Gravitational self-force: review/introduction

gravity meets Gravity.

UCD RELATIVITY GROUP

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Science Foundation Ireland sfi For what's next

IRISH RESEARCH COUNCIL
An Chomhairle um Thaighde in Éirinn

- Part I
The SF set-up: dealing with ‘point’ particles

- Part II
Solving the field equation: Teukolsky and a ‘closed form’ Green function

- Part III
Results: Self-force and reality

- **Part I**
The SF set-up: dealing with ‘point’ particles

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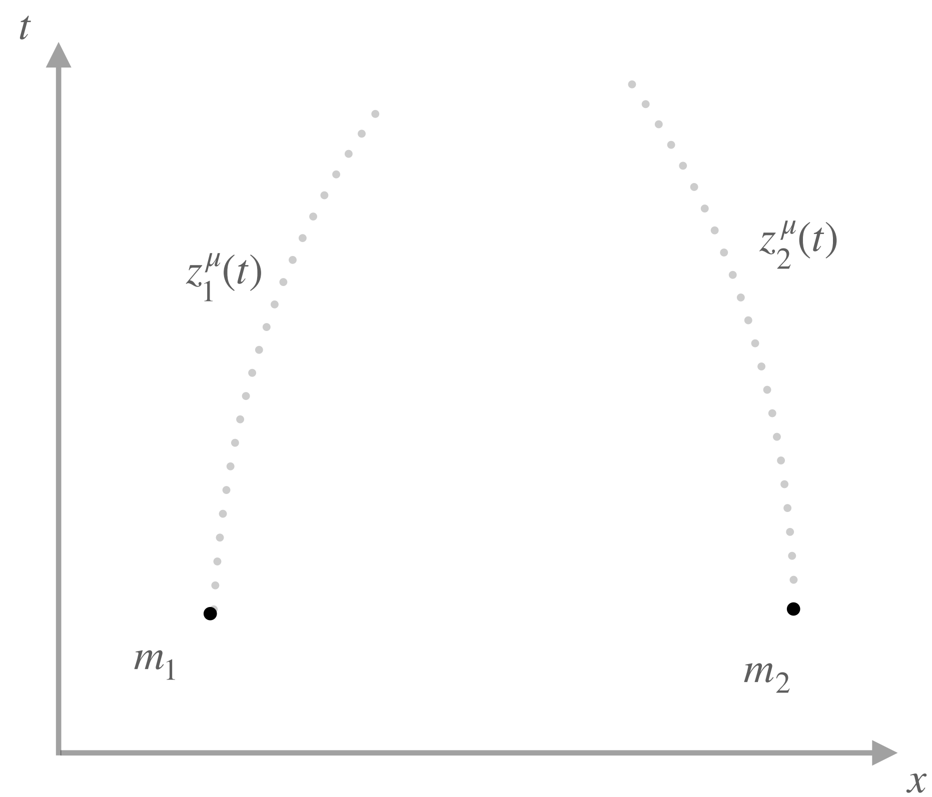
Results: Self-force and reality

- Introduction: The problem of motion, post-Minkowskian/post-Newtonian

'Point particles' endowed with multipole moments, fields constrained by EFEs



$$(1) \quad S = S_M[m_i] + S_{GR}[g]$$

See everyone's talks..

$$(2) \quad G_{\mu\nu}[g] = 8\pi T_{\mu\nu}$$

$$T_{\mu\nu} = \sum_{i=1,2} m_i \int d\tau \delta^4(x, z_i) u^i_\mu u^i_\nu$$

$$g_{\mu\nu} = \eta_{\mu\nu} + G h^{(1)}_{\mu\nu} + G^2 h^{(2)}_{\mu\nu} + \dots$$

Explicitly calculate $h^{(i)}_{\mu\nu}$, then solve geodesic equation in $g_{\mu\nu}$

