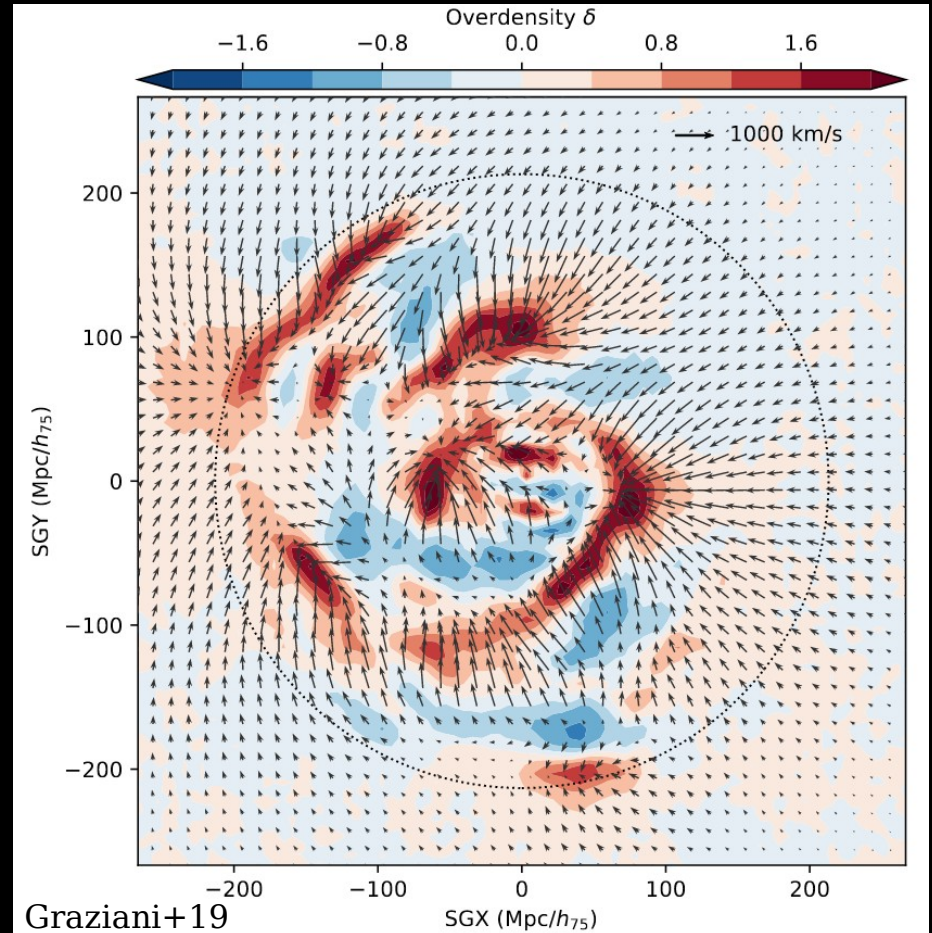


2 - Galaxy Flows

$$V_{\text{pec}} = c [z_{\text{obs}} - z_{\text{cos}}(d_L)] / [1 + z_{\text{cos}}(d_L)]$$

$$z_{\text{cos}} = f(d_L; H_0, \Omega_m, \Omega_\Lambda)$$

(Tully+16, Graziani+19, Kourkchi+20)



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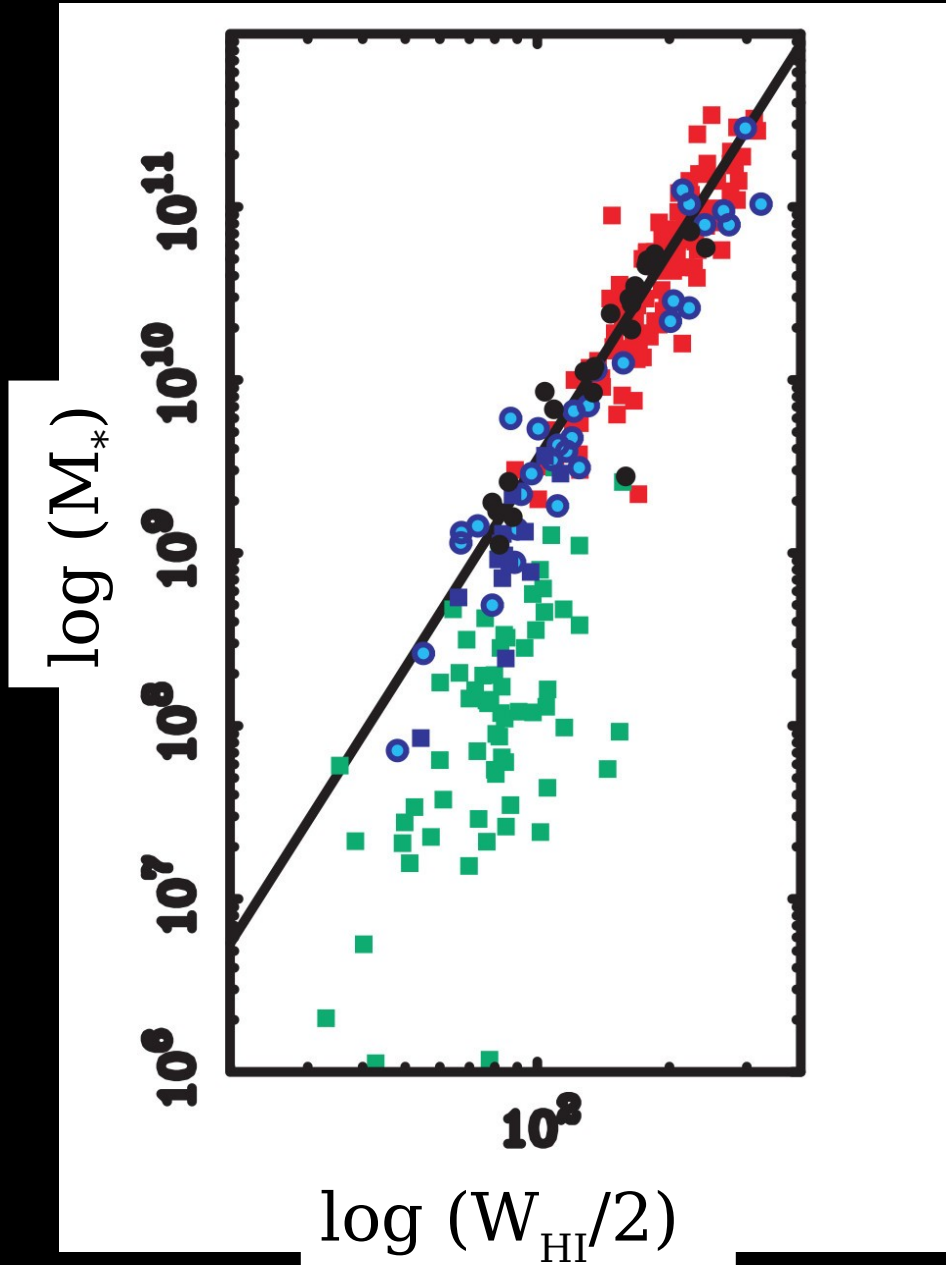
Need: High-resolution HI surveys (small galaxy samples)

Luminosity and HI linewidth are
proxies for more fundamental
physical quantities!

Objective: find the quantities that
give the tighter relation

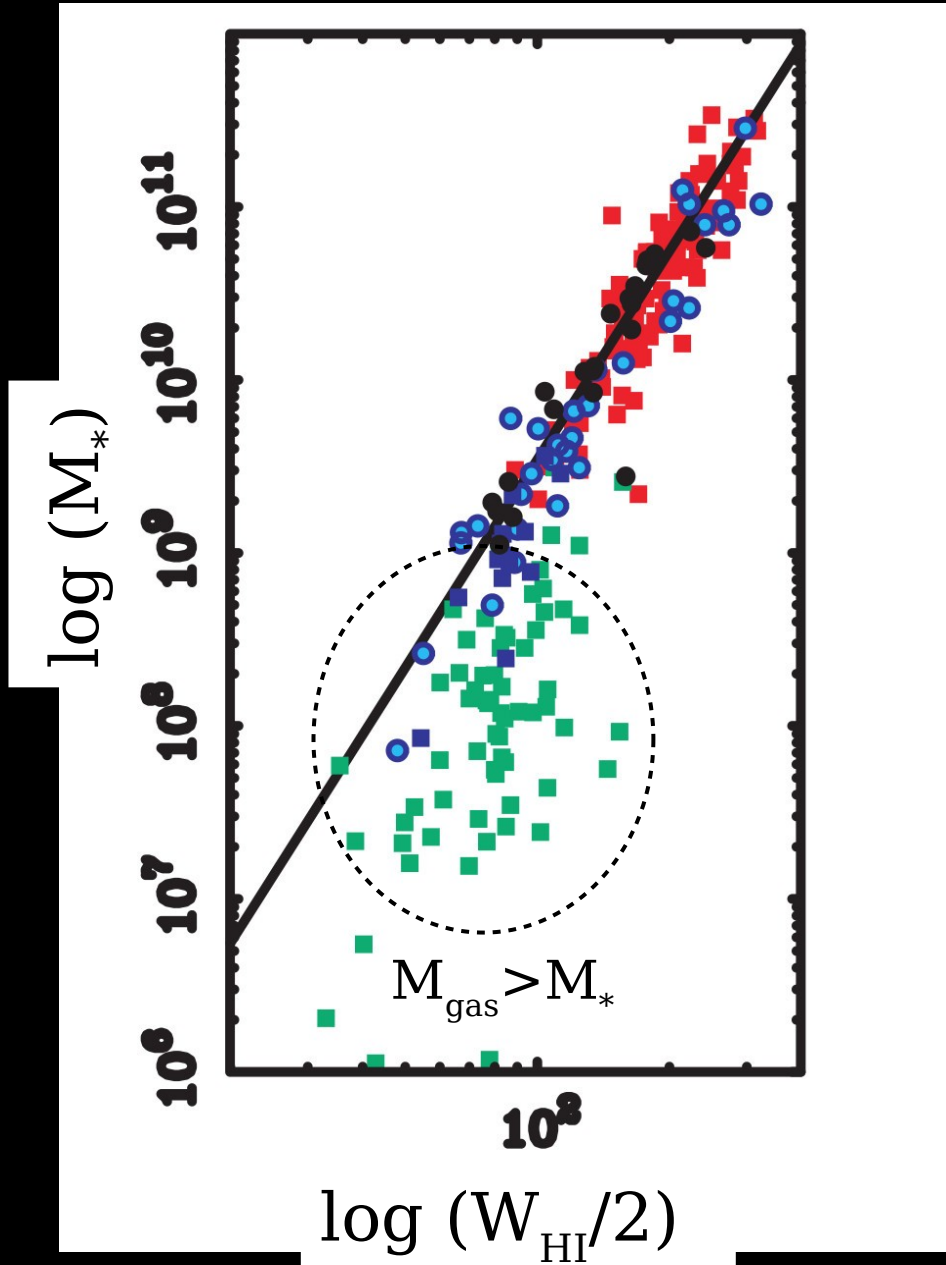
Optical/Near-IR Luminosity \rightarrow Stellar Mass

