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### Quantum machine learning for LHC data: Classification & Anomaly Detection

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### **Workflow: Model-dependent searches**





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### **Quantum machine learning**



The qubit:

$$\left|\psi\right\rangle = \alpha \left|0\right\rangle \! + \! \beta \left|1\right\rangle \\ \equiv \cos\left(\frac{\theta}{2}\right)\left|0\right\rangle \! + \! e^{i\phi}\sin\left(\frac{\theta}{2}\right)\left|1\right\rangle$$





The qubit:

$$\left|\psi
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Generic qubit operations (quantum gates)  $U = e^{-i\vec{\theta} \cdot \frac{\vec{\sigma}}{2}} \in SU(2):$ 

$$U(\theta,\phi,\lambda) = \begin{pmatrix} \cos\left(\frac{\theta}{2}\right) & -e^{i\lambda}\sin\left(\frac{\theta}{2}\right) \\ e^{i\phi}\sin\left(\frac{\theta}{2}\right) & e^{i(\phi+\lambda)}\cos\left(\frac{\theta}{2}\right) \end{pmatrix}$$

Construct all possible gates from  $U( heta,\phi,\lambda)$ 



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$$H = rac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \equiv U \begin{pmatrix} \pi \\ 2 \end{pmatrix}, 0, \pi$$



#### Single qubit gates

Decomposition of unitary up to global phase

 $U(\theta, \phi, \lambda) \sim R_z(\lambda) R_y(\theta) R_z(\phi)$ 

True for any other non-parallel axes.

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#### Multi-qubit gates

Controlled-X gate (CNOT)

$$CX_{q_0,q_1} = |0\rangle \langle 0| \otimes \mathbb{I}_{2 \times 2} + |1\rangle \langle 1| \otimes X$$
$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} = \begin{matrix} q_0 \\ q_1 \\ \hline \Phi \end{matrix}$$

#### Single qubit gates

Decomposition of unitary up to global phase

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#### Multi-qubit gates

Controlled-X gate (CNOT)

Hardware implementation: decompose quantum circuit to the physical (implemented) set of gates.

The above "building blocks" can construct any n-qubit quantum circuit by operating on **at most** two qubits at a time [DiV95].

### Quantum computer technologies



# Motivation

#### Why quantum machine learning? Why for HEP?

#### Practical and exploratory answer

Investigate a new set of ML techniques to assess for advantages. Why not?

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#### **Fundamental motivation**

Exploitation of information and correlations (quantum remnants) inherent in HEP data? Some natural applications: Quantum simulation of parton shower [Phys. Rev. D 106, 056002], Simulating Lattice Field Theory [arXiv:2302.00467]

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#### **Theoretical results**

Generalisation with few data, computational speed-ups, uncover correlations unrecognisable to classical methods, etc.

[M. Caro et al., Nature Communications 13, 4919 (2022)] [A. Abbas et al., Nature Computational Science 1, 403 (2021)] [Y. Liu et al., Nature Physics 17, 1013 (2021)] [H. Huang et al., Nature Communications 12, 2631 (2021)]

[H . Huang et al.,, Science 376, 1182 (2022)] [N. Pirnay et al., arXiv: 2212.08678 (2022)]

Among others...



#### Noisy intermediate scale quantum devices

- Circuit width: limited number of qubits.
- Circuit depth: limited number of operations per qubit (small decoherence times).
- Hardware noise.



Heavily dependent on the

Theoretically provable quantum

choice of the kernel.

advantages.

 $\mathcal{O}(N_{\mathrm{train}}^2)$ 

#### **Kernel methods**

Quantum Support Vector Machines



kernel-based training

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Quantum Neural Networks



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- Trainability issues (vanishing gradients)
- $\mathcal{O}(N_{\text{train}})$

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#### Current hardware limitations: feature reduction presently needed for realistic datasets.



SVM objective function is equivalent to (dual Lagrangian):

maximize 
$$L(c_1, \dots, c_n) = \sum_{i=1}^n c_i - \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n y_i c_i (\vec{x}_i \cdot \vec{x}_j) y_j c_j$$
  
subject to  $\sum_{i=1}^n c_i y_i = 0$ , and  $0 \le c_i \le C$ ,  $\forall i$ 

Kernel trick:  $(\vec{x}_i \cdot \vec{x}_j) \mapsto k(\vec{x}_i \cdot \vec{x}_j) = \phi(\vec{x}_i) \cdot \phi(\vec{x}_j)$ 



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Input Space





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Kernel trick:  $(\vec{x}_i \cdot \vec{x}_j) \mapsto k(\vec{x}_i \cdot \vec{x}_j) = \phi(\vec{x}_i) \cdot \phi(\vec{x}_j)$ 

Make the kernel quantum

$$|0\rangle = |0\rangle = U^{\dagger}(\vec{x}_{i}) = U(\vec{x}_{j}) = \Delta \Rightarrow K_{ij} = |\langle 0|U^{\dagger}(\vec{x}_{i})U(\vec{x}_{j})|0\rangle|^{2}$$
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\*Can be generalised to unsupervised learning

### **Quantum Neural Networks**

#### Variational quantum algorithm workflow





[M. Cerezo, et al. Nat. Rev. Phys. 3, 625–644 (2021)]

# Quantum Neural Networks

#### Variational quantum algorithm workflow



### **Results**

### Finding new-physics in dijet events with QML



#### Anomaly detection with quantum machine learning

**Background:** QCD dijet events.  $n^{\text{features}} = 300$  per jet  $\longrightarrow$  Too many for current hardware.

