Efficiently evaluating loop integrals in the EFTofLSS using QFT integrals with massive propagators

Diogo Bragança

w/ Babis Anastasiou, Leonardo Senatore, Henry Zheng

arXiv:2212.07421

QCD meets Gravity

16-12-2022

Outline

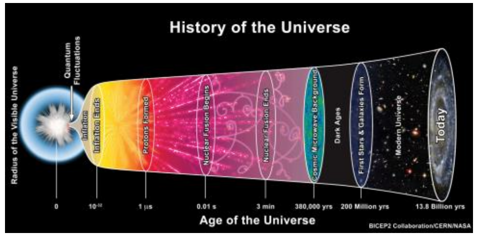
1. Intro: density perturbations in cosmology

2. Why the Effective Field Theory of Large-Scale Structure?

3. How to calculate loop corrections in the EFTofLSS?

4. Results from data analysis

5. All N-point functions at 1-loop



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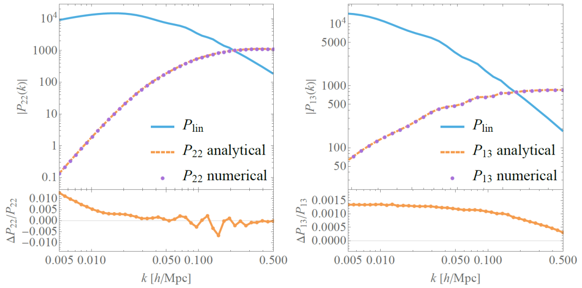
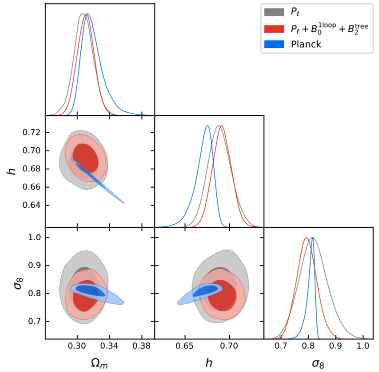




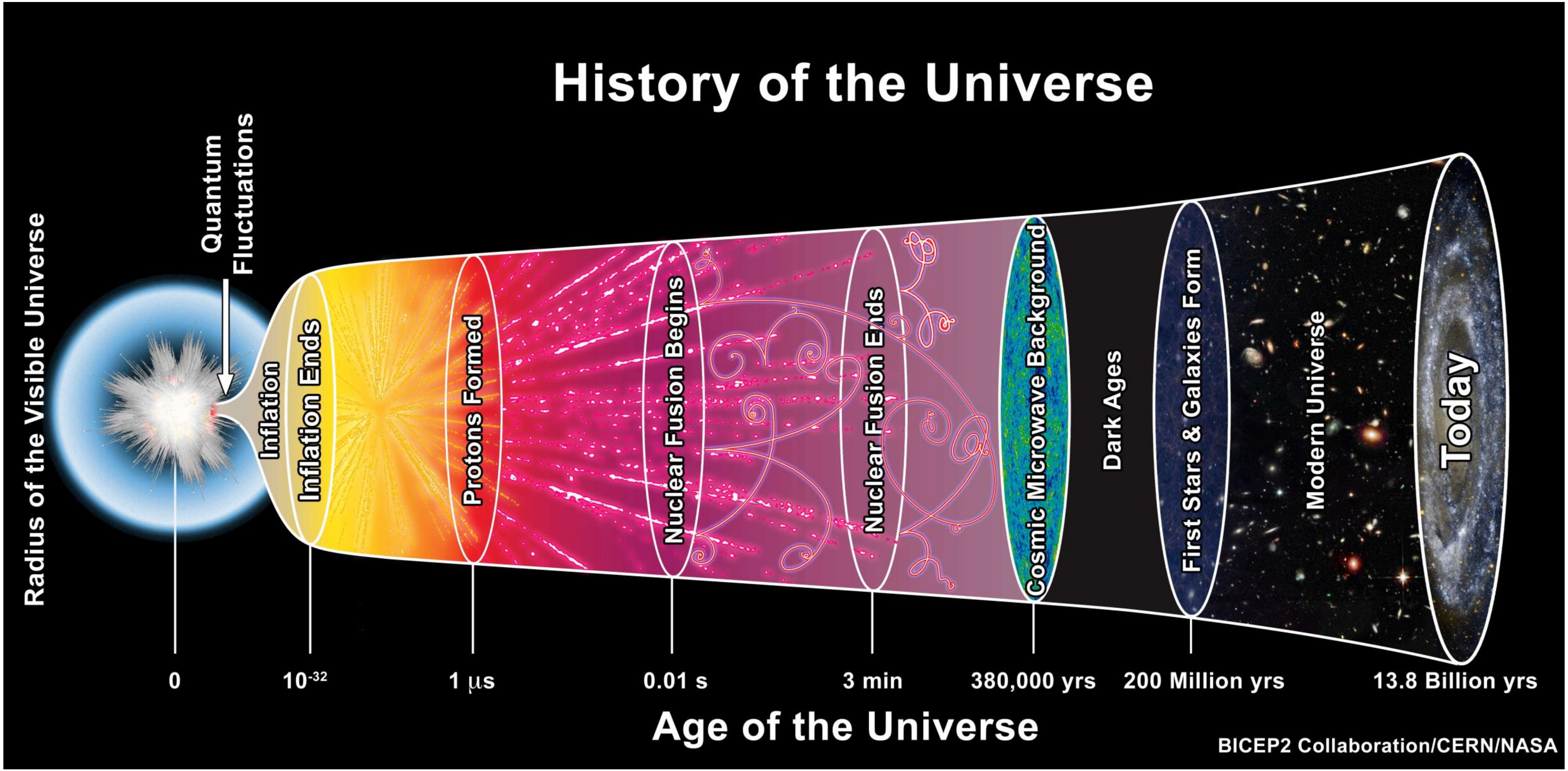
$$\left[-2\rho_4 - \frac{1}{2}\sum_{i,j=1}^{3}(\rho_i-\rho_4)\,\Pi_{ij}\,(\rho_j-\rho_4)\right]\int d^Dq\,\frac{1}{\mathcal{A}_1\mathcal{A}_2\mathcal{A}_3\mathcal{A}_4} = -\frac{1}{2}\int d^Dq\,\frac{2\mathcal{A}_4+\sum_{i,j=1}^{3}(\rho_i-\rho_4)\,\Pi_{ij}\,(\mathcal{A}_j-\mathcal{A}_4)}{\mathcal{A}_1\mathcal{A}_2\mathcal{A}_3\mathcal{A}_4}+\mathcal{O}(\epsilon)$$

Density perturbations in cosmology

Why are they important?

How do the primordial inhomogeneities evolve up until today?

Size of perturbations: $\delta \equiv \frac{\rho - \bar{\rho}}{\bar{\rho}}$



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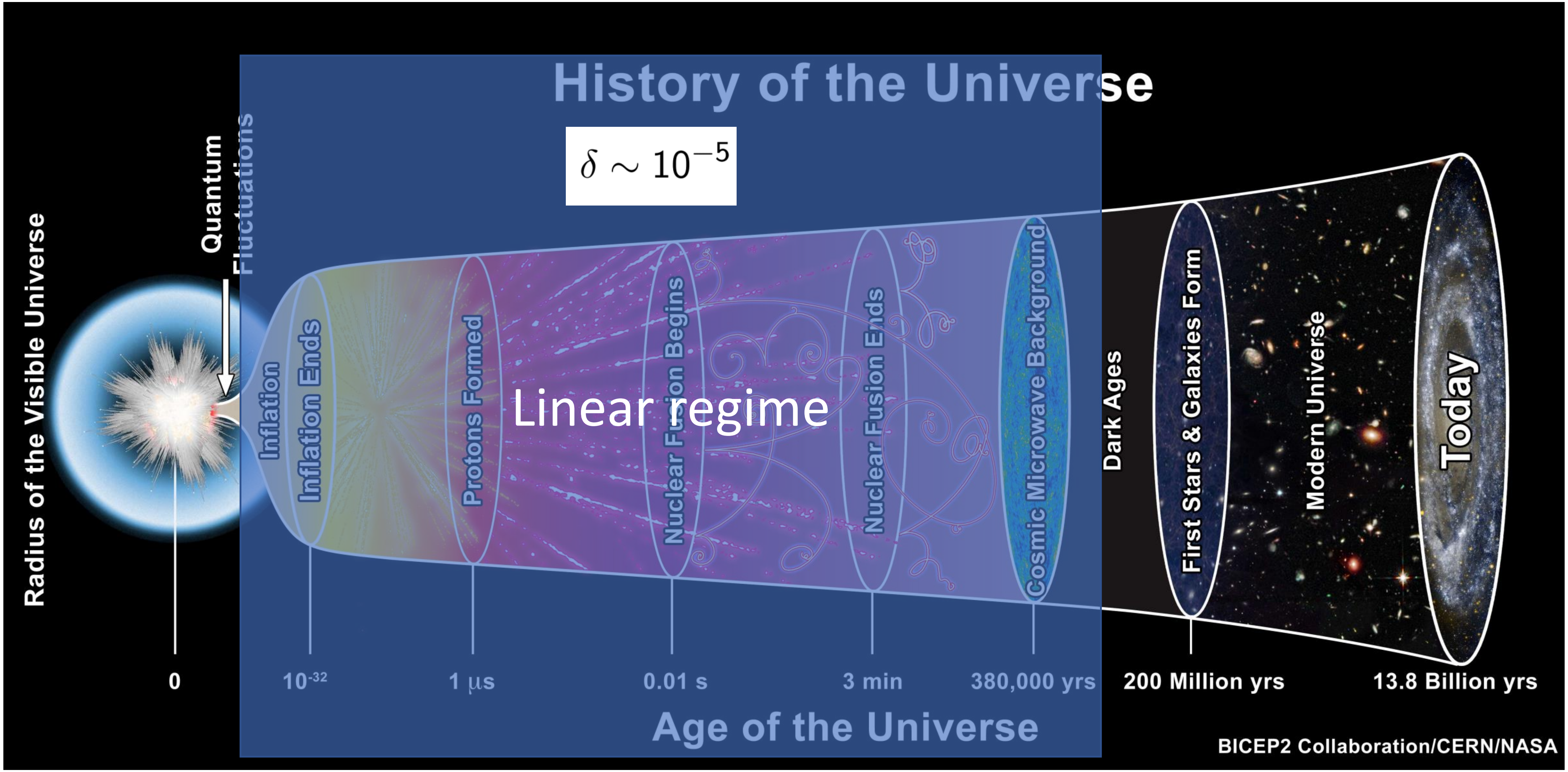