Stellar-Mass TF Relation



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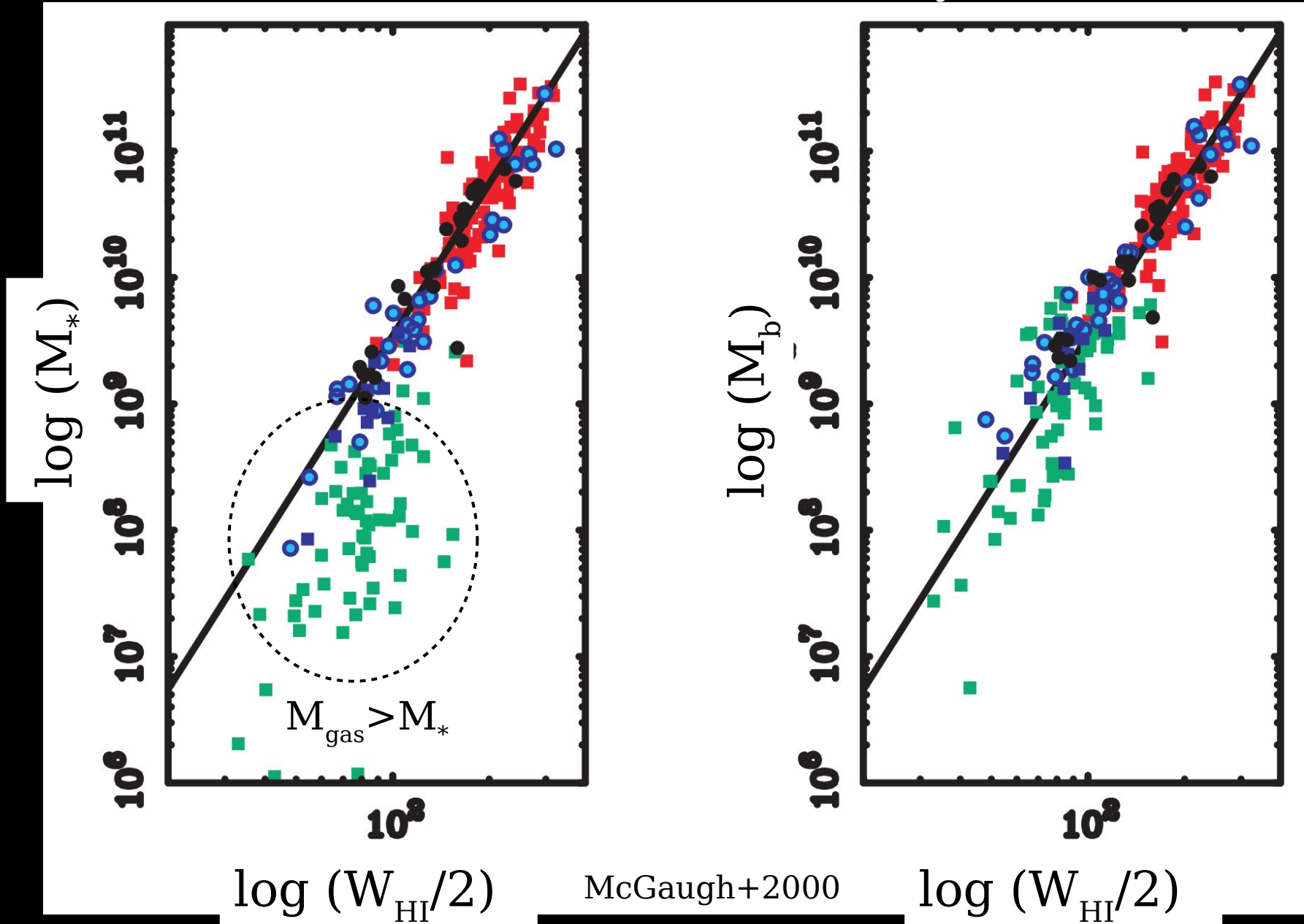
Stellar-Mass TF Relation



Baryonic Mass (stars+gas) is the key!

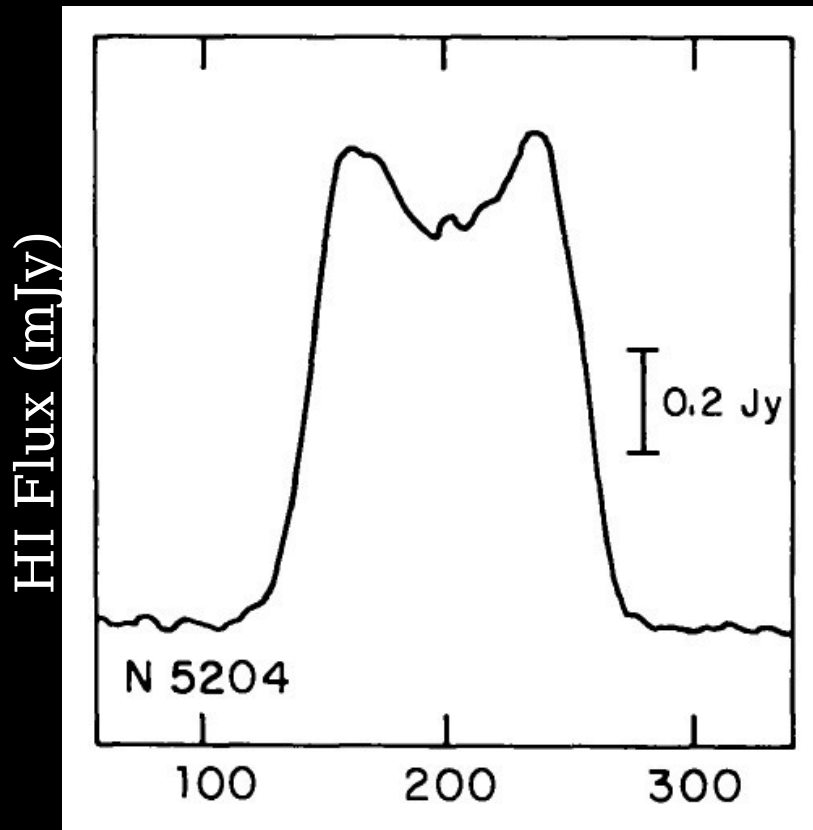
Stellar-Mass TF Relation

Baryonic TF Relation



McGaugh+2000

What's the HI line-width really measuring?

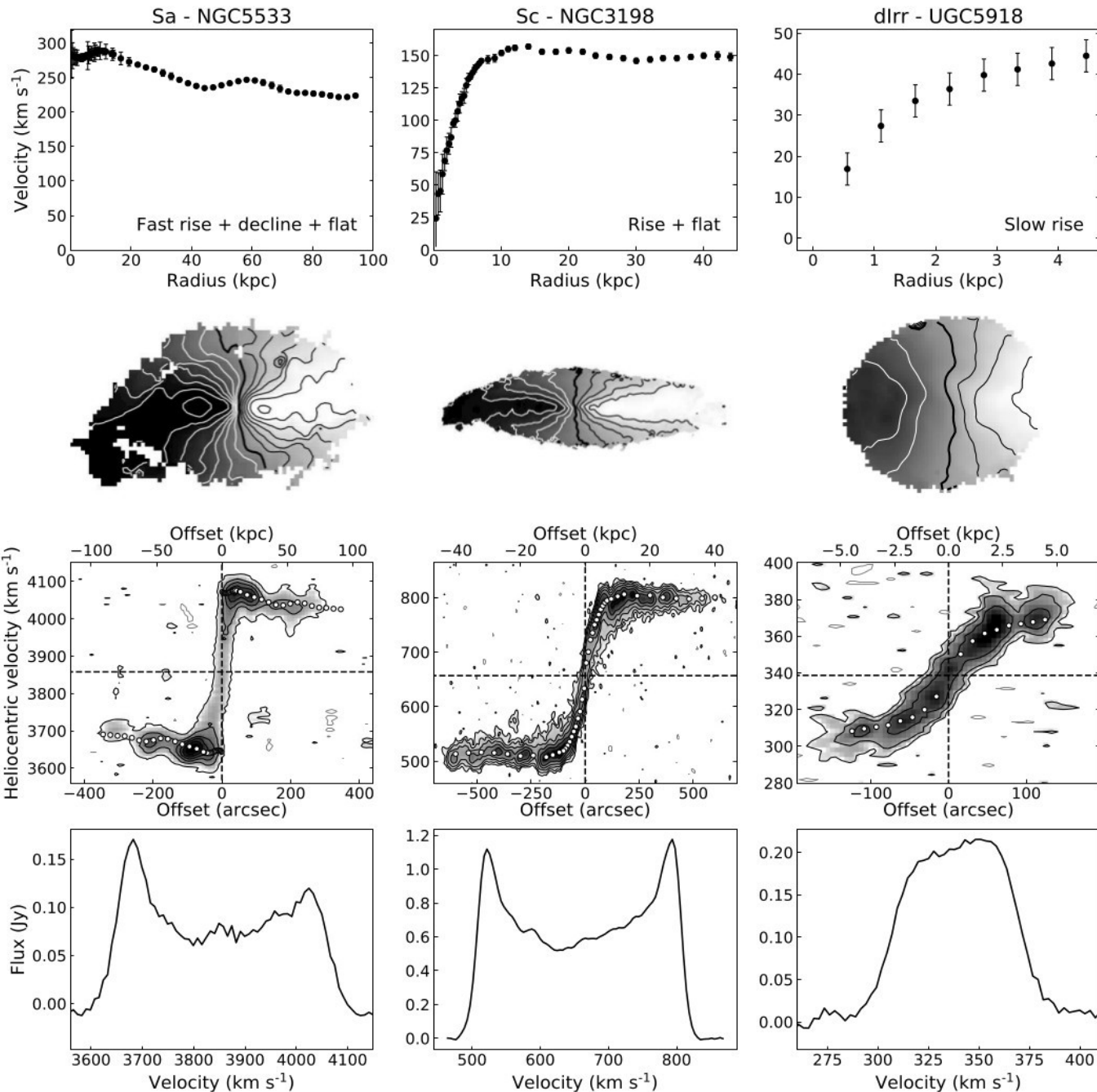


Line-of-Sight Velocity (km/s)

The HI line profile depends on $\Sigma_{\text{HI}}(R)$, $V_{\text{rot}}(R)$, inclination!

