Tidal effects in a Worldline Quantum Field Theory

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based on work with: Jan Plefka, Gustav Mogull, Gustav Jakobsen

Worldline Quantum Field Theory

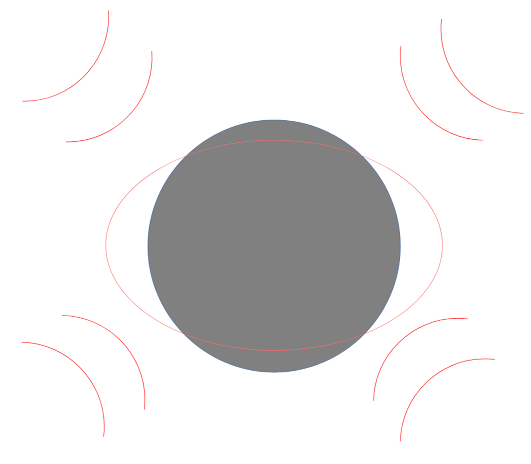

Figure 1: Deformation of a neutron star in an external tidal field

The Worldline-QFT[1] describes a scattering of two massive bodies, which leads to a deflection of the bodies and an emission of gravitational waves as shown in Fig.2. It works in an **EFT framework** in which a black hole is described as a **point particle** coupled to GR and neglects finite size effects:

$$S_{pm} = -\frac{m}{2}\int d\tau (g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu + 1) \quad (1)$$

The whole action of a two body scattering is given by the point particle actions of the bodies, the Einstein-Hilbert action and a gauge fixing term:

$$S = S_{EH} + S_{gf} + \sum_i S_{pm}^{(i)} \quad (2)$$

In a Post-Minkowskian expansion the **graviton** $h_{\mu\nu}$ is the first quantized field:

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu} \quad (3)$$

The second quantized field is given by the **deflection** z^μ from the unperturbed trajectory:

$$x_i^\mu(\tau) = b_i^\mu + v_i^\mu \tau + z_i^\mu(\tau) \quad (4)$$

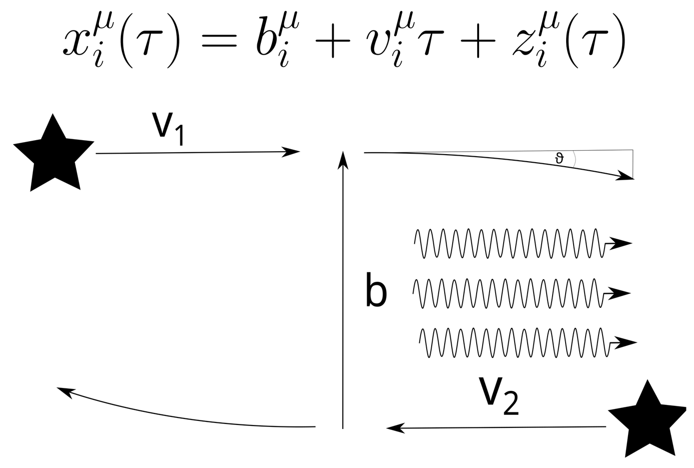

Figure 2: Scattering of two neutron stars with initial velocities v_i

$$\mathcal{A}_x = a_{1,x} + a_{2,x}\log\left(\frac{\gamma+1}{2}\right) + a_{3,x}\frac{\text{arccosh}\gamma}{\sqrt{\gamma^2-1}},$$

$$a_{1,E^2} = \frac{15}{128(\gamma-1)(\gamma+1)^4}\left(937\gamma^9 + 1551\gamma^8 - 2463\gamma^7 - 5645\gamma^6 + 20415\gamma^5 + 65965\gamma^4 - 349541\gamma^3 + 535057\gamma^2 - 360356\gamma + 92160\right),$$