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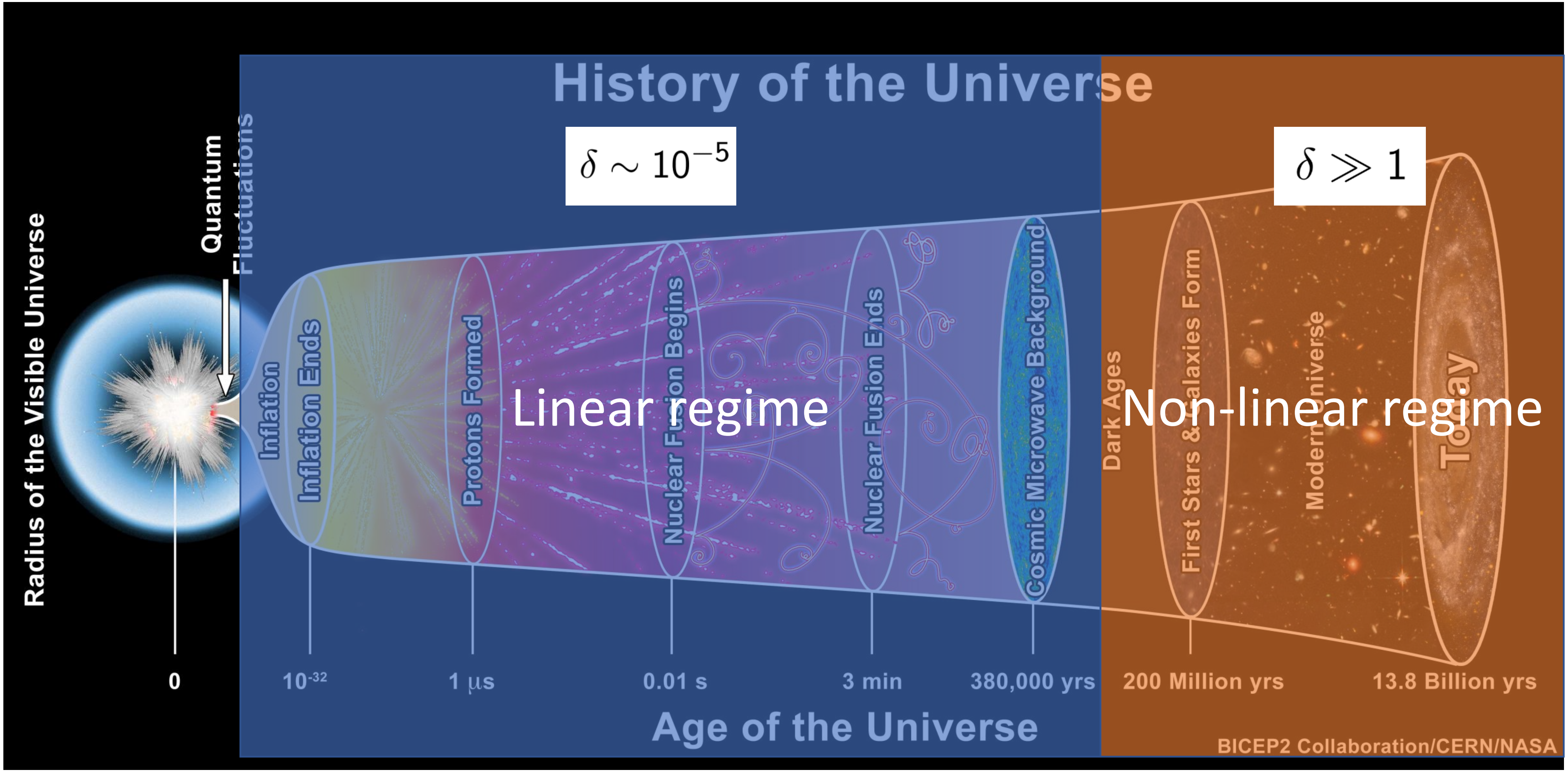

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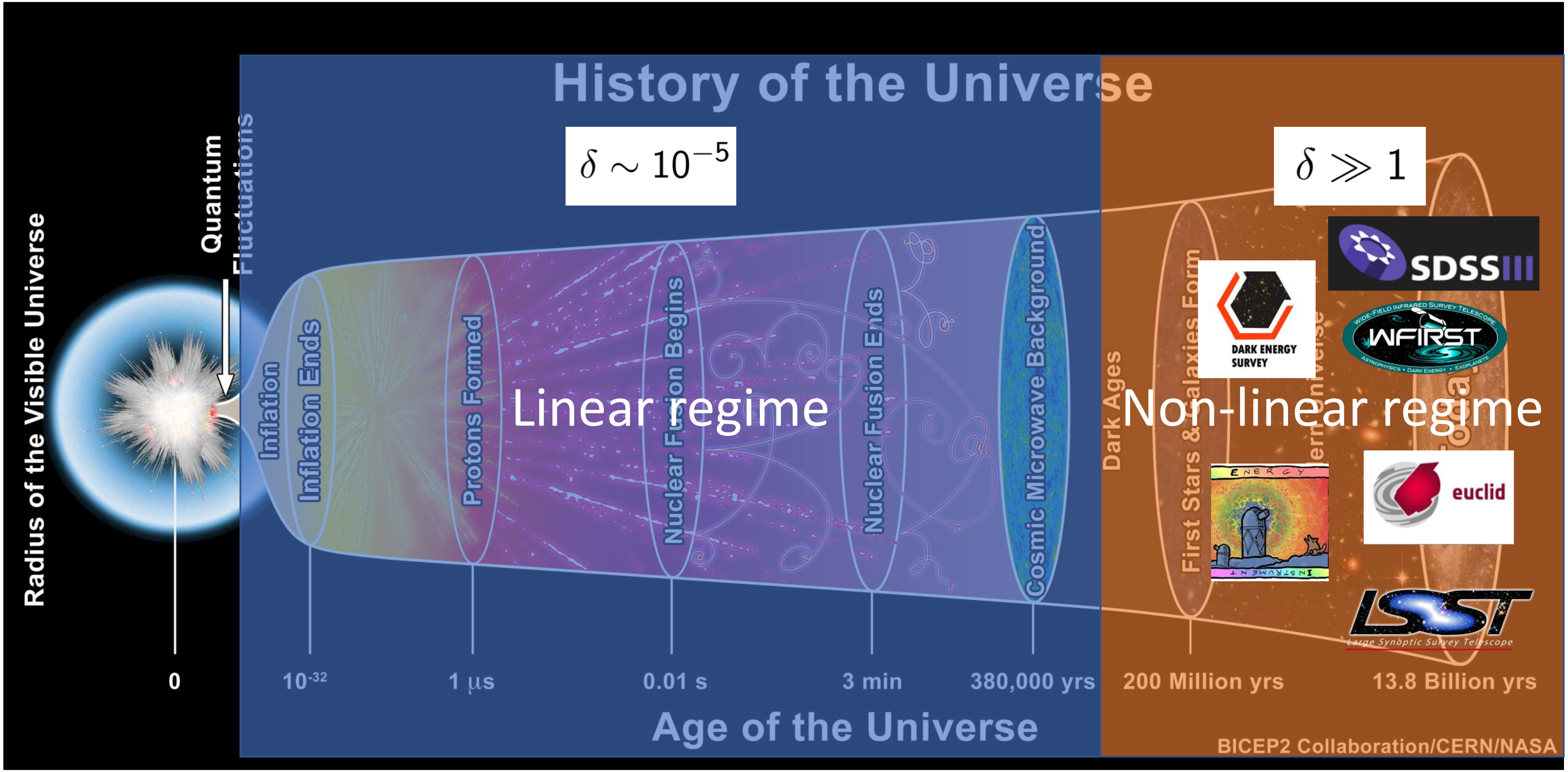