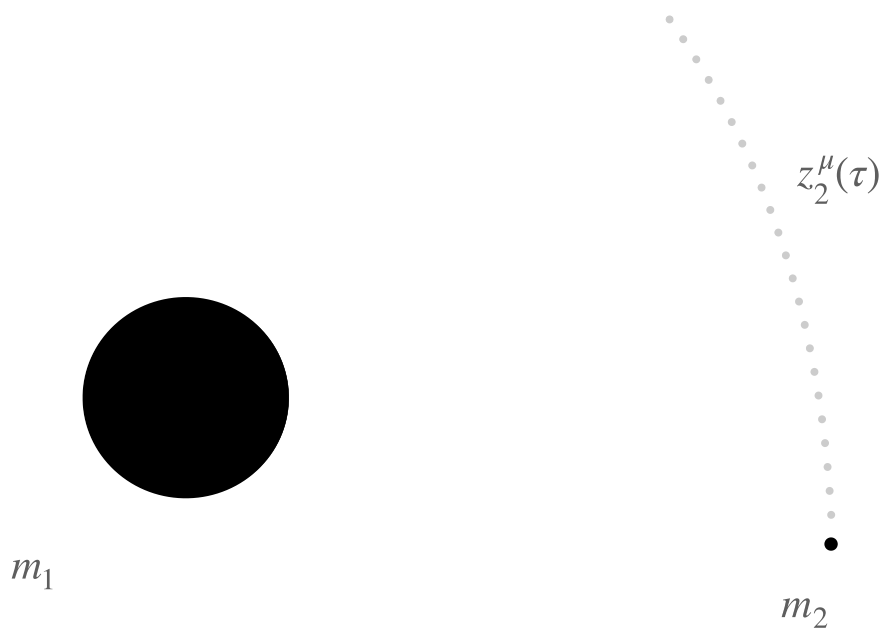
$$\square^0 \, [h^i]_{\mu\nu} = S[h^{i-1}, \dots] \;\; \rightarrow G_{\text{flat}}(x, x')$$

Both approaches rely on delicate treatments of point particle limit (??)

Subtraction of singular terms which *do not* affect the motion via, 'counter terms' in the action, regularisation techniques..

- The problem of motion, BH spacetime

'Point particle' endowed with multipole moments + Exact Kerr BH



$$(2) \quad G_{\mu\nu}[g] = 8\pi T_{\mu\nu}$$

$$T_{\mu\nu} = m_2 \int d\tau \delta^4(x, z_2) u_\mu u_\nu$$

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \epsilon h^{(1)}_{\mu\nu} + \epsilon^2 h^{(2)}_{\mu\nu} + \ldots$$

$$\epsilon = \frac{m_2}{m_1}$$

Explicitly calculate $h^{(i)}_{\mu\nu}$, solve geodesic equation

$$\bar{\nabla}^a \bar{\nabla}_a h^{(1)}_{\mu\nu} + 2\bar{R}_{\mu}{}^{a}{}_{\nu}{}^{b} h^{(1)}_{ab} = 16\pi T_{\mu\nu}$$

$$\bar{\nabla}^a \bar{\nabla}_a h^{(2)}_{\mu\nu} + 2\bar{R}_{\mu}{}^{a}{}_{\nu}{}^{b} h^{(2)}_{ab} = S[h^{(1)}_{\mu\nu}, h^{(1)}_{\mu\nu}]$$

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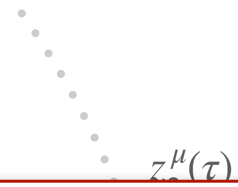