$$a_{1,B^2} = \frac{15}{256(\gamma+1)^4}\left(1559\gamma^8 + 3716\gamma^7 - 1630\gamma^6 - 11660\gamma^5 + 28288\gamma^4 + 155292\gamma^3 - 543442\gamma^2 + 535212\gamma - 180775\right),$$

$$a_{2,E^2} = 22532\left(21\gamma^4 - 14\gamma^2 + 9\right),$$

$$a_{2,B^2} = 157532\left(3\gamma^4 - 2\gamma^2 - 1\right),$$

$$a_{3,x} = -\frac{\gamma(2\gamma^2-3)}{4(\gamma^2-1)}a_{2,x}.$$

$$\mathcal{B}_{E^2} = \frac{45(\gamma-1)}{64(\gamma+1)^4}\left(42\gamma^8 + 210\gamma^7 + 315\gamma^6 - 105\gamma^5 - 944\gamma^4 - 1528\gamma^3 + 22011\gamma^2 - 33201\gamma + 16272\right),$$

$$\mathcal{B}_{B^2} = -\frac{45(\gamma-1)^2(105\gamma^5 + 630\gamma^4 + 1840\gamma^3 + 3690\gamma^2 - 17769\gamma + 15984)}{64(\gamma+1)^4}.$$

Tidal Feynman Rules

The point particle action neglects finite size effects, which are needed for an accurate description of neutron star scattering. To include **quadrupole** tidal effects in the WQFT a tidal action must be added to the point particle action:

$$S_{tidal} = \int d\tau_a c_{E^2}^{(a)} E_{\mu\nu}E^{\mu\nu} + c_{B^2}^{(a)} B_{\mu\nu}B^{\mu\nu} \quad (7)$$

with: $E_{\mu\nu} = R_{\alpha\mu\beta\nu}\dot{x}_a{}^\alpha \dot{x}_a{}^\beta, \quad B_{\mu\nu} = R^*_{\alpha\mu\beta\nu}\dot{x}_a{}^\alpha \dot{x}_a{}^\beta$ (8)

- Love numbers, which encode the tidal properties of the body, are incorporated in the action as Wilson coefficients c_{E^2/B^2}
- Action gives rise to new vertices. These are marked with a square in Feynman diagrams and **emit two or more gravitons from the worldline** as in Fig.3 and can emit an arbitrary number of z^μ fields
- Love numbers **vanish for non-spinning black holes** and tidal effects play no role for non-spinning black holes

Leading Order Waveform

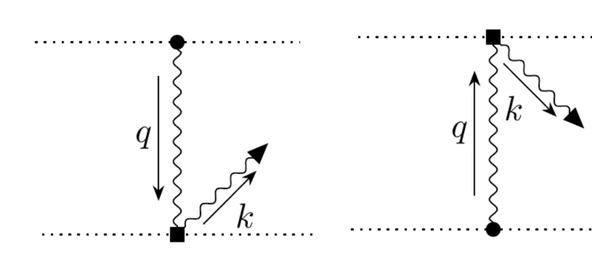

Figure 3: The two diagrams contributing to tidal effects to the leading order waveform

The waveform in the WQFT is given by a Fourier transformation of the expectation value of a single graviton:

$$f(u,\hat{x}) = \frac{8}{\kappa}G\int_\Omega e^{-ik\cdot x}\epsilon^{\mu\nu}k^2\langle h_{\mu\nu}(k)\rangle \quad (9)$$

- Tidal effects do **not contribute to the wave memory** at leading order:
$$\Delta f_{tidal} = f|_{u=-\infty}^{\infty} = 0 \quad (10)$$
- Tidal effects do **not contribute to radiation of angular momentum** at leading order
- The waveform can also be used to calculate the radiated Momentum (in a v expansion)
$$P^\mu_{\text{rad}} = \frac{1}{32\pi G}\int du\sigma[\dot{f}_{ij}]^2\rho^\mu \quad (11)$$
where $f = f_{ij}\epsilon^{ij}$.