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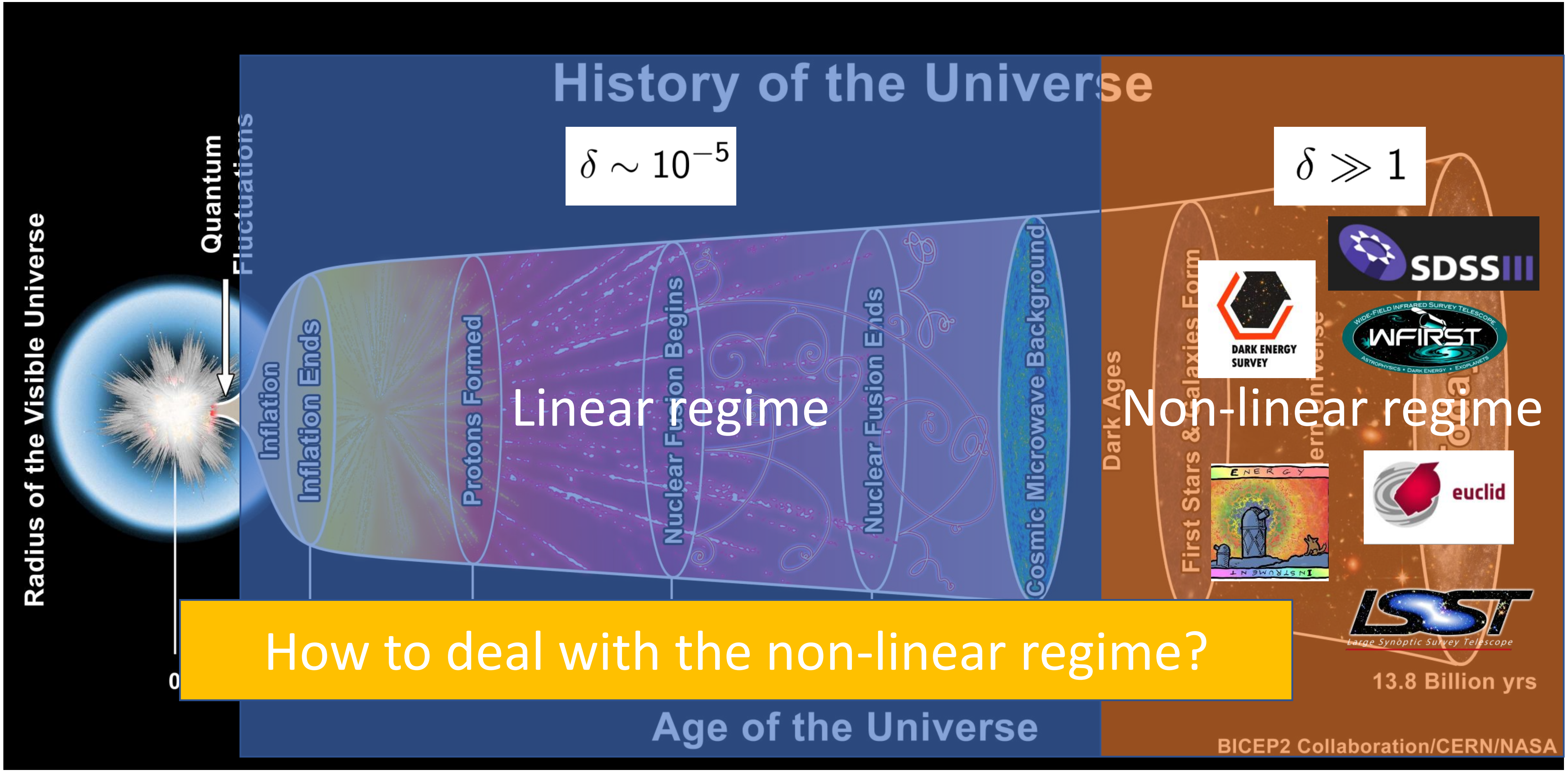

The standard solution: perturbation theory

3 equations:

- Continuity equation (conservation of mass)
- Euler equation
- Poisson equation

$$\partial_t \rho = -\nabla_{\boldsymbol{r}} \cdot (\rho \boldsymbol{u})$$

$$(\partial_t + \boldsymbol{u} \cdot \nabla_{\boldsymbol{r}})\boldsymbol{u} = -\frac{\nabla_{\boldsymbol{r}} P}{\rho} - \nabla_{\boldsymbol{r}} \Phi$$

$$\nabla_{\boldsymbol{r}}^2 \Phi = 4\pi G \rho$$

Assumptions:
- Only consider cold dark matter (CDM)
- It is a perfect fluid (no pressure)

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$$H \equiv \frac{\dot{a}}{a} \qquad \boldsymbol{r} = a(t)\boldsymbol{x}$$

- 0^{th} order $\frac{\partial \bar{\rho}}{\partial t} + 3H\bar{\rho} = 0$
- 1^{st} order $\partial_t \delta = -\frac{1}{a}\nabla \cdot \boldsymbol{v}$

$$(\partial_t + H)\boldsymbol{v} = -\cancelto{=0}{\frac{\nabla \delta P}{a\bar{\rho}}} - \frac{1}{a}\nabla \delta \Phi$$

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Linear system is easy to solve

Why the Effective Field Theory of Large-Scale Structure?

How does it solve the naïve perturbation theory shortcomings?

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Linear solution: $\delta^{(1)}(a, k) \propto a \propto t^{2/3}$

Size of perturbations: $\delta \equiv \dfrac{\rho - \bar{\rho}}{\bar{\rho}}$

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Non-linear equations:

- Continuity equation (conservation of mass) $\partial_t \delta = -\frac{1}{a}\nabla \cdot ((1+\delta)\boldsymbol{v})$
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$$\delta(a,\vec{k}) = \sum_{n=1}^{\infty} a^n\, \delta^{(n)}(\vec{k})$$

$$\delta^{(n)}(\vec{k}) = \int_{\vec{q}_1 \ldots \vec{q}_n} \delta_D(\vec{k} - \sum \vec{q}_i) F_n(\vec{q}_1, \ldots, \vec{q}_n)\delta^{(1)}(\vec{q}_1)\ldots\delta^{(1)}(\vec{q}_n)$$

Problems with perturbation theory

General solution:

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2. Perturbations are not small. Is PT even valid?

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- **Good expansion parameter (small)**
- Short scale interactions induce an **effective cosmological fluid** with pressure and viscosity (Baumann *et al.* arXiv:1004.2488)
- Parameters of effective fluid exactly provide the **counterterms** to renormalize the standard theory (Carrasco *et al.* arXiv:1206.2926)
- Parameters can be fitted to simulations and/or observations

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New theory: Effective field theory of Large-Scale Structure

How to use EFTofLSS efficiently?

$$\langle \delta^{(i)}(k_1)\delta^{(j)}(k_2)\rangle = (2\pi)^3\delta^3(k_1+k_2)P_{ij}(k_1)$$

Need **statistics** to compare with data

Power spectrum

$$\langle \delta(k_1)\delta(k_2)\rangle = (2\pi)^3\delta^3(k_1+k_2)P(k_1)$$

Bispectrum

$$\langle \delta(k_1)\delta(k_2)\delta(k_3)\rangle = (2\pi)^3\delta^3(k_1+k_2+k_3)B(k_1,k_2,k_3)$$

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At 1-loop:

$$P(k) = P_{\rm lin}(k) + P_{13}(k) + P_{22}(k) + P_{\rm ct}(k)$$

Loop diagrams

$$P_{22}(k) = 2\int_q \left[F_2(\boldsymbol{q},\boldsymbol{k}-\boldsymbol{q})\right]^2 P_{\rm lin}(q)P_{\rm lin}(|\boldsymbol{k}-\boldsymbol{q}|)$$

$$P_{13}(k) = 6P_{\rm lin}(k)\int_q F_3(\boldsymbol{q},-\boldsymbol{q},\boldsymbol{k})P_{\rm lin}(q)\,,$$

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At 1-loop:

$$B(k_1,k_2,k_3) = B_{\rm tree} + B^{I}_{321} + B^{II}_{321} + B_{411} + B_{222} + B_{\rm ct}$$

Loop diagrams

$$B_{222}(k_1,k_2,k_3) = 8\int_{\boldsymbol{q}} F_2(\boldsymbol{q},\boldsymbol{k}_1-\boldsymbol{q})F_2(\boldsymbol{k}_1-\boldsymbol{q},\boldsymbol{k}_2+\boldsymbol{q})F_2(\boldsymbol{k}_2+\boldsymbol{q},-\boldsymbol{q})$$
$$\times P_{\rm lin}(q)P_{\rm lin}(|\boldsymbol{k}_1-\boldsymbol{q}|)P_{\rm lin}(|\boldsymbol{k}_2+\boldsymbol{q}|)\,,$$
$$B^{I}_{321}(k_1,k_2,k_3) = 6P_{\rm lin}(k_1)\int_{\boldsymbol{q}} F_3(-\boldsymbol{q},-\boldsymbol{k}_2+\boldsymbol{q},-\boldsymbol{k}_1)F_2(\boldsymbol{q},\boldsymbol{k}_2-\boldsymbol{q})P_{\rm lin}(q)P_{\rm lin}(|\boldsymbol{k}_2-\boldsymbol{q}|)$$
$$+\,5\text{ perms}\,,$$
$$B^{II}_{321}(k_1,k_2,k_3) = F_2(\boldsymbol{k}_1,\boldsymbol{k}_2)P_{\rm lin}(k_1)P_{13}(k_2) + 5\text{ perms}\,,$$
$$B_{411}(k_1,k_2,k_3) = 12P_{\rm lin}(k_1)P_{\rm lin}(k_2)\int_{\boldsymbol{q}} F_4(\boldsymbol{q},-\boldsymbol{q},-\boldsymbol{k}_1,-\boldsymbol{k}_2)P_{\rm lin}(q) + 2\text{ cyclic perms}\,.$$

Calculating the loop integrals – example with power spectrum

$$P(k) = P_{\rm lin}(k) + P_{13}(k) + P_{22}(k) + P_{\rm ct}(k)$$

$$P_{22}(k) = 2\int_q \left[F_2(\boldsymbol{q}, \boldsymbol{k}-\boldsymbol{q})\right]^2 P_{\rm lin}(q) P_{\rm lin}(|\boldsymbol{k}-\boldsymbol{q}|)$$

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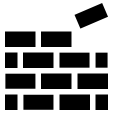
The following strategy is adopted:



Decompose P_{lin} into sum of **predetermined basis functions**

- Basis functions are **cosmology independent.**
- Cosmology dependence is encoded in the **coefficients** of each basis function.