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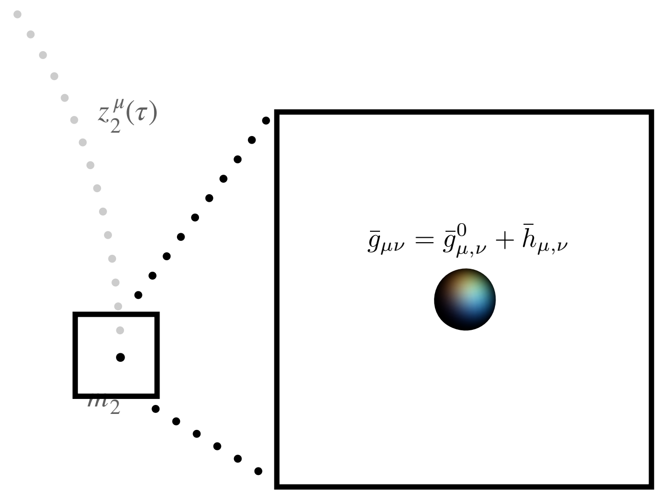
Point particle divergence (UV)

(1) Define a singular potential $h^S_{\mu\nu}$

- $\lim_{x\to z(\tau)} h^{\rm ret}_{\mu\nu} - h^{\rm S}_{\mu\nu}$ is finite
- $h^S_{\mu\nu}$ is causal
- $h^S_{\mu\nu}$ produces no net force

-Detweiler-Whiting singular field
[*Detweiler, Whiting 2003*]

(2)



Match effective global field to a local expansion of Kerr+tidal field

- Determines subtraction term in effective field + self-force

[*Mino, Sasaki & Tanaka 1997* (1SF), *Pound*++ 2016…(2SF)]

Explicitly

$$\bar\nabla^a \bar\nabla_a h^{(1)}_{\mu\nu} + 2R_\mu{}^a{}_\nu{}^b h^{(1)}_{ab} = 16\pi T_{\mu\nu}$$

$$\bar\nabla^a \bar\nabla_a h^{(2)}_{\mu\nu} + 2\bar{R}_\mu{}^a{}_\nu{}^b h^{(2)}_{ab} = S[h^{(1)}_{\mu\nu}, h^{(1)}_{\mu\nu}]$$

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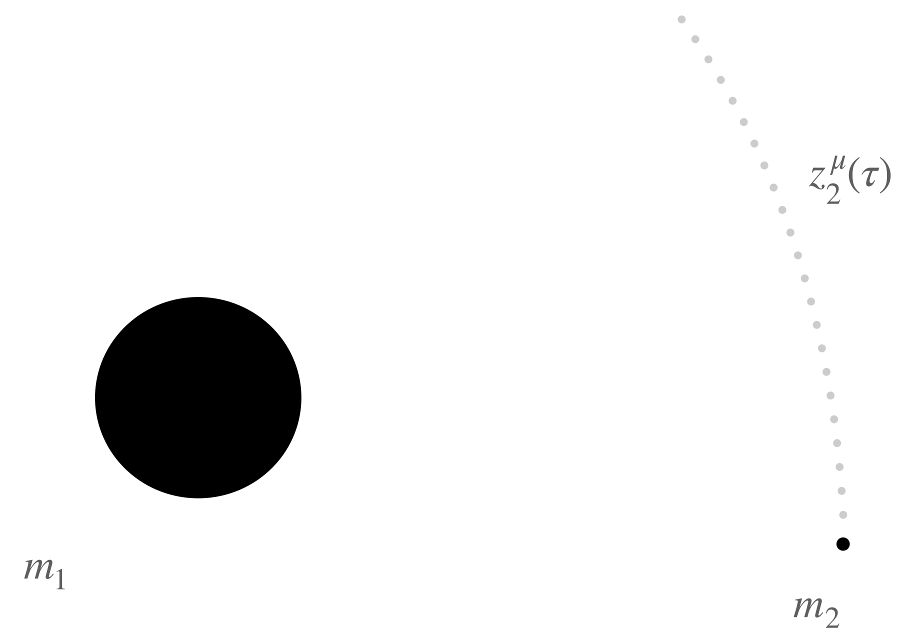

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+

subtraction terms determined from matching

$$u^a \bar{\nabla}_a u^\mu = -\frac{1}{2}(\bar{g}^{\mu\nu} - h^{\mu\nu}_R)(2h^R_{\nu\rho;\sigma} - h^R_{\rho\sigma;\nu})u^\rho u^\sigma$$

$$h^R_{\mu\nu} = \epsilon h^{R,(1)}_{\mu\nu} + \epsilon^2 h^{R,(2)}_{\mu\nu}$$

$\longrightarrow$ All small mass-ratio black hole binaries!