Need to spatially resolve HI distribution and kinematics!

Rotation Curves for Different Galaxies



Rotation Curve:

V_{rot} vs Radius

Velocity Field:

$V_{\text{l.o.s.}} \propto V_{\text{rot}} \sin(i) \cos(\theta)$

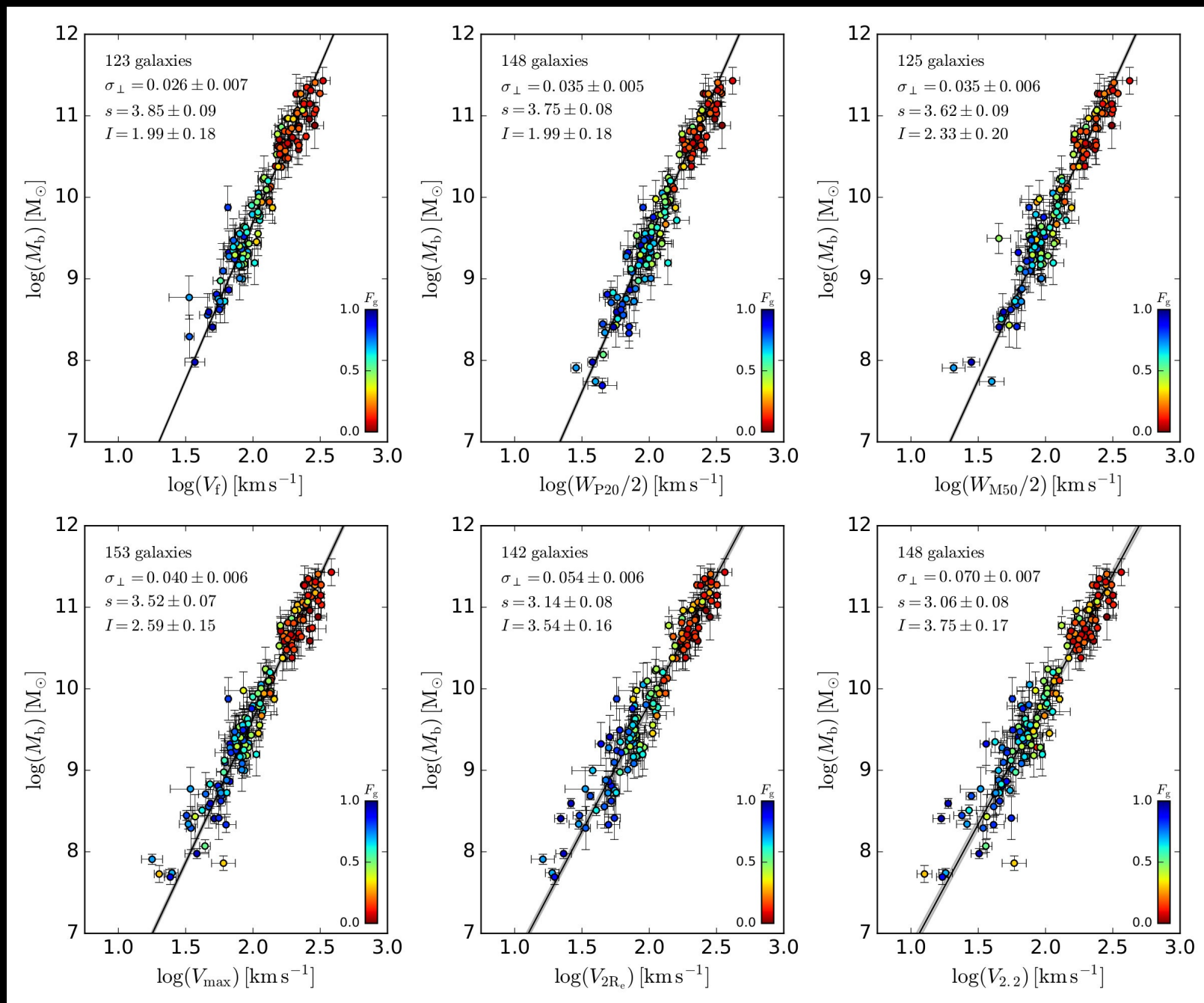
Major-Axis PVD:

$V_{\text{rot}} \sin(i)$ vs Radius

Global HI profile:

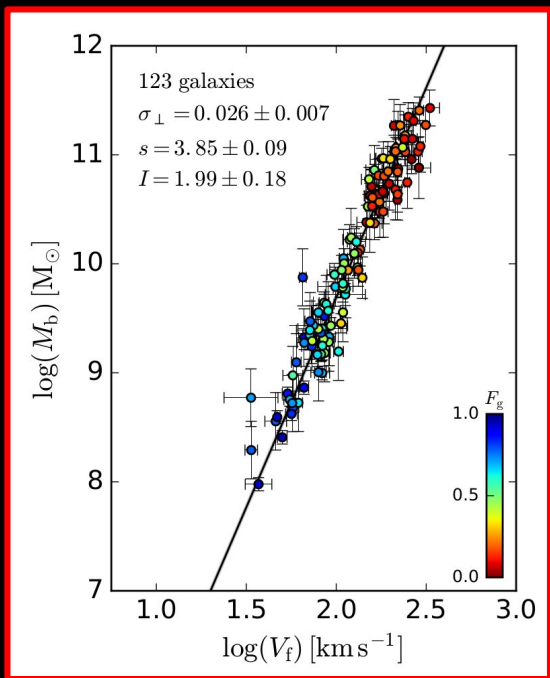
Cimatti, Fraternali, Nipoti (2019)

BTFR for different velocity definitions



Lelli+2019

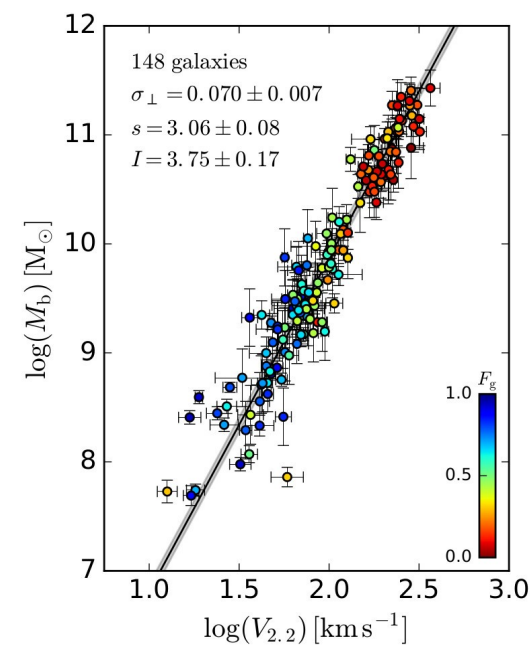
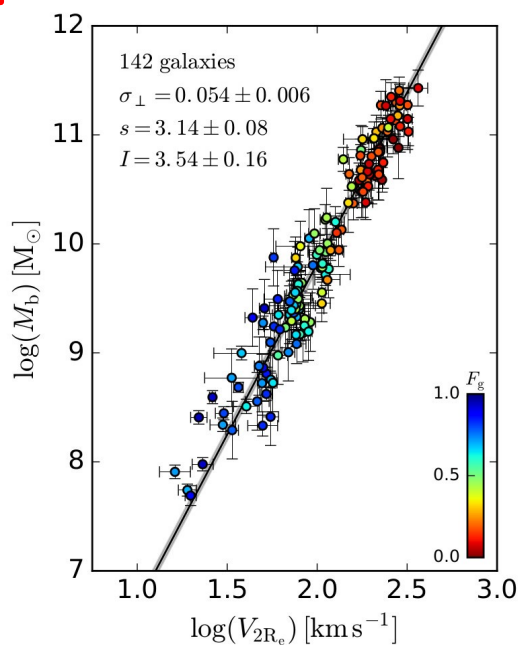
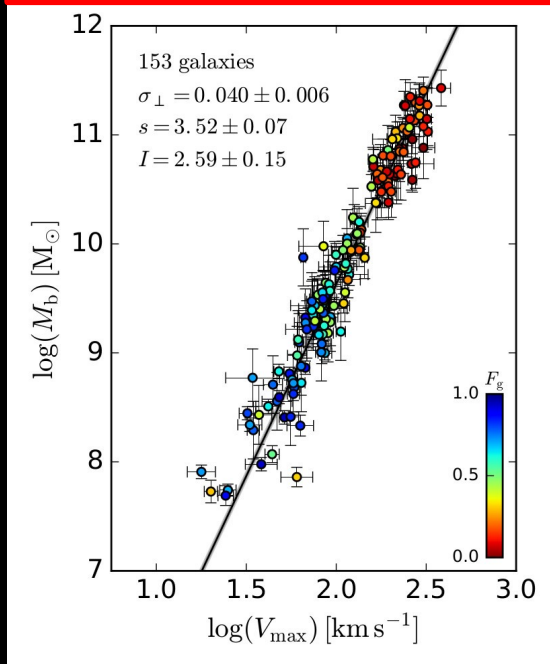
BTFR for different velocity definitions



The mean rotation velocity along flat part (V_f) gives the tightest and steepest BTFR!