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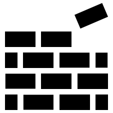

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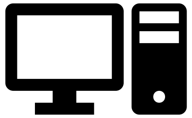

Calculate the loops for each combination of basis functions, obtaining tensors

- Tensor rank depends on the specific diagram.
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Calculate the loops for each combination of basis functions, obtaining tensors

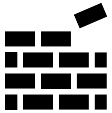
- Tensor rank depends on the specific diagram.
- Tensors are cosmology independent.

Contract the tensors with the cosmology-dependent coefficients

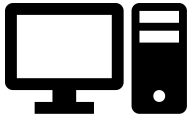
- This directly gives the integral.
- Instead of a numerical integration we are doing a **matrix multiplication.**

Calculating the loop integrals – previous method

FFTLog
Simonovic *et al.* arXiv:1708.0813



Decompose P_{lin} into sum of **predetermined basis functions**



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$$\bar{P}_{\rm lin}(k_n) = \sum_{m=-N/2}^{m=N/2} c_m\, k_n^{\nu+i\eta_m}$$

Coefficients are quicky calculated using FFTLog

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$$\int_{\boldsymbol{q}} \frac{1}{q^{2\nu_1} |\boldsymbol{k} - \boldsymbol{q}|^{2\nu_2}} \equiv k^{3-2\nu_{12}} \mathsf{I}(\nu_1, \nu_2)$$

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$$\bar{P}_{22}(k) = k^3 \sum_{m_1,m_2} c_{m_1} k^{-2\nu_1} \cdot M_{22}(\nu_1,\nu_2) \cdot c_{m_2} k^{-2\nu_2}$$

- Works well for 1-loop power spectrum

However:

- ~50 basis functions required (matrices become very heavy in bispectrum)
- Analytically very challenging past 1-loop power spectrum (dependence in k is not analytic)
- So far, a parameter inference using FFTLog with full 1-loop bispectrum in real data has not been done (see Philcox *et al.* arXiv:2206.02800 for an approximation using simulations)

Calculating the loop integrals – previous method

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New method is required!

Calculating the loop integrals – new method (this talk)

Analytic decomposition
w/ Anastasiou, Senatore, Zheng arXiv:2212.07421

$$f(k^2, k^2_{\rm peak}, k^2_{\rm UV}, i, j) \equiv \frac{\left(k^2/k_0^2\right)^i}{\left(1 + \frac{(k^2 - k^2_{\rm peak})^2}{k^4_{\rm UV}}\right)^j}$$

Decompose P_{lin} into sum of **predetermined basis functions**

$$P_{\rm fit}(k) = \frac{\alpha_0}{1 + \frac{k^2}{k^2_{{\rm UV},0}}} + \sum_{n=1}^{N-1} \alpha_n f(k^2, k^2_{{\rm peak},n}, k^2_{{\rm UV},n}, i_n, j_n) = \sum_{n=0}^{N-1} \alpha_n f_n(k^2)$$

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$$P_{\text{fit}}(k) = \frac{\alpha_0}{1 + \frac{k^2}{k^2_{\text{UV},0}}} + \sum_{n=1}^{N-1} \alpha_n f(k^2, k^2_{\text{peak},n}, k^2_{\text{UV},n}, i_n, j_n) = \sum_{n=0}^{N-1} \alpha_n f_n(k^2)$$

Calculate the loops for each combination of basis functions, obtaining tensors

$$L_B(n_1, d_1, n_2, d_2, k^2, M_1, M_2) \equiv \int_q \frac{|\boldsymbol{k} - \boldsymbol{q}|^{2n_1} q^{2n_2}}{(|\boldsymbol{k} - \boldsymbol{q}|^2 + M_1)^{d_1} (q^2 + M_2)^{d_2}}$$

Contract the tensors with the cosmology-dependent coefficients

Calculating the loop integrals – new method (this talk)

Analytic decomposition
w/ Anastasiou, Senatore, Zheng arXiv:2212.07421

$$f(k^2, k^2_{\mathrm{peak}}, k^2_{\mathrm{UV}}, i, j) \equiv \frac{\left(k^2/k_0^2\right)^i}{\left(1 + \frac{(k^2 - k^2_{\mathrm{peak}})^2}{k^4_{\mathrm{UV}}}\right)^j}$$

Decompose P_{lin} into sum of **predetermined basis functions**

$$P_{\mathrm{fit}}(k) = \frac{\alpha_0}{1 + \frac{k^2}{k^2_{\mathrm{UV},0}}} + \sum_{n=1}^{N-1} \alpha_n f(k^2, k^2_{\mathrm{peak},n}, k^2_{\mathrm{UV},n}, i_n, j_n) = \sum_{n=0}^{N-1} \alpha_n f_n(k^2)$$

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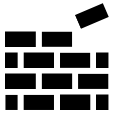
Contract the tensors with the cosmology-dependent coefficients

$$\bar{P}_{22}(k) = \boldsymbol{\alpha}^T M^{(22)}(k^2)\boldsymbol{\alpha}$$

- Works well for 1-loop power spectrum and 1-loop bispectrum
- 16 basis functions required (matrices are much more amenable)
- Differential equation techniques can be used for 2-loop power spectrum (see Samuel's talk yesterday)
- Parameter inference using this method with full 1-loop bispectrum in real data **has already been done** (see D'Amico *et al.* arXiv:2206:08327)

First step: one must have a decent fit

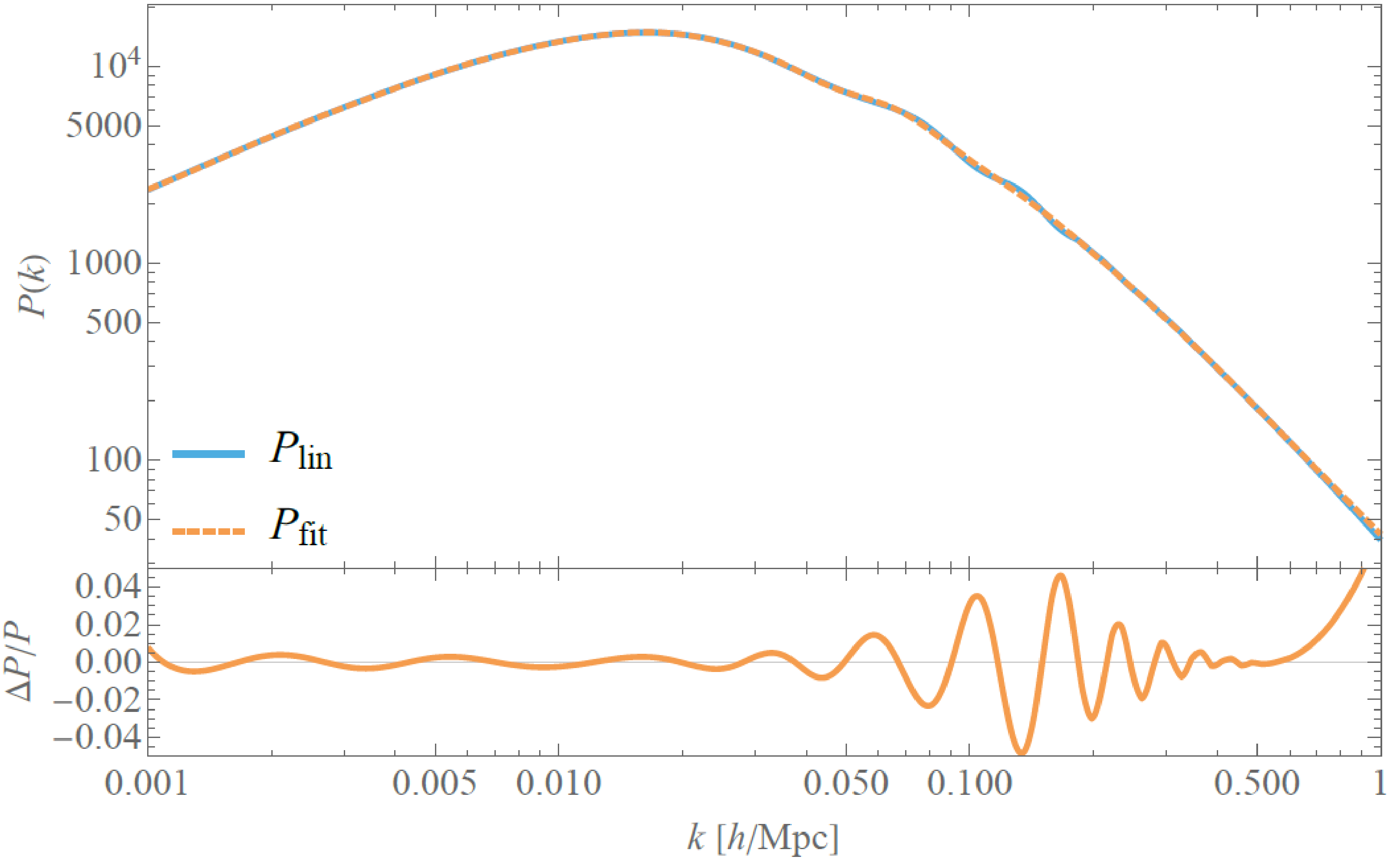
Analytic decomposition
w/ Anastasiou, Senatore, Zheng
arXiv:2212.07421