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Results: Self-force and reality

- Teukolsky approach — Lightcone structure & 'gauge invariance'

Newman-Penrose

- Kerr spacetime two radial null vectors, typically written l^μ, n^μ: $\quad l \cdot l = n \cdot n = 0, \quad l \cdot n = -1$
- Rotational degrees of freedom captured by 2 *complex* null vectors $m^\mu, \bar{m}^\mu$

$$g_{ab} = l_{(a}n_{b)} - m_{(a}\bar{m}_{b)}$$

$$R_{abcd} = \Psi_0 n_a \bar{m}_b n_c \bar{m}_d + \Psi_1 n_a l_b n_c \bar{m}_d + \Psi_2 n_a \bar{m}_b m_c l_d + \Psi_3 n_a l_b m_c l_d + \Psi_4 l_a m_b l_c m_d$$

$$\boxed{\begin{array}{ll} \Psi_0 - & s = 2 \\ \Psi_1 - & s = 1 \\ \Psi_2 - & s = 0 \\ \Psi_3 - & s = -1 \\ \Psi_4 - & s = -2 \end{array}}$$

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$$\Psi_2 = \frac{-1}{(r + ia\cos\theta)^3}$$

Teukolsky 1973:

Perturbations : $\Psi_i + \epsilon \psi_i$

- ψ_0, ψ_4-gauge invariant
- Capture all dynamics of the field
- Each satisfy there own field equation

$$O\psi_0 = T_0$$

$$\zeta^{-4} O \zeta^4 \psi_4 = T_4 \qquad \zeta = r - ia\cos\theta$$

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or

$$\bar{\nabla}^a \bar{\nabla}_a h^{(1)}_{\mu\nu} + 2\bar{R}_{\mu\ \nu}^{\ a\ b} h^{(1)}_{ab} = 16\pi T_{\mu\nu}$$

O- separable differential operator!

$${}_s\psi = \sum_{lm} \int d\omega e^{-i\omega t} {}_sR_{lm\omega}(r) \ {}_sS_{lm}(\theta, \phi; a\omega)$$

Just need to solve
a radial ODE

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Spheroidal wave functions

$${}_sS_{lm}(\theta, \phi; a\omega) = {}_sY_{lm}(\theta, \phi) + \left(\alpha_{lm} \; {}_sY_{lm-1}(\theta, \phi) + \beta_{lm} \; {}_sY_{lm+1}(\theta, \phi)\right) a\omega + \ldots$$

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Some facts:

- The field at infinity (e.g. outgoing radiation, scattering amplitudes) ψ_4 is sufficient

- also encode local conservative information:
  - can reconstruct full $h_{\mu\nu}$ from knowledge of $\psi_{0,4}$

$$h_{\mu\nu} = \nabla_a \zeta^4 \nabla_b C^a{}_{(\mu}{}^b{}_{\nu)}[{}_s\psi] + \nabla_{(\mu}\xi_{\nu)} + \mathcal{N}T_{\mu\nu} + g_{\mu\nu}[\delta M, \delta a]$$

[Wald, Cohen & Kegeles, Chrzanowski, Steward 70s]
[Price, Whiting; Acksteiner, Andersson, Backdahl ++; Green, Hollands, Zimmerman +; Dolan, CK, Wardell]

- How do we calculate Teukolsky solutions? e.g. a free field with physical boundary conditions?