Radiated Momentum

$$P^\mu_{\text{rad,tidal}} = \frac{G^3\pi m_1^2 m_2^2}{|b|^7}\left[\left(c_{E^2}^{(1)}\mathcal{A}_{E^2} + c_{E^2}^{(2)}\mathcal{B}_{E^2} + c_{B^2}^{(1)}\mathcal{A}_{B^2} + c_{B^2}^{(2)}\mathcal{B}_{B^2}\right)\frac{\gamma v_2^\mu - v_1^\mu}{\sqrt{\gamma^2-1}} + (1\leftrightarrow 2)\right] \quad (12)$$

In-In-Formalism [2]

Taking into account a classical system obeys causality, it is required to use the so called in-in-formalism [3], rather than the usual in-out-formalism. In this formalism fields propagate from time $t = -\infty$ to $t = \infty$ and propagate then back to $t = -\infty$. For the calculation in a WQFT this is simply done by replacing Feynman propagators by **retarded propagators**, which point **from cause to action**.
The WQFT provides a very direct access to observables through the calculation of expectation values via Feynman diagrams. To take the classical limit it is enough to consider **only tree level diagrams**.

Tidal Effects to the Deflection

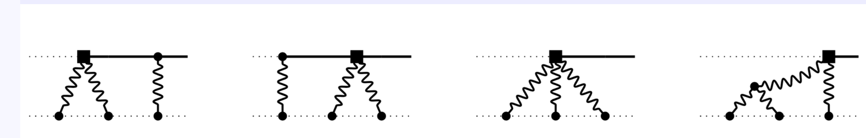

Figure 4: Example graphs contributing to finite size effects of $\Delta p_1^{(3)}$

The WQFT provides a **direct computation of the deflection via the z^μ-field**

$$\Delta p_1^\mu = -m_1\omega^2\langle z_1^\mu(\omega)\rangle|_{\omega=0} \quad (13)$$

and is computed by Feynman diagrams with an outgoing z_1^μ-field, as in Fig. 4. The leading order in finite size effects is given by diagrams with a single tidal vertex.

Conservative Deflection

A pure conservative scattering can be calculated by an evaluation of the appearing loop integrals in the so called **potential region**, which prevents propagators to go on-shell and excludes radiative effects. The kinematic of the bodies gets simplified:

$$(p_i + \Delta p_i)^2 = \Delta p_i^2, \quad \Delta p_1^\mu = -\Delta p_2^\mu \quad (14)$$

- part of the higher order deflection can be calculated by lower orders in the PM expansion $\Delta p_i^\mu = \sum_n G^n p_i^{(n)\mu}$
- the $i0$ prescription of propagators can be neglected in all calculations
- self-interaction diagrams do not contribute

Radiative Effects

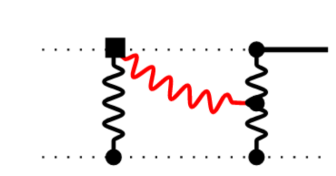

Figure 5: A graph with a radiative contribution. The graviton propagator which can go on shell is marked red

In the WQFT radiative effects are easily incorporated by the **usage of retarded propagators** pointing from cause to action. Additional to the conservative deflection it gives also the radiated momentum

$$P_{rad}^\mu = p_1^\mu + p_2^\mu, \quad (15)$$

which is given in eq.(12) in agreement with previous results. In comparison to other common formalisms the advantage of the WQFT is that this is all done in a single calculation, and there is **no need to differ between conservative and radiative effects**. The result is consistent with the leading order waveform result.

References

[1] Gustav Mogull, Jan Plefka, and Jan Steinhoff.
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[2] L. V. Keldysh.
Diagram technique for nonequilibrium processes.
Zh. Eksp. Teor. Fiz., 47:1515–1527, 1964.

[3] Gustav Uhre Jakobsen, Gustav Mogull, Jan Plefka, and Benjamin Sauer.
All Things Retarded: Radiation-Reaction in Worldline Quantum Field Theory.
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