**BSM anomalies:** Graviton & New Scalar Boson  $\longrightarrow$  Multi-jet final state  $G \rightarrow W^-W^+$   $A \rightarrow HZ \rightarrow ZZZ$ 

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### Kernel-based quantum anomaly detection

Unsupervised quantum kernel machine  $K_{ij} = |\langle 0|U^{\dagger}(\vec{x}_i)U(\vec{x}_j)|0\rangle|^2$ 

#### Designed data encoding circuit



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[K.A. Wozniak\*, VB\*, E. Puljak\*, et al., arXiv: 2301.10780]

# **Kernel-based quantum anomaly detection**

Unsupervised quantum kernel machine  $K_{ij} = |\langle 0|U^{\dagger}(\vec{x}_i)U(\vec{x}_j)|0\rangle|^2$ 



#### Instance of significant and consistent quantum performance advantage!

Very exciting and first of its kind result (HEP + Anomaly detection)!

# Quantum circuit properties vs. performance

#### Performance vs. circuit architectures

Analysing circuit depth (expressibility) and amount entanglement

Importance of intrinsically quantum properties of the feature map.



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#### Performance vs. circuit architectures

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Importance of intrinsically quantum properties of the feature map.

Up to **five times** the performance of the classical model for 16 qubits!



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### Quantum hardware runs

Submit jobs to a real machine (ibm\_toronto) using IBMQ cloud. (CERN quantum-hub)

Map algorithm to hardware qubits.

Minimal instance 100 + 100 (train + test) datapoints.



Superconducting qubits connectivity topology

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Kernel Machine Run	AUC	$\langle {\rm tr} \rho^2 \rangle$
Hardware $L = 1$ Ideal $L = 1$	$0.844 \\ 0.999$	0.271(6) 1
Hardware $L = 3$ Ideal $L = 3$	$\begin{array}{c} 0.997 \\ 1.0 \end{array}$	0.15(2) 1
Classical	0.998	-

Purity of fully mixed state:  $1/2^{n_{\rm q}}\approx 0.39\times 10^{-2}$  (decoherence = loss of "quantumness")

 $\langle \mathrm{tr} \rho^2 \rangle = \langle K(x_i, x_i) \rangle$  $\rho(x_i) = U(x_i) |0\rangle \langle 0| U^{\dagger}(x_i)$ 

# Proposed data encoding circuit realistic and suitable for current devices

QML for (un)supervised learning: Fundamentally different data representation and processing.

Promising results identifying a **significant and consistent advantage** in anomaly detection!

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Identify HEP problems that quantum models could have a natural inductive bias.

Parallel rapid development of quantum hardware.

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#### For more details checkout:

- K.A. Wozniak<sup>\*</sup>, VB<sup>\*</sup>, E. Puljak<sup>\*</sup>, et al., Quantum anomaly detection in the latent space of proton collision events at the LHC, arXiv:2301.10780
- J. Shuhmacher, L. Bogia, VB, et al. Unravelling physics beyond the standard model with classical and quantum anomaly detection, arXiv: 2301.10787
- VB, S. González-Castillo, et al., Higgs analysis with quantum classifiers, EPJ Web Conf., 251 (2021) 03070

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### **Questions?**

# **Backup slides**



# Quantum clustering for anomaly detection

Construct clusters in the Hilbert space

Quantum distance calculation from clusters:



Minimise the distance with **quantum** (QK-means) or hybrid/**classical** (QK-medians) optimisation algorithms

# Quantum clustering for anomaly detection

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#### **Quantum K-medians**



<sup>[</sup>K.A. Wozniak\*, VB\*, E. Puljak\*, et al., arXiv: 2301.10780]

Minimise the distance with **quantum** (QK-means) or hybrid/classical (QK-medians) optimisation algorithms

Quantum and classical anomaly detection has similar performance.

### **Convolutional autoencoder architecture**



## **Expressibility and entanglement capability**

Expressibility [S. Sim, et al., Adv. Quantum Technol. 2 (2019) 1900070]



Expressibility & Entanglement capability of our data encoding circuit



[K.A. Wozniak\*, **VB**\*, E. Puljak\*, et al., arXiv:2301.10780]

- The normalised data samples are split into training, validation, and testing data sets.
- Classification power metric: Receiver Operating Characteristic (ROC) curve.
- More compact metric: Area Under Curve (AUC) of the ROC curve.
- More practical metric: working point of an analysis  $\epsilon_B(\epsilon_S^*)$



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