Decompose P_{lin} into sum of
predetermined basis functions

$$P_{\rm fit}(k) = \frac{\alpha_0}{1+\frac{k^2}{k^2_{\rm UV,0}}} + \sum_{n=1}^{N-1} \alpha_n f(k^2, k^2_{{\rm peak},n}, k^2_{{\rm UV},n}, i_n, j_n) = \sum_{n=0}^{N-1} \alpha_n f_n(k^2)$$

$$f(k^2, k^2_{\rm peak}, k^2_{\rm UV}, i, j) \equiv \frac{\left(k^2/k_0^2\right)^i}{\left(1+\frac{(k^2-k^2_{\rm peak})^2}{k^4_{\rm UV}}\right)^j}$$



Next: Loop integral computation strategy

Calculate the loops for each combination of basis functions, obtaining tensors

- **Goal**: general expression for

$$\boxed{L(n_1, d_1, n_2, d_2, n_3, d_3, k_1^2, k_2^2, k_3^2, M_1, M_2, M_3) \equiv \int_q \frac{|\boldsymbol{k}_1 - \boldsymbol{q}|^{2n_1} q^{2n_2} |\boldsymbol{k}_2 + \boldsymbol{q}|^{2n_3}}{(|\boldsymbol{k}_1 - \boldsymbol{q}|^2 + M_1)^{d_1} (q^2 + M_2)^{d_2} (|\boldsymbol{k}_2 + \boldsymbol{q}|^2 + M_3)^{d_3}}}$$

- **Strategy**:
  - IBP to get master integrals (triangle, bubble, tadpole)
  - Evaluate master integrals
- **Key differences** with QCD:
  - 3d instead of 4d – simpler integrals
  - Complex masses in general – need to be careful with **branch cuts**

Bubble master integral

- Integral given by

$$\boxed{B_{\text{master}}(k^2, M_1, M_2) = \int \frac{d^3\boldsymbol{q}}{\pi^{3/2}} \frac{1}{(q^2 + M_1)(|\boldsymbol{k} - \boldsymbol{q}|^2 + M_2)}}$$

Use Schwinger parametrization

$$\frac{i}{A} = \int_0^\infty ds(1 + i\epsilon)\,\exp(iA(1 + i\epsilon)s)$$

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- Calculation depends on relative sign of the imaginary part of the masses

Same sign

$$= \sqrt{\pi}\int_0^1 dx \frac{1}{\sqrt{x(1-x)k^2 + M_1 x + M_2(1-x)}}$$

Bubble master integral

- Integral given by

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$$B_{\text{master}}(k^2, M_1, M_2) = \frac{\sqrt{\pi}}{k}\left[i\log\left(2\sqrt{x(1-x)+m_1x+m_2(1-x)}+i(m_1-m_2-2x+1)\right)\right]_{x=0}^{x=1} - \text{discontinuities}\,,$$

$$m_1 = M_1/k^2 \quad m_2 = M_2/k^2$$

Numerically tricky to evaluate: how to know the branch cut was crossed?

Bubble master integral – branch cuts

$$B_{\text{master}}(k^2, M_1, M_2) =$$
$$\frac{\sqrt{\pi}}{k}\left[i\log\left(2\sqrt{x(1-x)+m_1x+m_2(1-x)}+i(m_1-m_2-2x+1)\right)\right]_{x=0}^{x=1}$$
$$-\text{discontinuities}\,,$$

- Define argument of the log $A(x, m_1, m_2) \equiv 2\sqrt{x(1-x)+m_1x+m_2(1-x)}+i(m_1-m_2-2x+1)$

Bubble master integral – branch cuts

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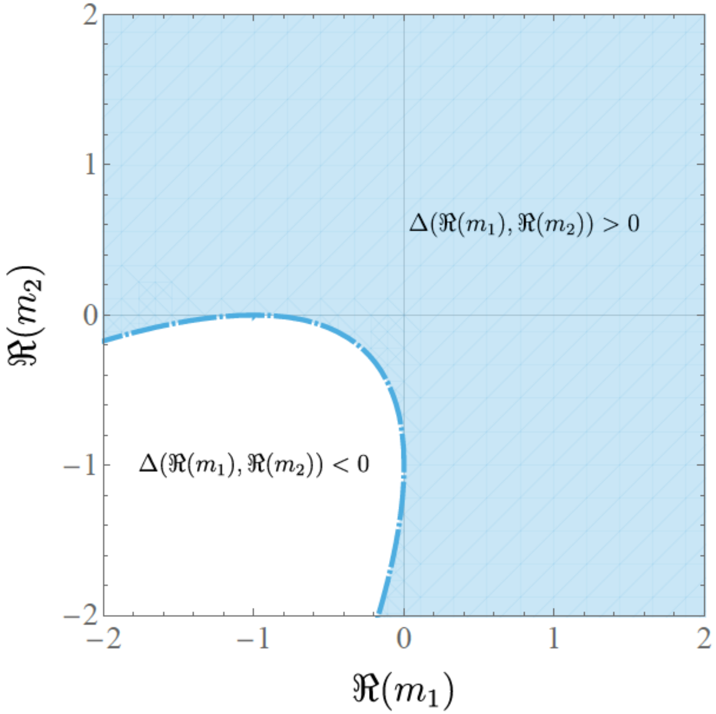
- Define argument of the log $A(x, m_1, m_2) \equiv 2\sqrt{x(1-x)+m_1x+m_2(1-x)}+i(m_1-m_2-2x+1)$

There is one branch cut
$\Leftrightarrow \Im\left(A(1, m_1, m_2)\right) > 0$ and $\Im\left(A(0, m_1, m_2)\right) < 0$

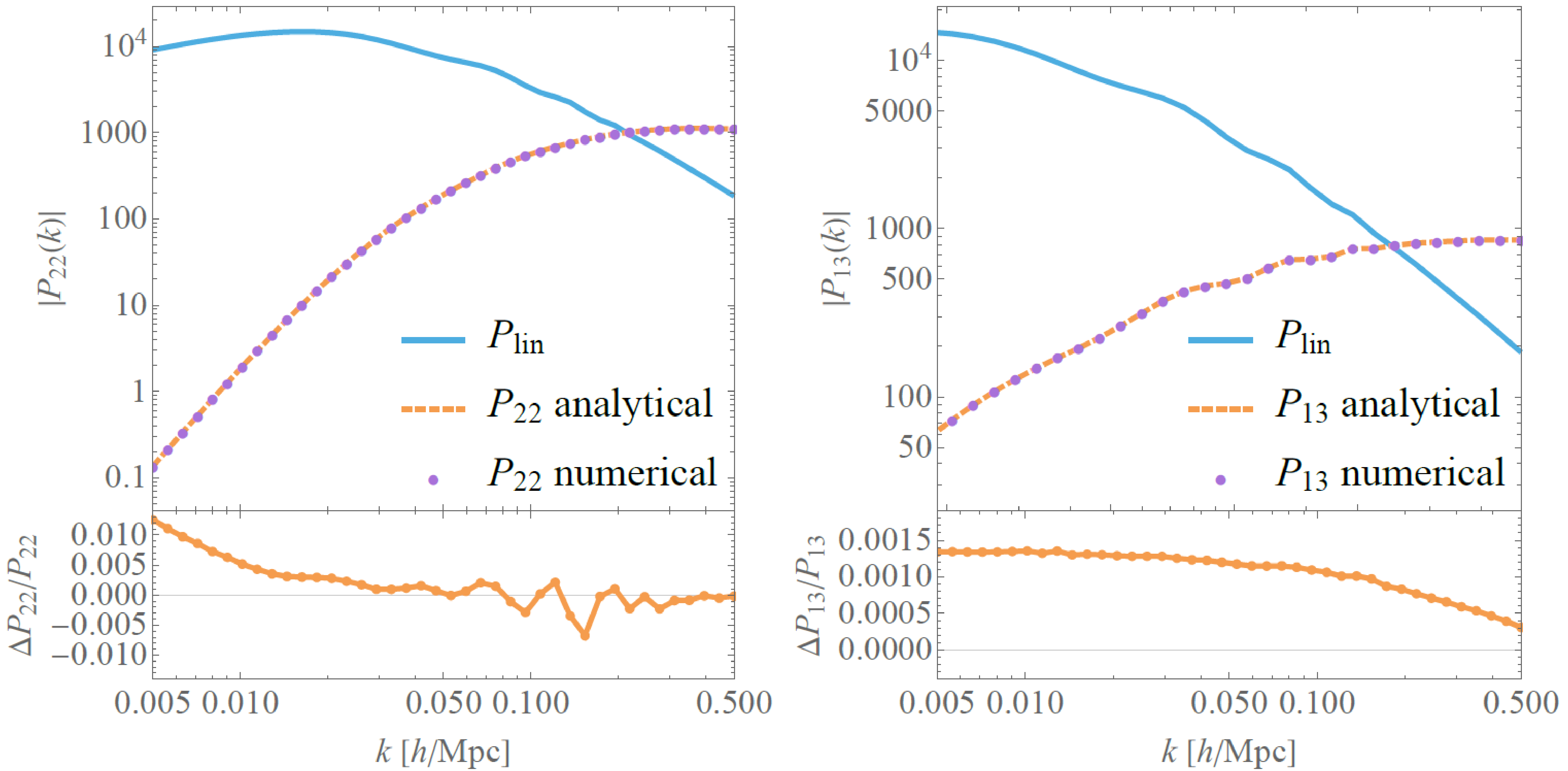
$$B_{\text{master}}(k^2, M_1, M_2) = \frac{\sqrt{\pi}}{k} i[\log\left(A(1,m_1,m_2)\right) - \log\left(A(0,m_1,m_2)\right) - 2\pi i H(\text{Im}\, A(1,m_1,m_2))H(-\text{Im}\, A(0,m_1,m_2))]$$