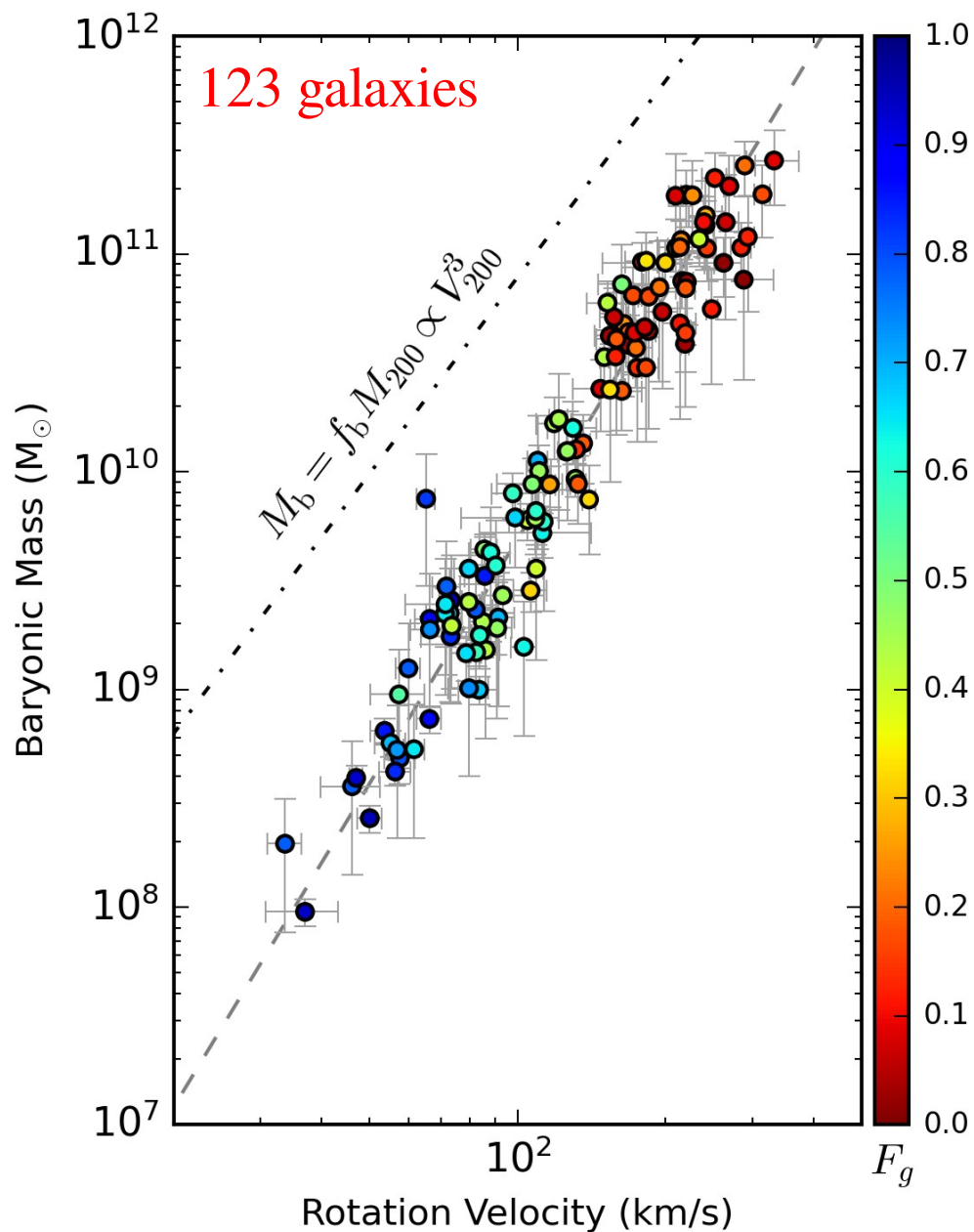
(Verheijen 2001; Noordermeer & Verheijen 2007; McGaugh 2005; Ponomareva+2017; Lelli+2019)

Counter intuitive: Baryons important near the center but M_b best correlate with V_f (mostly set by the dark matter halo)!



Lelli+2019

Baryonic TF relation in a Λ CDM context



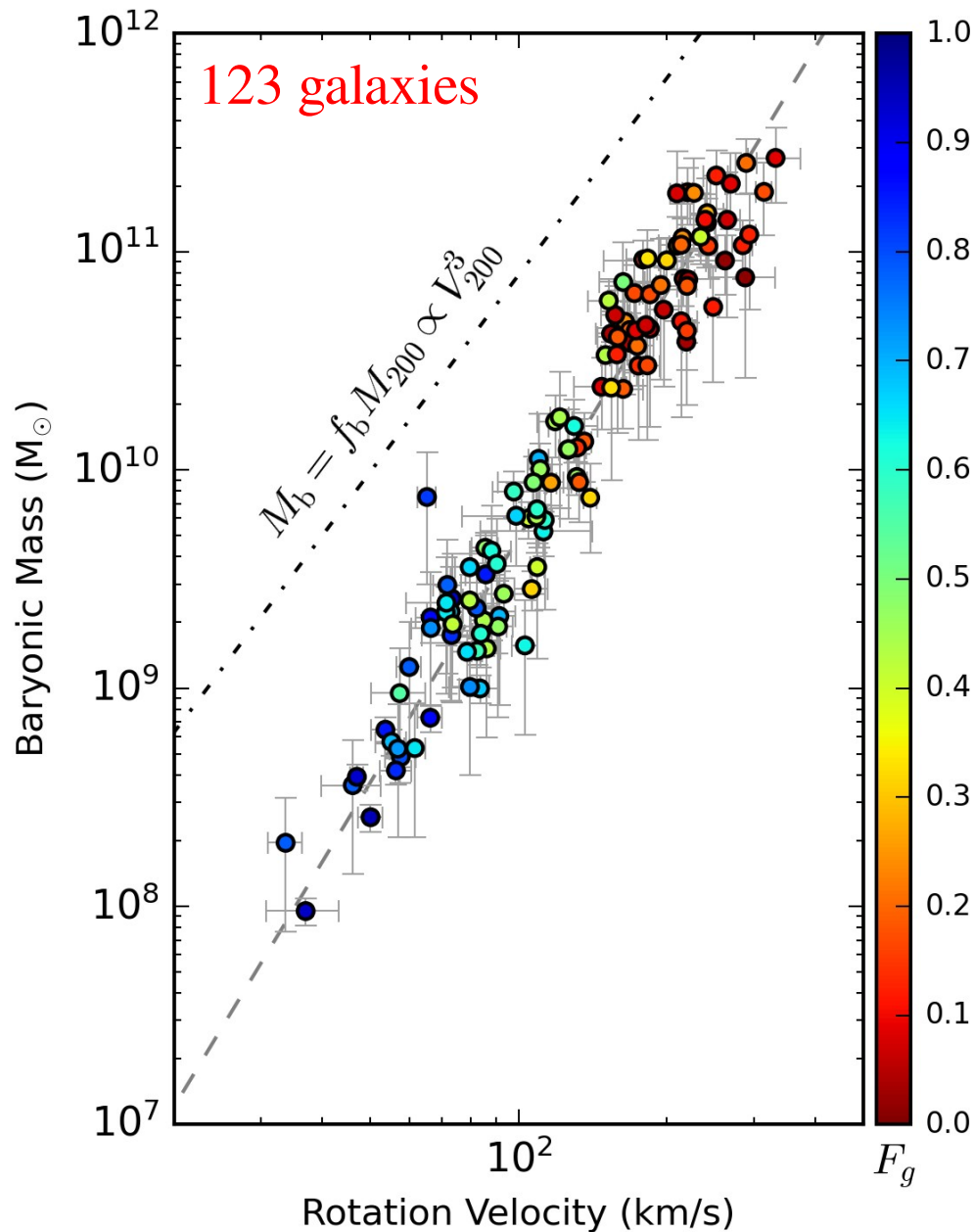
Lelli+2016, 2019

From basic arguments (McGaugh 2012):

$$M_b = \sqrt{\frac{2}{\Delta}} \frac{1}{GH_0} f_b f_v^{-3} V_f^3$$

$$f_b = \frac{M_b}{M_\Delta} \quad f_v = \frac{V_f}{V_\Delta}$$

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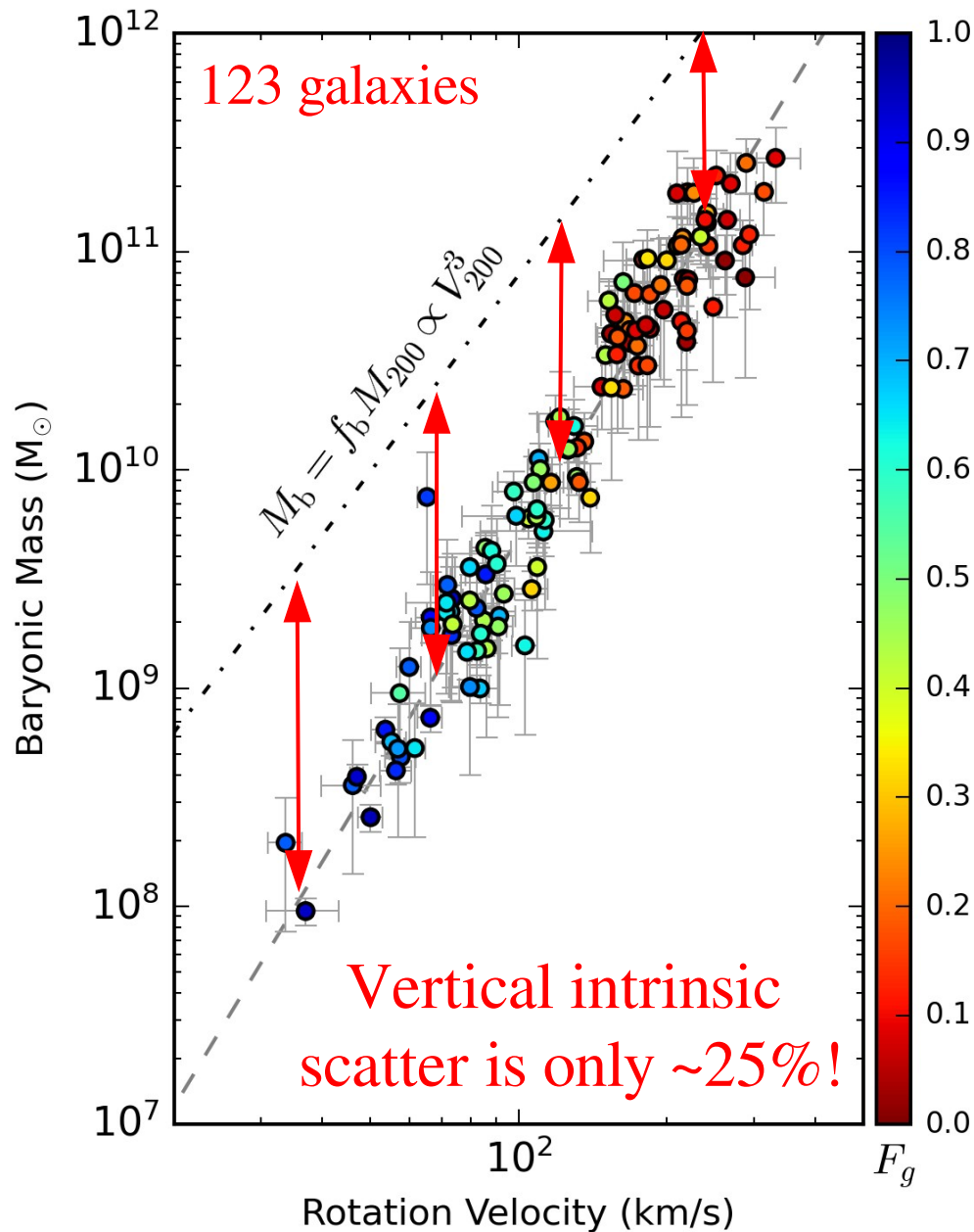
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Simplest working hypotheses:

$$f_b = f_{\text{cosmic}} = \Omega_b / \Omega_m \quad \text{From CMB}$$

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To get lower normalization:

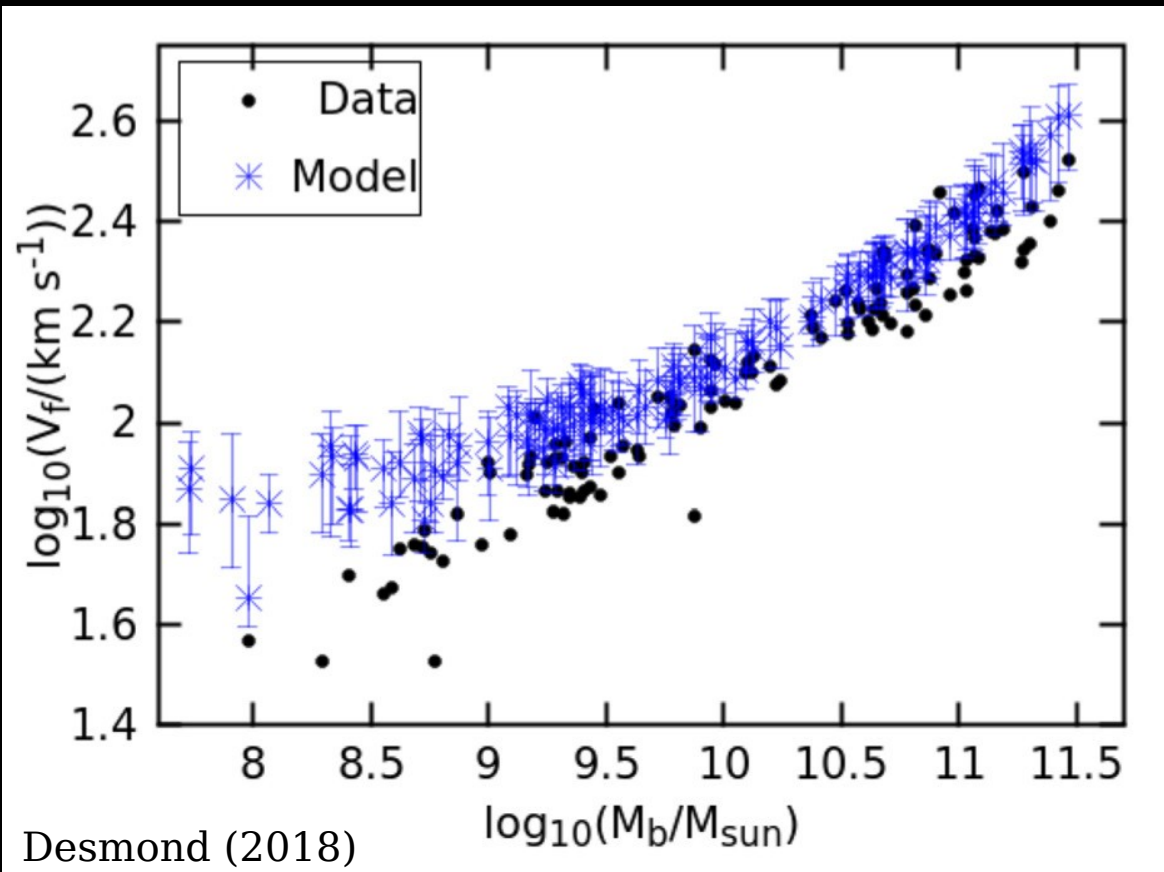
$$f_b < f_{\text{CMB}} \rightarrow \text{missing baryons problem}$$

To get slope of about 4:

$$f_b \text{ must systematically vary with } V_f$$

\rightarrow fine-tuning problem for Λ CDM!

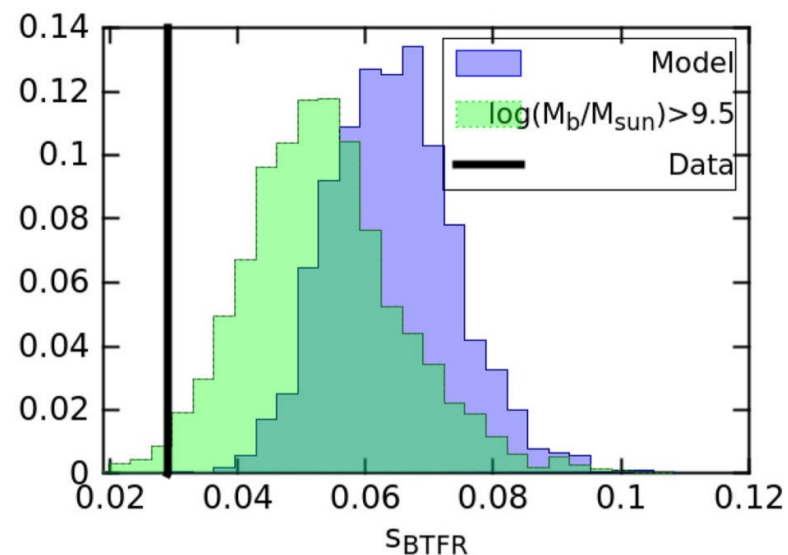
BTFR from semi-empirical Λ CDM models



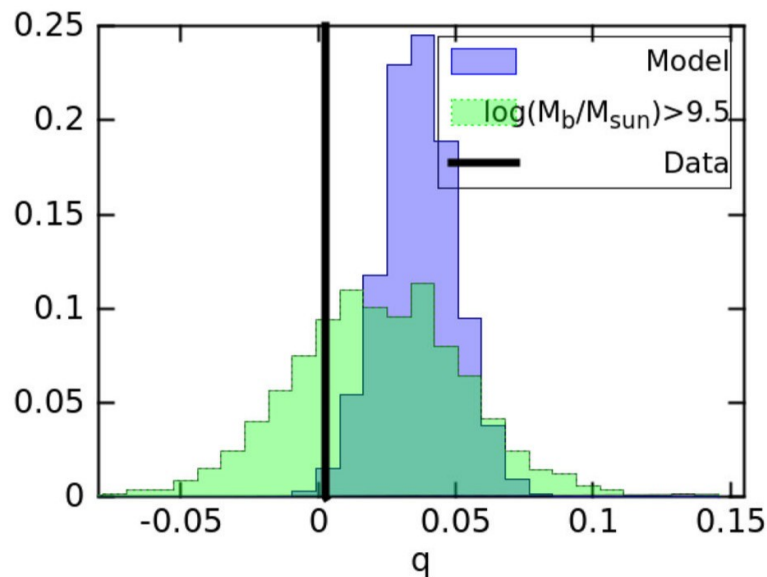
Scatter on BTFR should emerge from

- Scatter in $f_b = M_b/M_{200}$
(galaxy formation is stochastic!)
- Scatter in $f_v = V_f/V_{200}$
(halo properties, e.g., M_{200} - C_{200} relation)

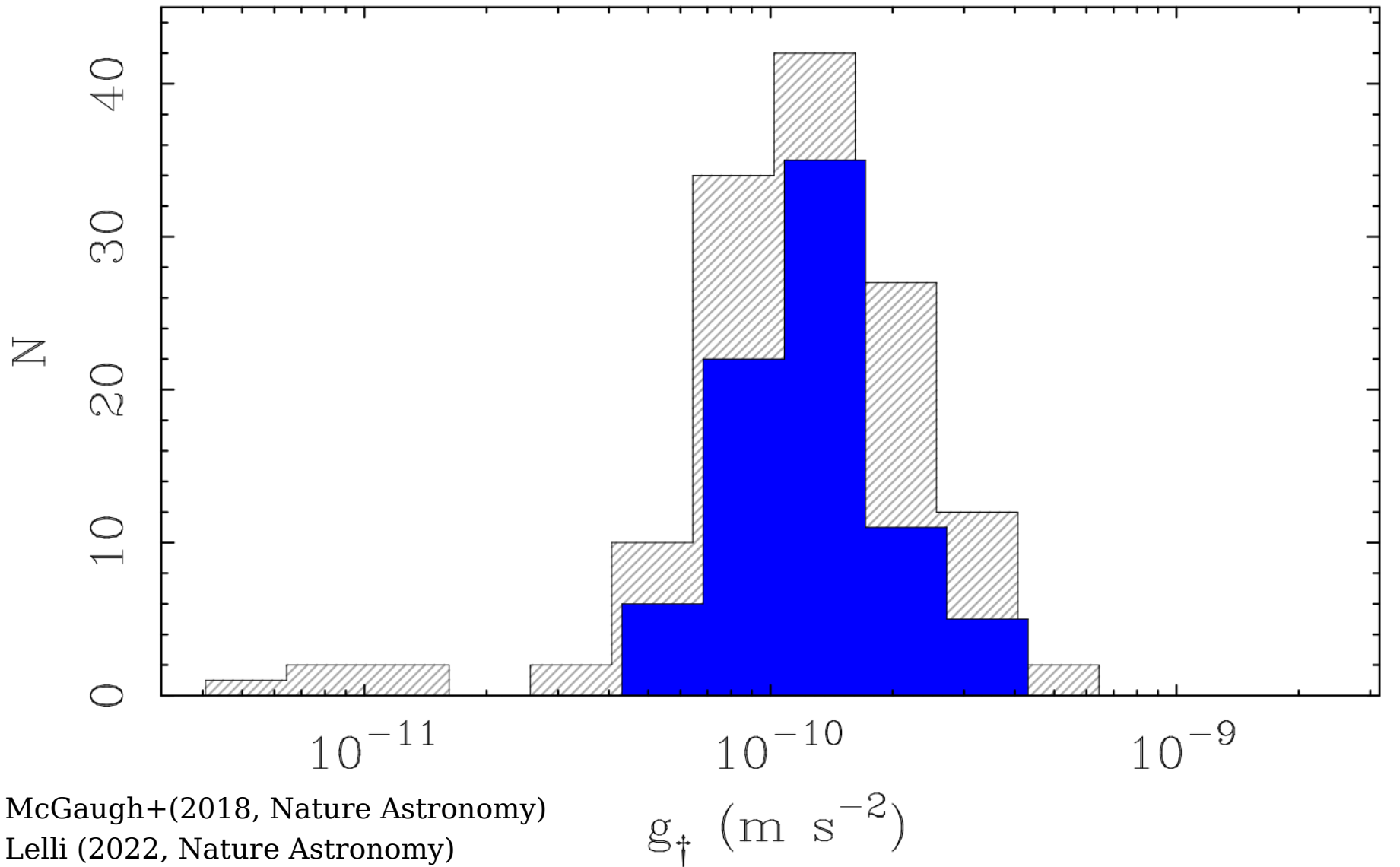
Scatter is 3.6σ too high!