Following work by [Leaver 80s], Mano Suzuki and Takasugi [MST ~96] wrote down the homogeneous (vacuum) solutions

$$R^{\text{in}}_{lm\omega}(r) = C^{\text{in}}_{lm\omega} \sum_{n=-\infty}^{\infty} a^{\nu}_{n} \; {}_2F_1(a, b, c, 1 - r/2M)$$

$$R^{\text{up}}_{lm\omega}(r) = C^{\text{up}}_{lm\omega} \sum_{n=-\infty}^{\infty} a^{\nu}_{n} \; U(d, e, r\omega)$$

$$G^{\text{ret}}_{lm\omega}(r, r') = \frac{R^{\text{in}}_{lm\omega}(r')R^{\text{up}}_{lm\omega}(r)}{W[R^{\text{in}}_{lm\omega}(r), R^{\text{in}}_{lm\omega}(r)]}\theta(r - r') + \frac{R^{\text{in}}_{lm\omega}(r)R^{\text{up}}_{lm\omega}(r')}{W[R^{\text{in}}_{lm\omega}(r), R^{\text{in}}_{lm\omega}(r)]}\theta(r' - r)$$

At 1SF

$$\psi = \int drG^{\text{ret}}_{lm\omega}(r, r')T(r') \propto \{\partial^2_r G^{\text{ret}}_{lm\omega}(r, r_p), \partial_r G^{\text{ret}}_{lm\omega}(r, r_p), G^{\text{ret}}_{lm\omega}(r, r_p)\}$$

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$$\psi = \int drG^{\rm r}_{l}$$

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At 1SF

$$\psi = \int drG^{\text{r}}_{l}$$

$$\begin{aligned} T &= \mathscr{T}[T_{\mu\nu}] \propto \partial_r^2[\delta(r-r_p)] \\ &= \delta''(r-r_p) + \delta'(r-r_p) + \delta(r-r_p) \end{aligned}$$

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$$a_n^{\nu} \sim (GM\omega)^{|n|}$$

$$R^{\rm up}_{lm\omega}(r) = C^{\rm in}_{lm\omega} \sum_{n=-\infty}^{\infty} a_n^{\nu}\, U(a,b,r\omega)$$

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—Valid to all orders in G (or $1/c$)

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At 1SF

- Extensively used by PN-SF calculations [MST, Fujita, Bini, Damour, Munna, Evans, Hopper, CK, Ottewill, Wardell..]
- Basis used for *only* 1SF results for generic orbits in Kerr [van de Meent, Nasipak]
- Can provide 100-1000s digits of numerical accuracy [Shah+, Evans, Forseth, Hopper, Munna]
- useful for 2SF??

—Valid to all orders in G (or $1/c$)

The work often is getting from spectral harmonics/ω-domain to t-domain! *(See Bautista, Friday afternoon)*

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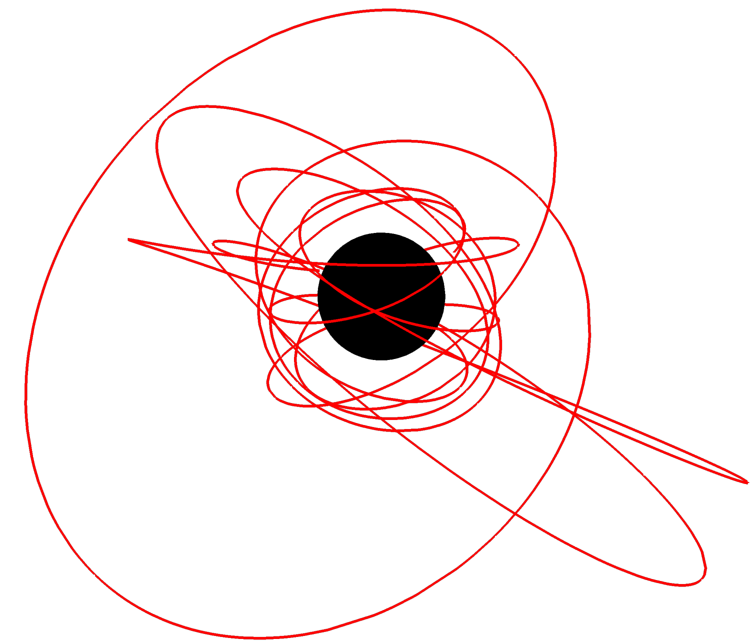
Results: Self-force and reality