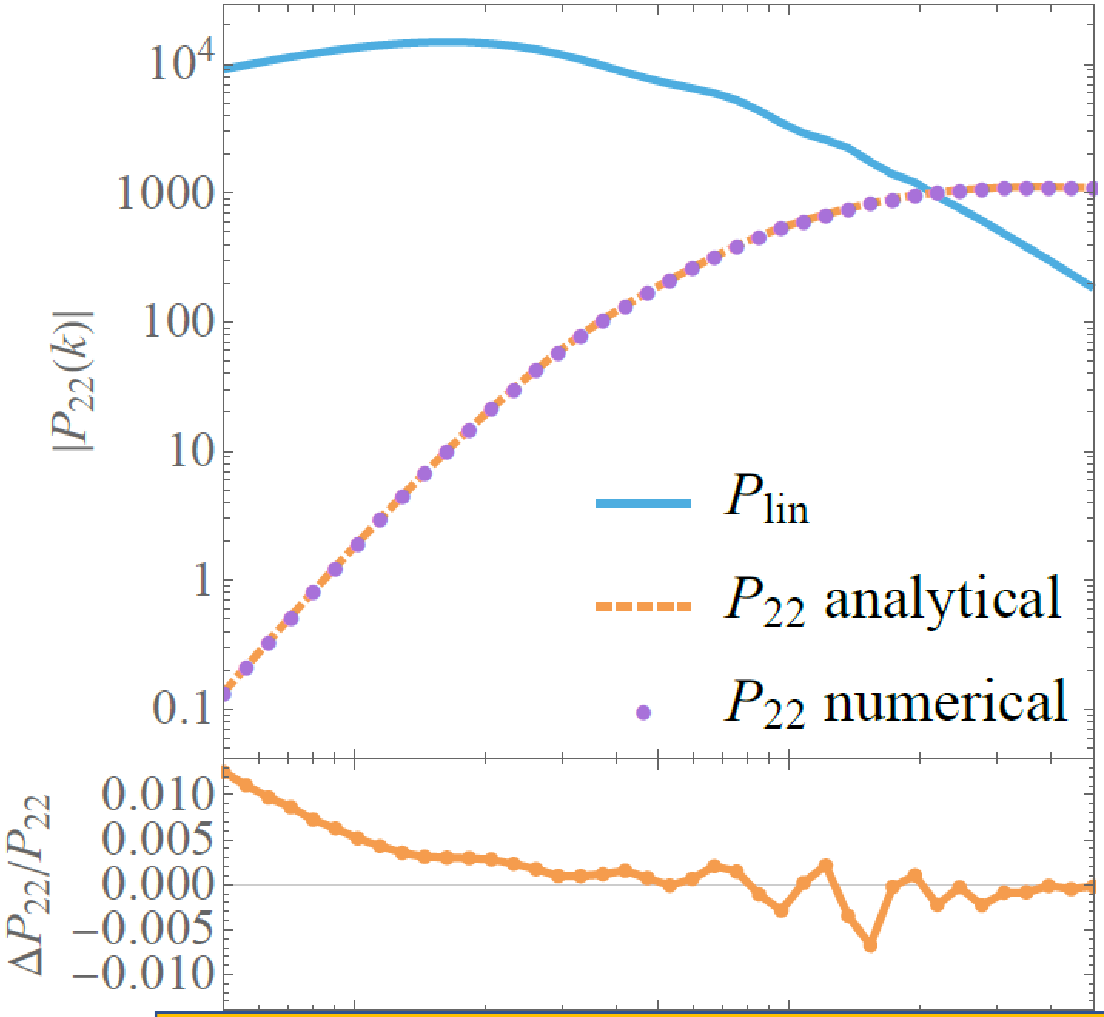
- For opposite sign, the exact same expression is obtained!

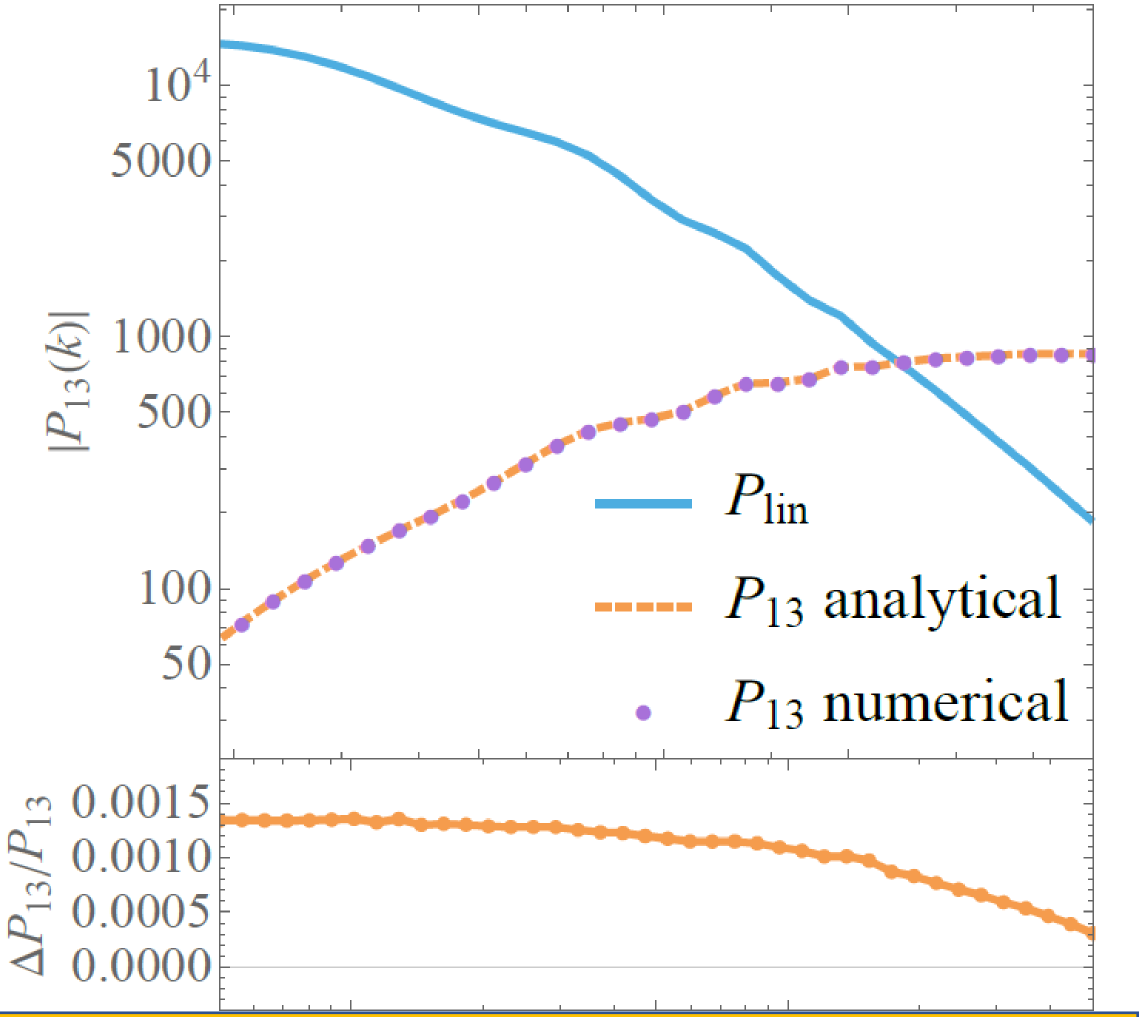
- Extremely efficient to evaluate numerically.

Matter power spectrum: comparison with numerical integration



Matter power spectrum: comparison with numerical integration





Excellent agreement, method works!

Triangle master integral

- Integral given by

$$\boxed{T_{\text{master}}(k_1^2, k_2^2, k_3^2, M_1, M_2, M_3) = \int \frac{d^3\boldsymbol{q}}{\pi^{3/2}} \frac{1}{(q^2 + M_1)(|\boldsymbol{k}_1 - \boldsymbol{q}|^2 + M_2)(|\boldsymbol{k}_2 + \boldsymbol{q}|^2 + M_3)}}$$

Use Schwinger parametrization

$$\frac{1}{A} = \int_0^\infty ds \exp(-As)$$

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- Calculation depends on relative sign of the imaginary part or of the real part of the masses.
- Choosing **masses with positive real part** dramatically simplifies the derivation

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$$T_{\text{master}} = \left[c_1 F_{\text{int}}(R_2, z_+, z_-, x_+) + c_2 F_{\text{int}}(R_2, z_+, z_-, x_-)\right]_{y=0}^{y=1}$$

$$F_{\text{int}}(R_2, z_+, z_-, x_0) = \frac{\sqrt{\pi}}{2} \int_0^1 dx \frac{1}{\sqrt{R_2(x - z_+)(x - z_-)}(x - x_0)}$$

- Parameters are functions of kinematics and masses

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Numerically tricky to evaluate: how to know the branch cut was crossed?