Curvature is 3σ too strong!

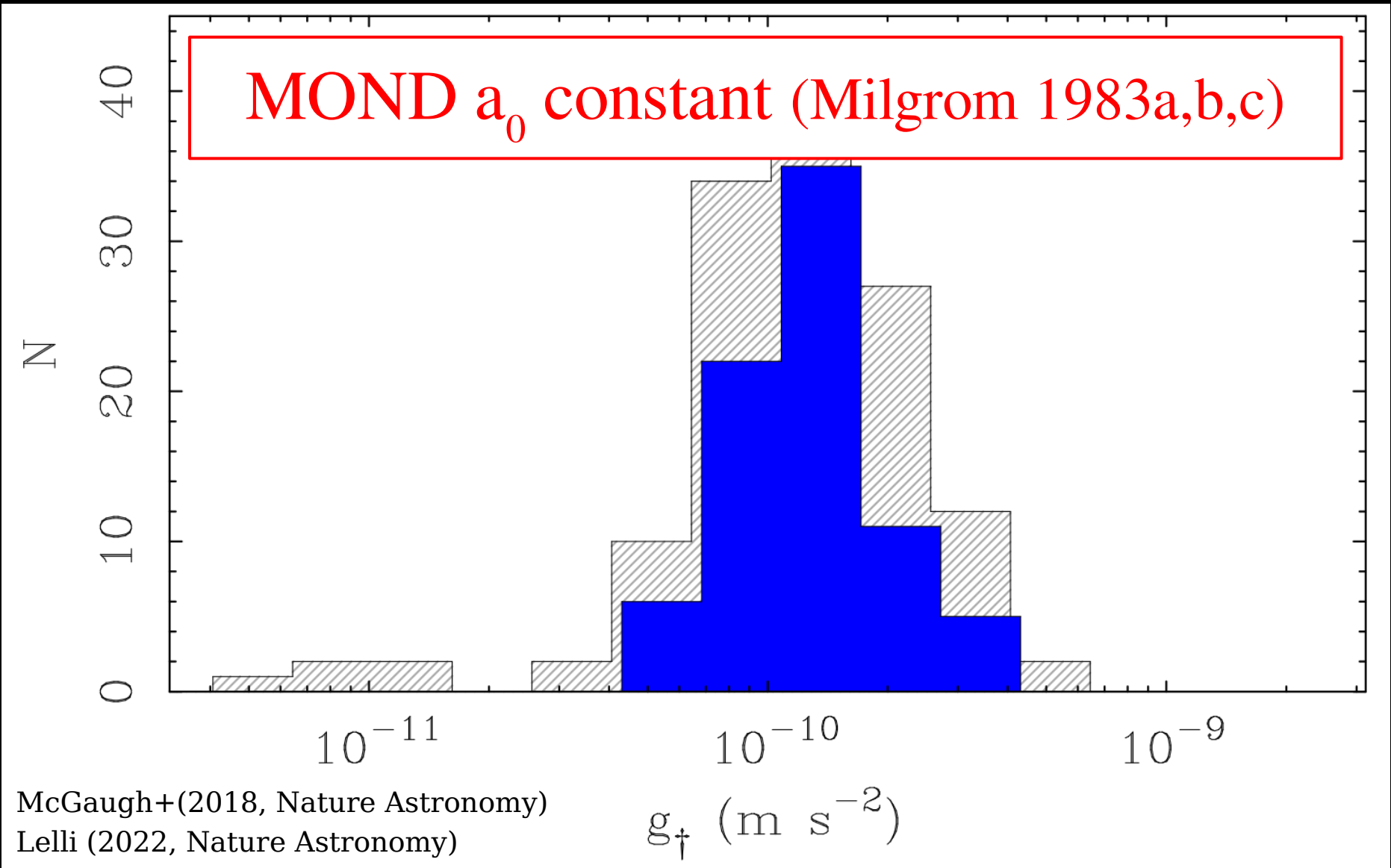


Slope $\sim 4 \rightarrow$ Acceleration Scale



On dimensional grounds: $g_{\dagger} = V_f^4 / (G_N M_b)$

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Take-home points on the TF relation

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→ $H_0 \simeq 75 \pm 2$ km/s/Mpc (Tully+16, Schombert+19, Kourkchi+22)

→ Galaxy flows: mass distribution within 200 Mpc (Graziani+19, Kourkchi+20)

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→ Galaxy flows: mass distribution within 200 Mpc (Graziani+19, Kourkchi+20)

2. Baryonic TF relation (M_b vs V_f) is most fundamental

→ Link between baryons and DM in galaxies (McGaugh+2000, Verheijen+2001)

→ Small intrinsic scatter (25%): fine-tuning problem in Λ CDM (Lelli+2016)

→ Slope $\simeq 4$: Acceleration scale of $\sim 10^{-10}$ m/s² (McGaugh+2018, Lelli 2022)

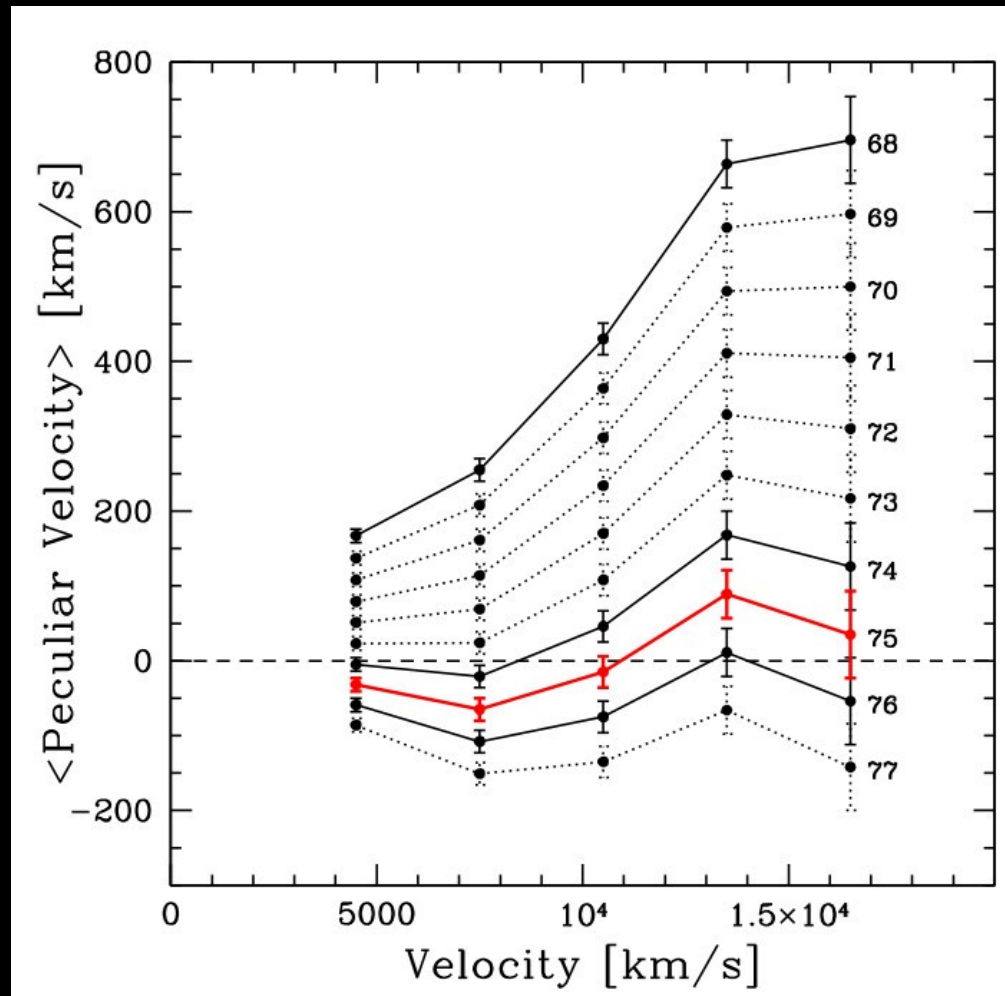
→ Consistent with a-priori predictions of MOND (Milgrom 1983a,b,c)

More Slides

Peculiar Velocities & Hubble Constant

$$V_{\text{pec}} = (V_{\text{mod}} - H_0 d_L) / (1 + H_0 d_L / c) \quad V_{\text{mod}} = f(z, d_L, \Omega_m, \Omega_\Lambda)$$