- Status report: Where is self-force at

Scientific motivation: Extreme mass-ratio inspirals as a source for LISA

- mass ratios $10^{-4} < \epsilon < 10^{-6}$
- eccentricities $0 \leq e < .9?$
- Kerr spin $0 \leq a < .99?$
- linear order in secondary spin s
- arbitrary orientations of both spins
- $\sim 1/\epsilon$ orbital cycles



GW phase through an inspiral: $\phi = \dfrac{\phi_0}{\epsilon} + \dfrac{\phi_{1/2}}{\epsilon^{1/2}} + \phi_0 + O(\epsilon)$

ϕ_0 : dissipative 1SF (e.g. fluxes)

$\phi_{1/2}$: orbital resonances

ϕ_1 : cons 1SF, diss 2SF + linear-in-spin fluxes

Bound orbit implies discrete frequency spectrum:

$$\omega = m\Omega_\varphi + n\Omega_r + k\Omega_\theta$$

$$\int d\omega \to \sum_{mnk}$$

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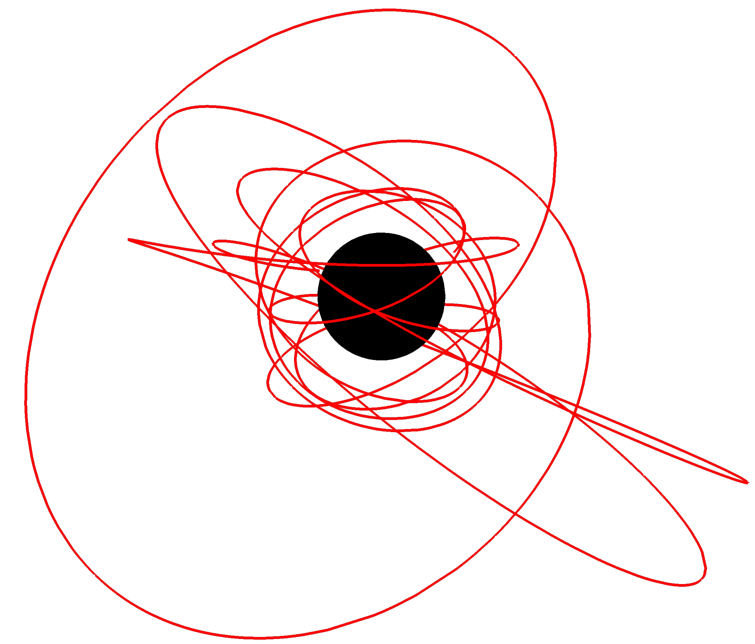

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WHAT IS KNOWN

	Schwarzschild	Kerr
0SF	Analytic solutions for motion, with spin	Analytic solutions for motion, with spin
1SF	**Numerics**: Routine, high accuracy for both cons+diss **Weak-field**: extensive ~20 PN results +spin squared	Methods fairly well understood, best approach debated. **Numerics**: some data for generic orbits exists. With spin emerging **Weak-field**: dissipative PN results for generic, ~7-10PN conservative results.
2SF	• Full quasi-circular inspiral. Intriguing agreement with NR ** • ….	— — — — —

Review Barack and Pound 2018 + many many references..
** [next slide..]