Triangle master integral

$$\sqrt{ab} = s(a,b)\sqrt{a}\sqrt{b}$$

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- Arctan branch cut structure

$$\lim_{\epsilon\to 0}\arctan(x\,i) - \arctan(x\,i-\epsilon) = \pi\,,\ |x|>1$$

$$\lim_{\epsilon\to 0}\arctan(x\,i+\epsilon) - \arctan(x\,i-\epsilon) = \frac{\pi}{2}\,,\ |x|=1$$

- Branch cut when $A^2 \leq -1$, which describes an arc.

- Define argument of arctan $A(z, z_+, z_-, x_0) \equiv \frac{\sqrt{z_+ - z}\sqrt{x_0 - z_-}}{\sqrt{x_0 - z_+}\sqrt{z - z_-}}$

Triangle master integral

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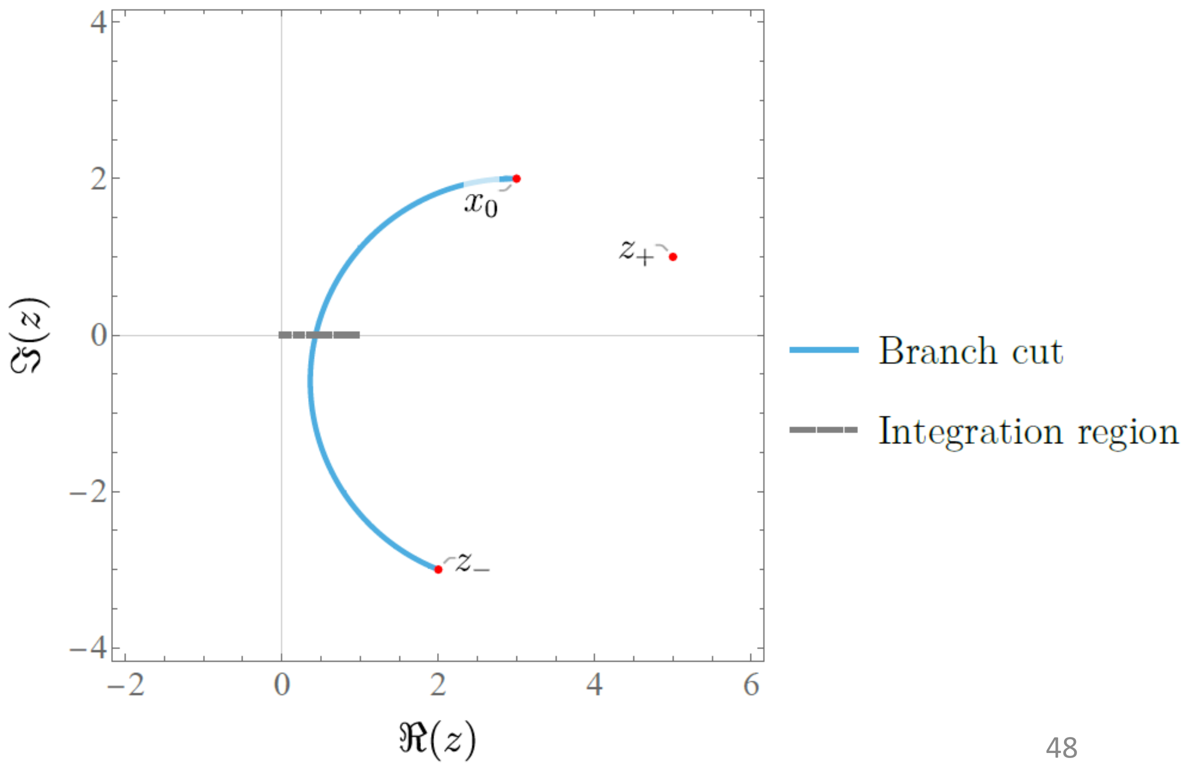

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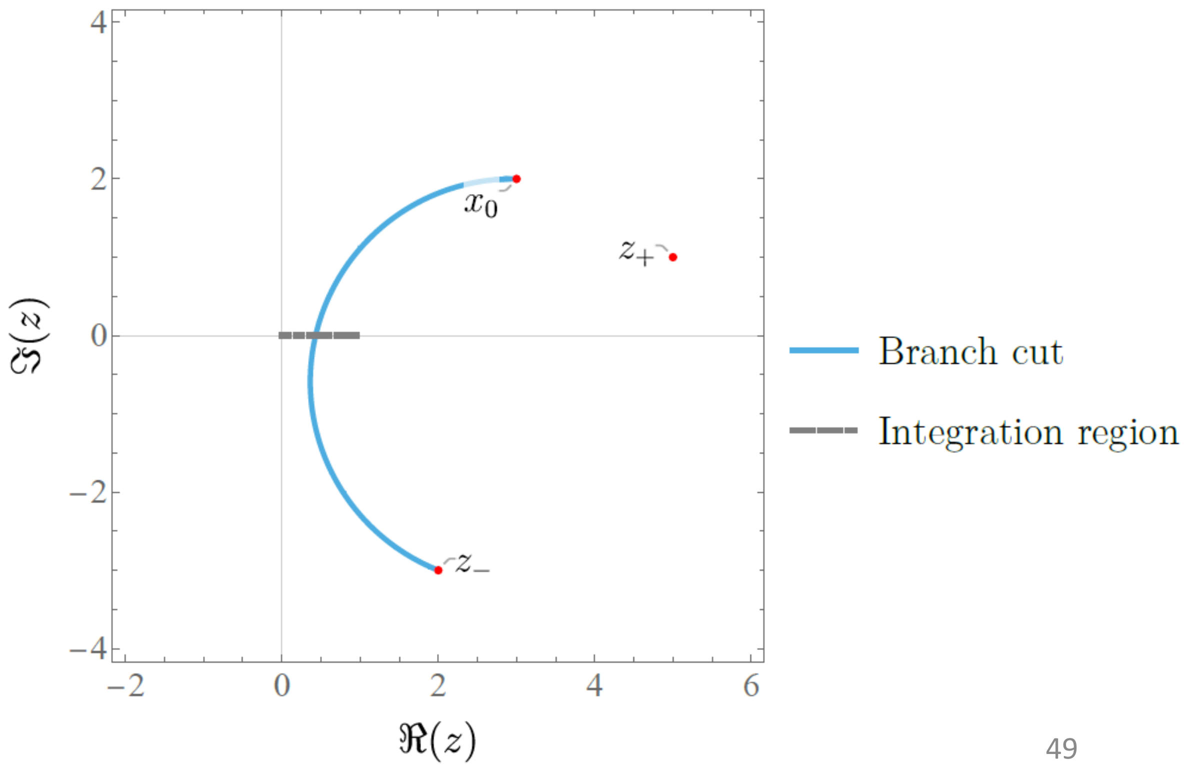
- Branch cut when $A^2 \leq -1$, which describes an arc.
- Crossing if arc **intersects** integration region.

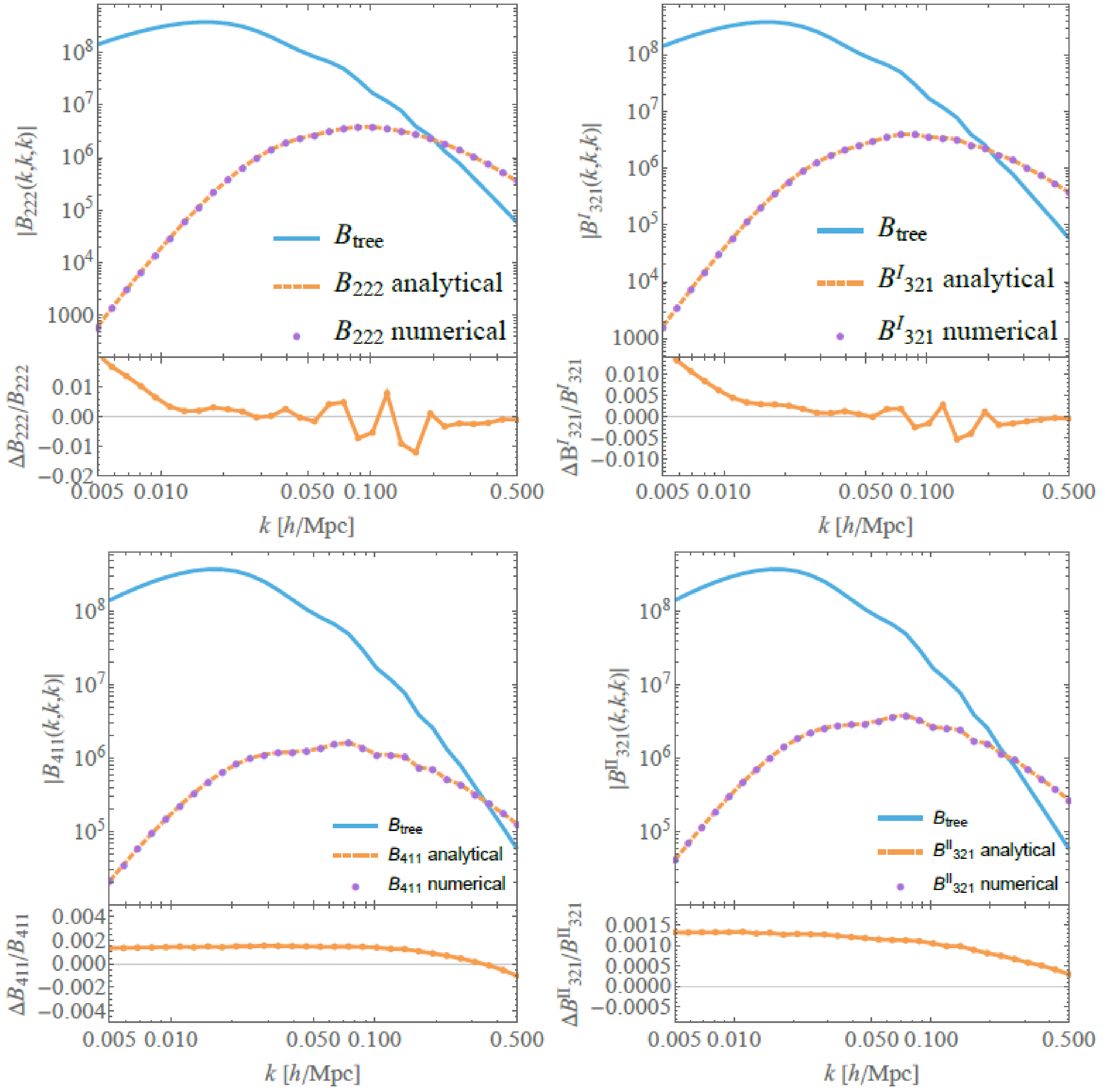
- Possible to know where are the crossings only from values of $x_0, z_-,$ and z_+!
- Direction of crossing depends on $\Re\frac{dA}{dz}$
- Can be numerically implemented