Fix Ω_m and Ω_Λ (or equivalently q_0), vary H_0 and get different V_{pec}

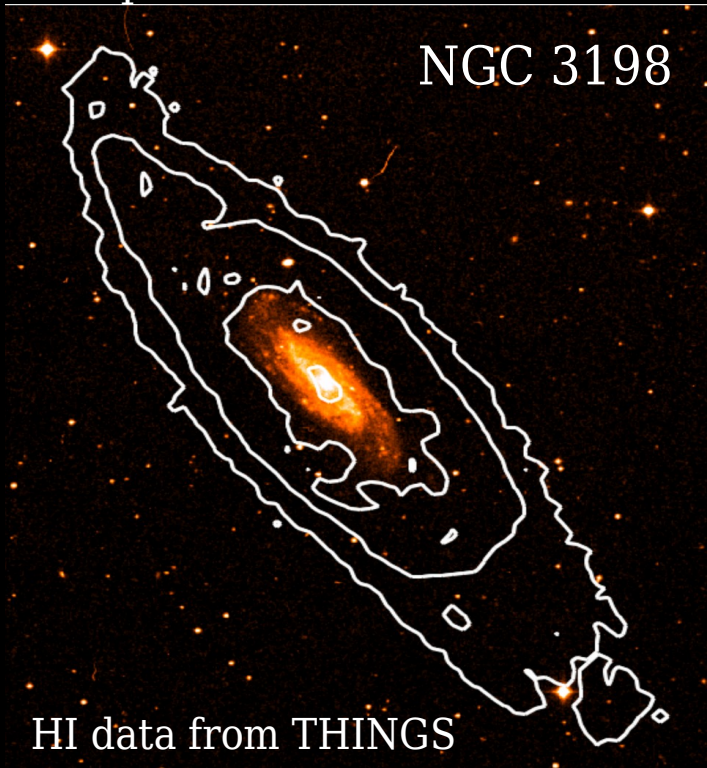


Cosmicflows:
10k galaxies

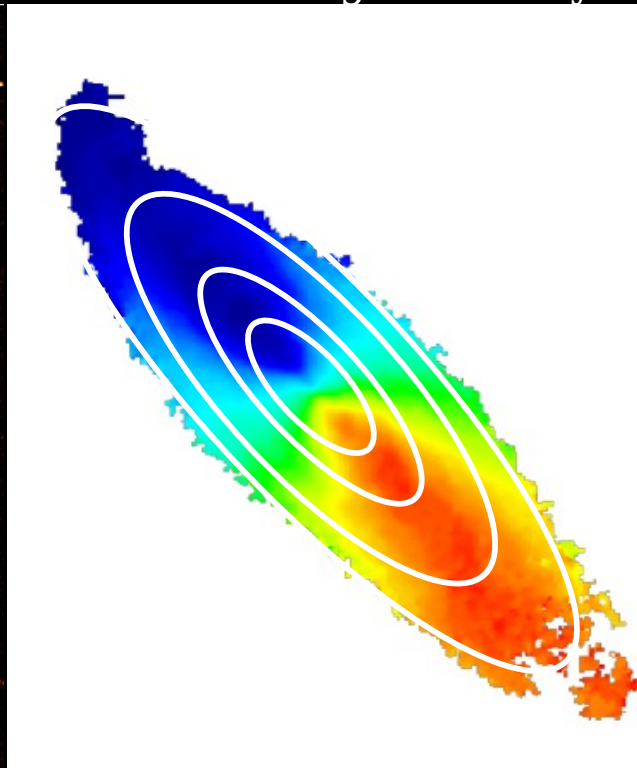
Tully+2016:
 $H_0 = 75 \pm 2$

HI distribution and kinematics

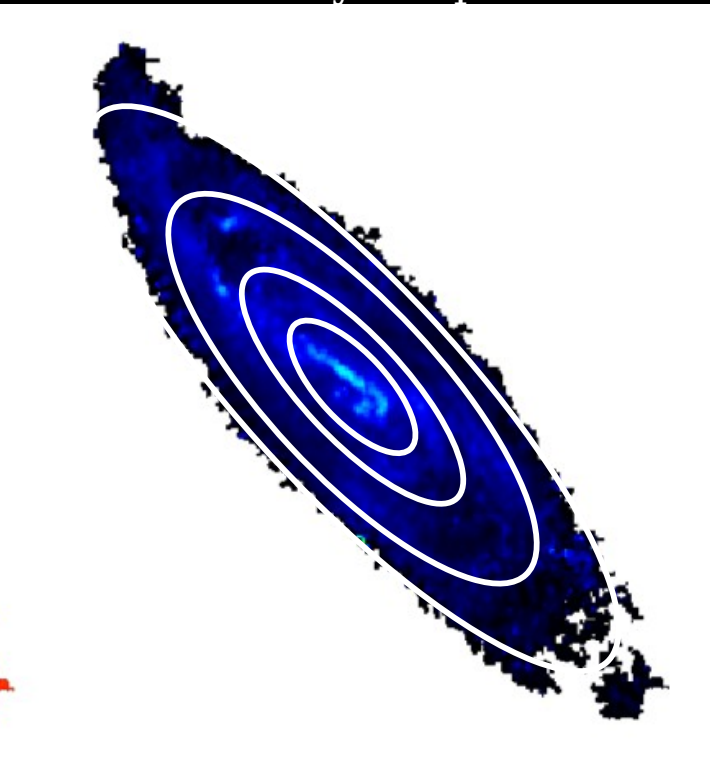
Optical + HI Distribution



HI line-of-sight velocity



HI velocity "dispersion"



How to derive a rotation curve:

- Divide galaxy into a set of concentric rings
- Deprojection from **sky plane** to **galaxy plane**

$$V_{\text{l.o.s.}} = V_{\text{sys}} + V_{\text{rot}} \sin(i) \cos(\theta)$$

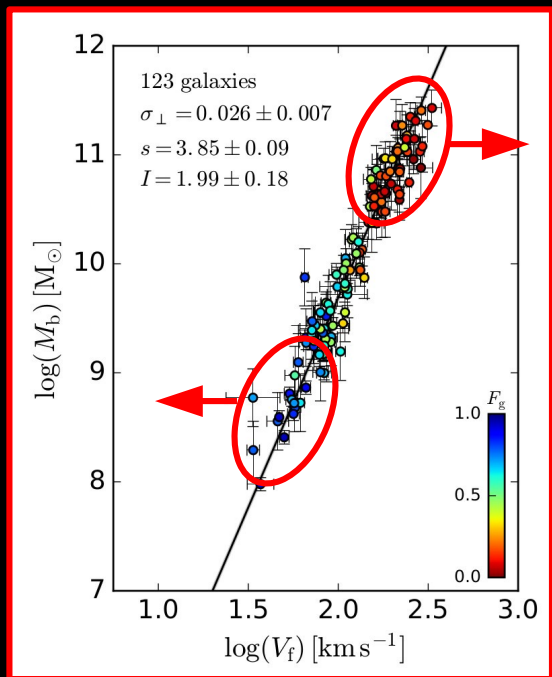
$$\cos(\theta) = \text{fnc}(\text{center, position angle})$$

i = disk inclination angle

θ = azimuthal angle

V_{sys} = systemic velocity

BTFR for different velocity definitions



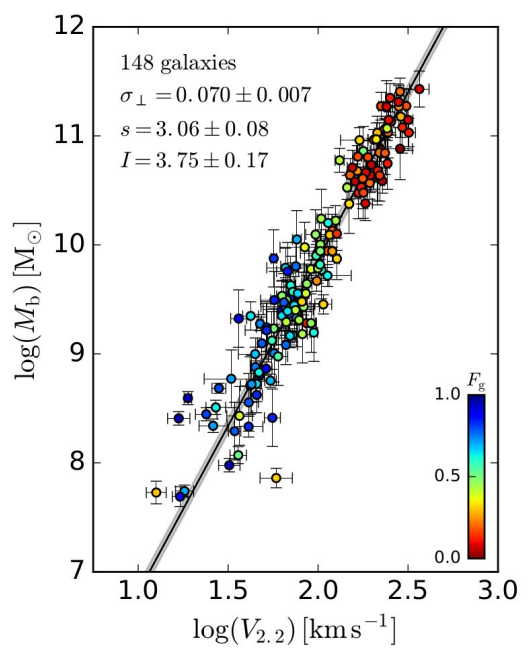
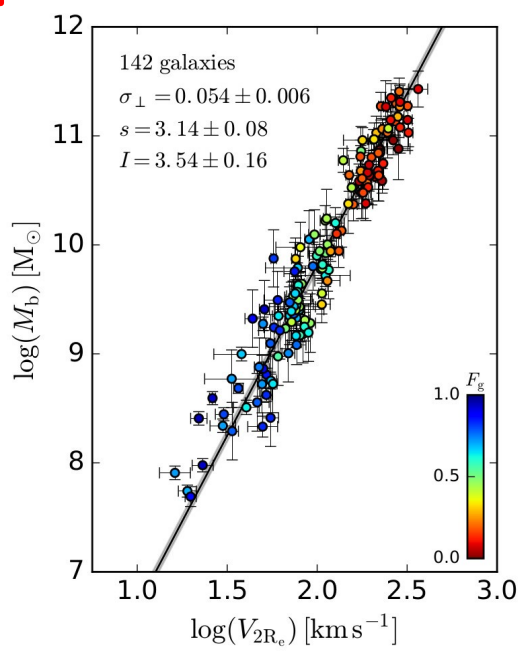
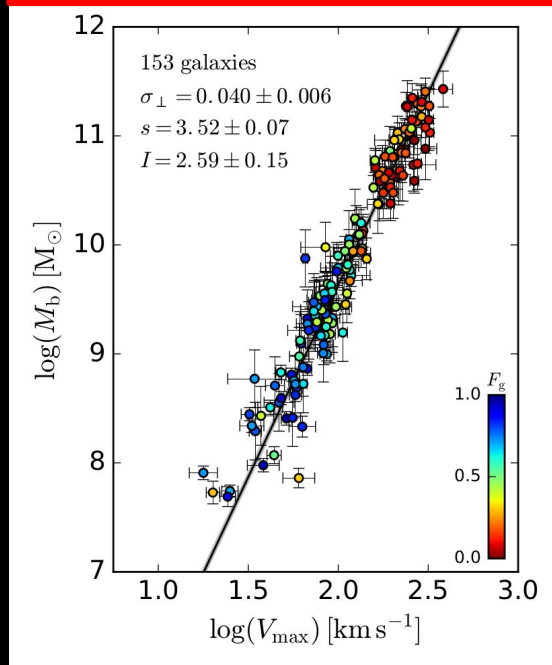
Why M_b - V_{flat} relation is steeper?

Rotation curve shapes!