- Status report: Where is self-force at

Resolving gauge issues in Kerr spacetime [Barack, Merlin, Pound, van de Meent, Green, Hollands, Zimmerman, Toomani, Spiers, Pound; Dolan, CK, Wardell]

Numerical developments [Barack, van de Meent, Macedo, Leather, Warburton, Wardell, Zenginoğlu, Nasipak]

post-Newtonian self-force [Bini, Damour, Geralico, Steinhoff, Munna, Evans, Fujita, Wardell, CK, Ottewill]

Overlaps with EOB/PN [Khalil, Antonelli, Vines, Steinhoff, Bini, Damour, Geralico, Wardell, CK, Ottewill, Vines, Akcay, van de Meent]

2SF project [Pound, Miller, Warburton, Wardell, Moxon, Durkan, Upton, Spiers, Bonetto, Sam, Le Tiec]

Spinning secondary [Mathews, Wardell, Witzany, Drummond, Hughes, Piovano, Maselli, Pani, Skoupy, Lukes-Gerakopolous..]

Waveform generation and accelleration [Lynch, Warburton, van de Meent, Katz, Chua, Speri, Hughes]

Resonances [Nasipak, van de Meent, Speri, Gair, Hughes, Lynch]

Regularisation [Heffernan, Upton, Pound]

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** [next slide..]

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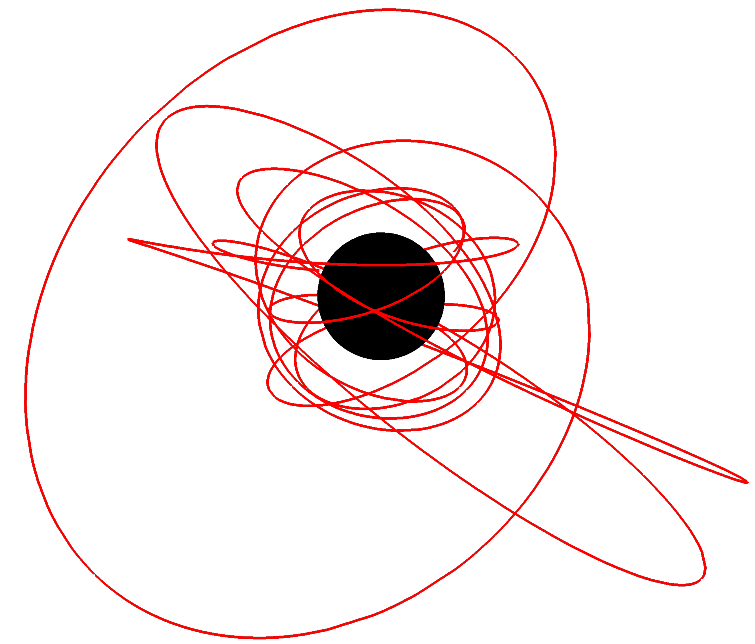

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WHAT IS MISSING?

	Schwarzschild	Kerr
0SF		
1SF	• Generic spin orientations • higher multipole moments on secondary • **Scattering?** Numerics emerging, see next talk	• Full understanding of gauge issues using Teukolsky • Dealing with large computation times of generic case • Full treatment of resonances • Conservative post-Newtonian information in more interesting cases + spinning secondary.
2SF	• Teukolsky approach • efficient non-linear source • more regular gauges? • analytic results (PN) • transition to plunge • eccentric orbits • Conservative gauge invariants	Everything..

- Recent 2SF results: not just extreme mass-ratios

Flux comparison with numerical relativity

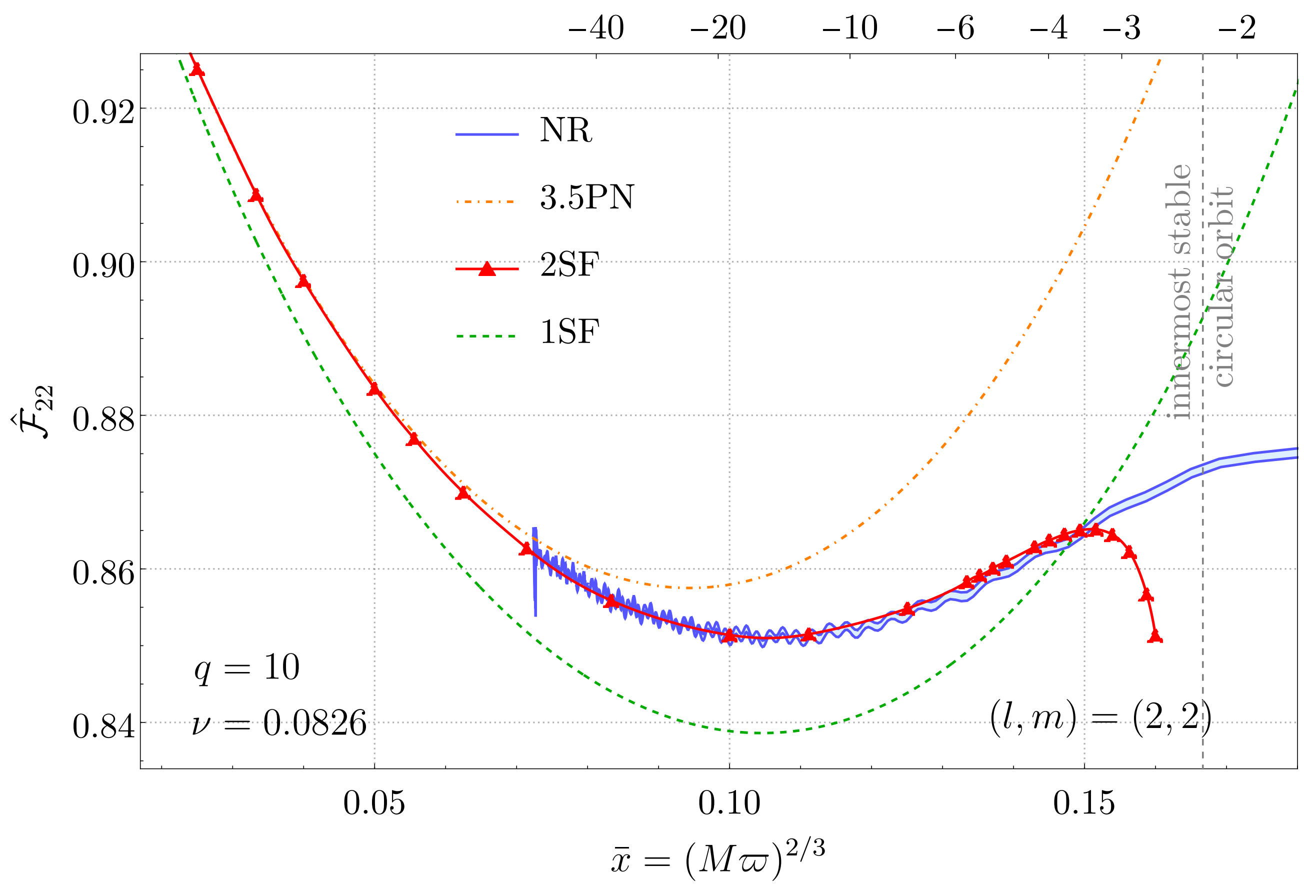

Figure 1 [*Pound, Wardell, Warburton, Miller, Durkan*]

$m_1 : m_2 = 1 : 10$

Series of papers, also including explicit waveform comparisons
[*Pound, Wardell, Warburton, Miller*; + *Durkan*; + *Durkan, Le Tiec*; +*Durkan, Albertini, Nagar*]

- Advertisements:

Black Hole Perturbation Toolkit: bhptoolkit.org.

- Teukolsky solvers: homogeneous + point particle, *Mathematica*, *c*++ implementations
- SpinWeightedSpheroidalHarmonics package
- Kerr Geodesics
- data repositories, numerical data, post-Newtonian series (redshifts, fluxes, gauge invariants..)
- Fast inspiral models

Capra Conference on Radiation Reaction in General Relativity
-July 3-7 Copenhagen, Niels Bohr Institute 2023 (caprameeting.org)