At high M_b : declining RCs $\rightarrow V_{\text{in}} > V_{\text{flat}}$

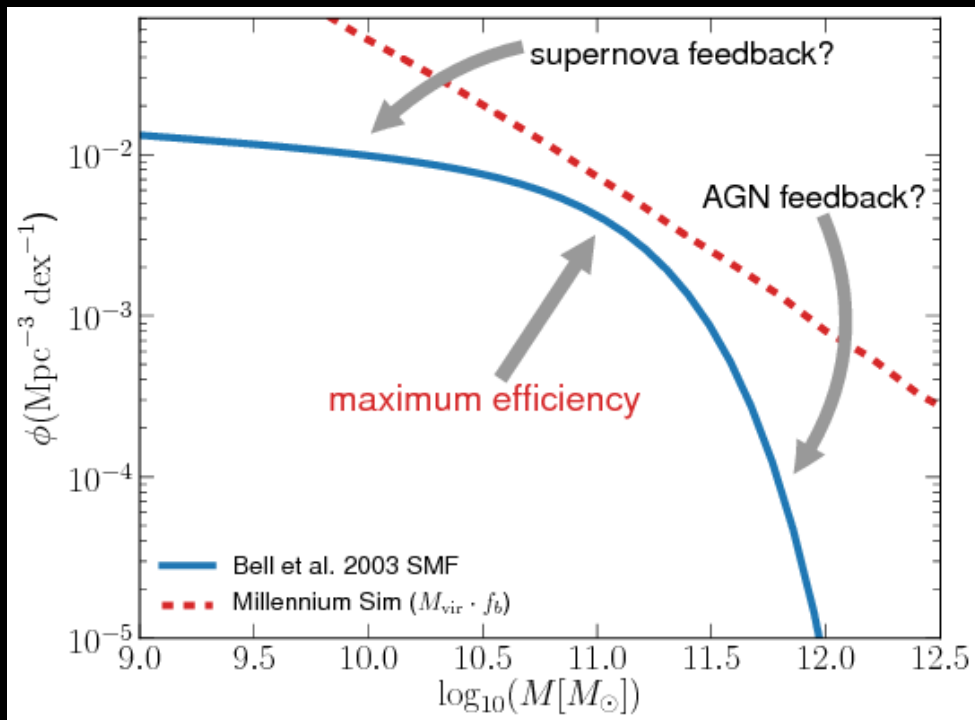
At low M_b : rising RCs $\rightarrow V_{\text{in}} < V_{\text{flat}}$

Inner velocities give shallower BTFR



Lelli+2019

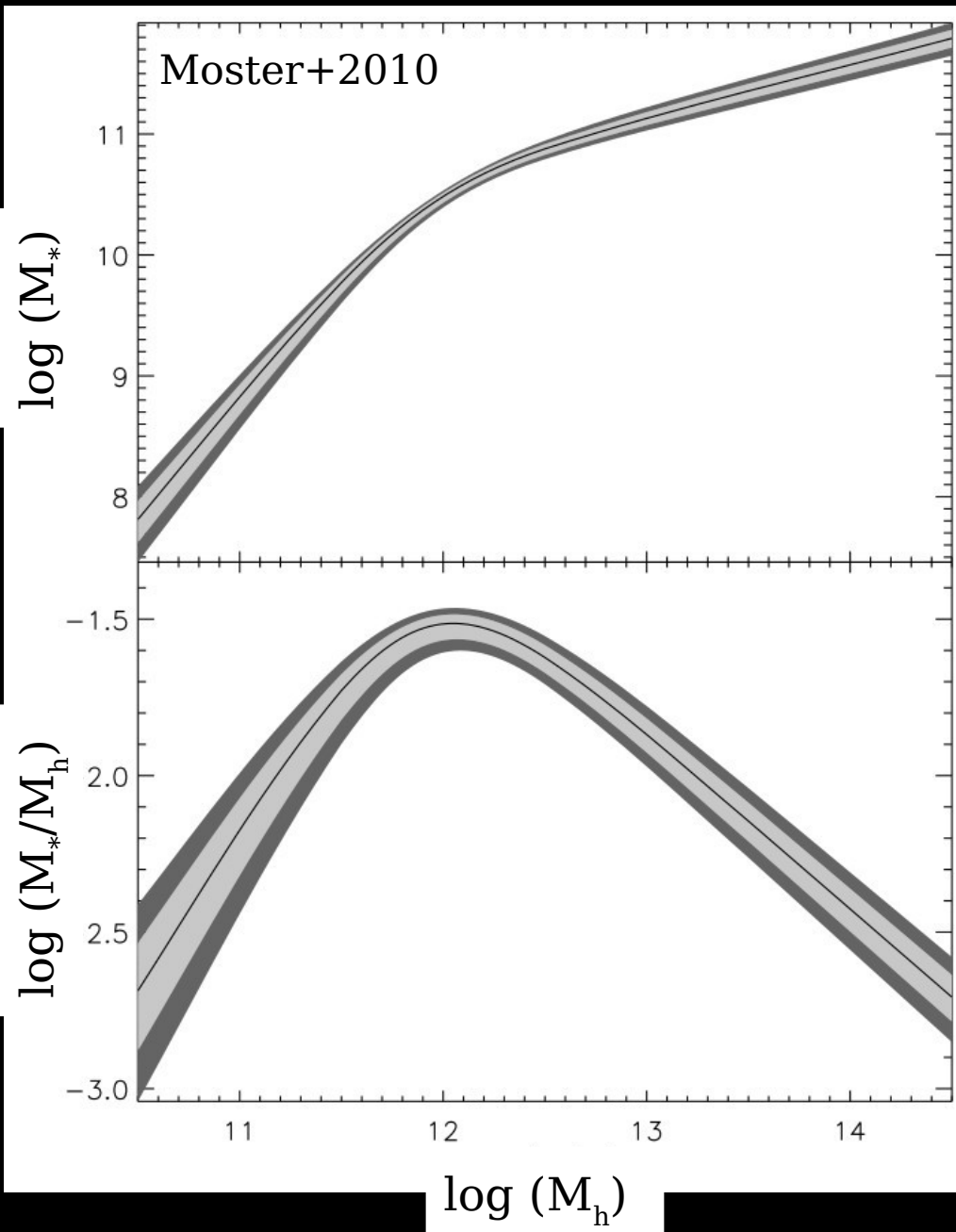
The Stellar Mass Function Problem



A constant M_*/M_h cannot reproduce the observed stellar mass function!

Basics of Abundance Matching (AM):

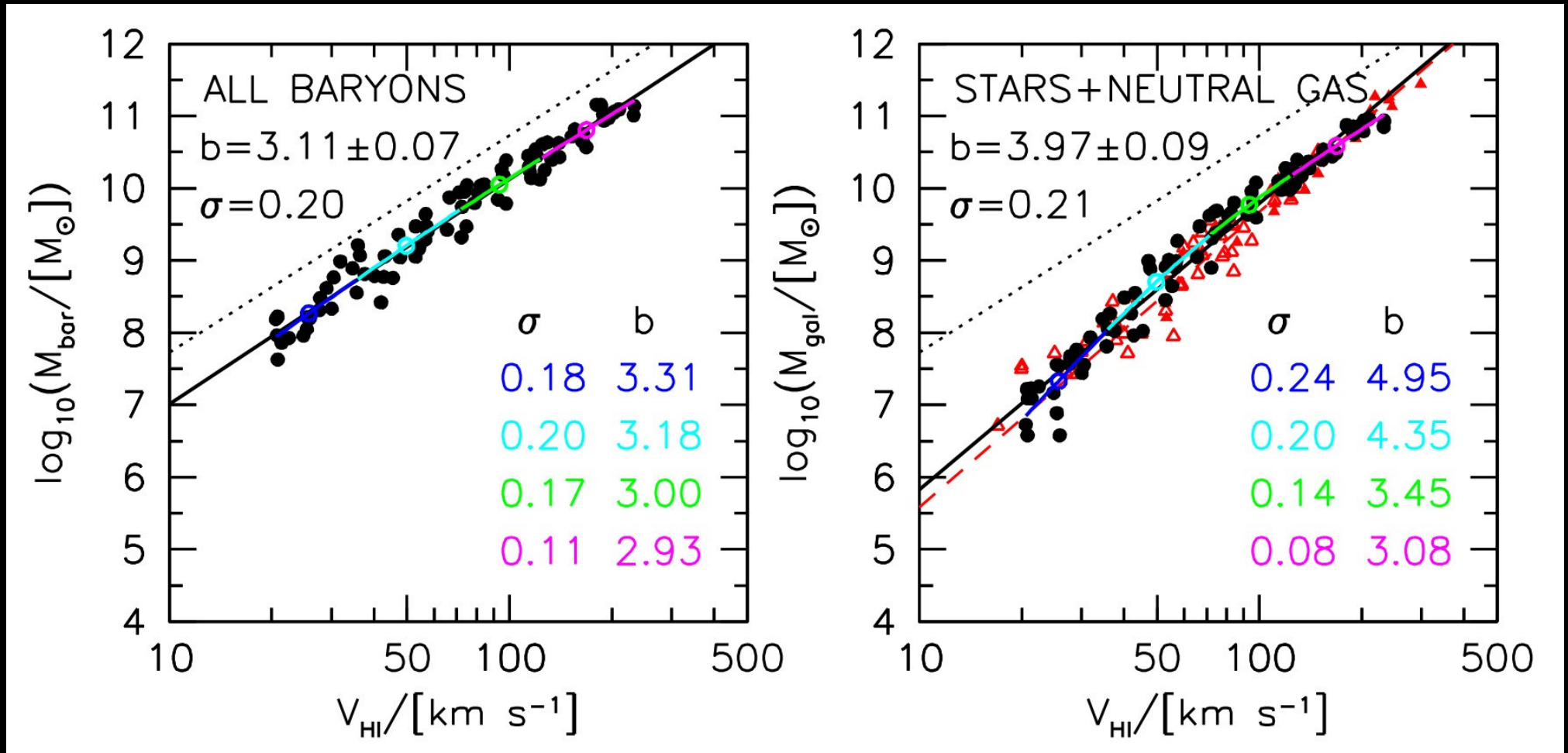
- Order galaxies and halos by mass
- Assign the most massive galaxy to the most massive halo, and so on.
- Derive M_* - M_h relationship



BTFR from hydrodynamical simulations

If we could measure hot gas...

What we can actually measure!



NIHAO zoom-in cosmological simulations of galaxy formation (Dutton+2017)
BTFR curvature has almost disappeared and the scatter is small.