- Define argument of arctan $A(z, z_+, z_-, x_0) \equiv \frac{\sqrt{z_+-z}\sqrt{x_0-z_-}}{\sqrt{x_0-z_+}\sqrt{z-z_-}}$



Matter bispectrum: comparison with numerical integration

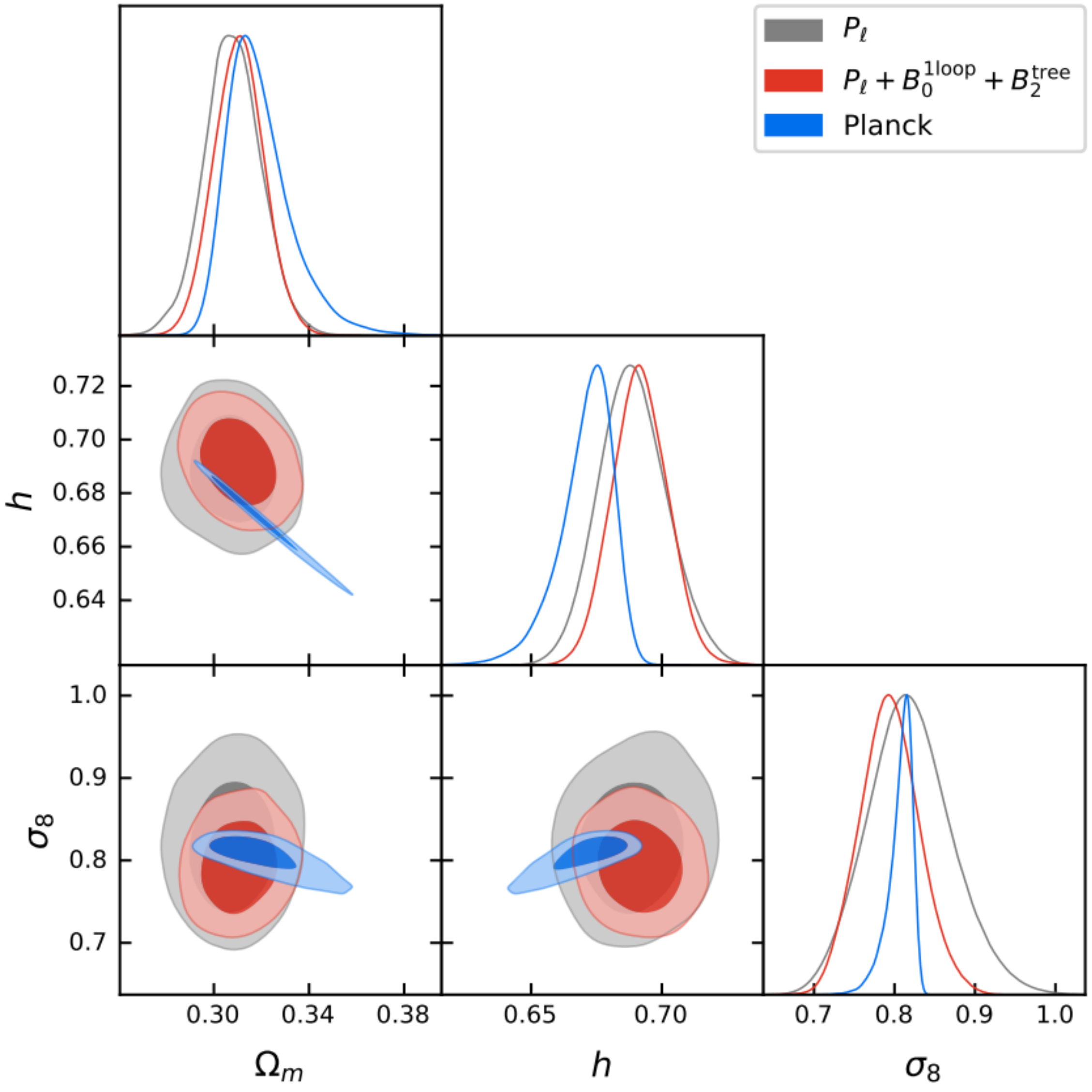
- B_{222} matches well within 1%
- Other diagrams are even better
- We can now make **parameter inference** using 1-loop power spectrum and bispectrum because the computation of the loop corrections is **extremely fast.**



Results from real data analysis

Results using this method with BOSS

Cosmological parameter inference



D'Amico, Donath, Lewandowski, Senatore, Zhang **arXiv:2206.08327**

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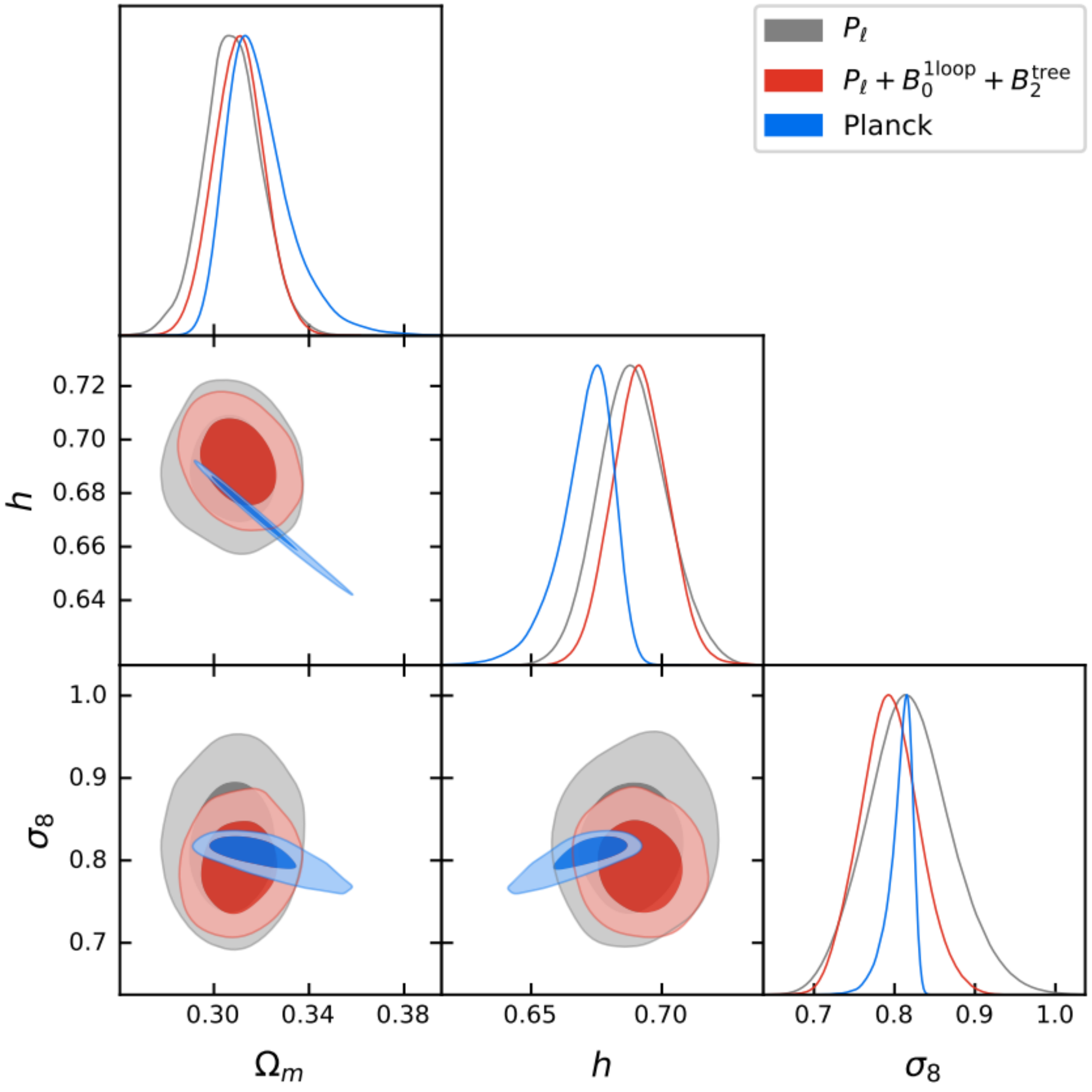

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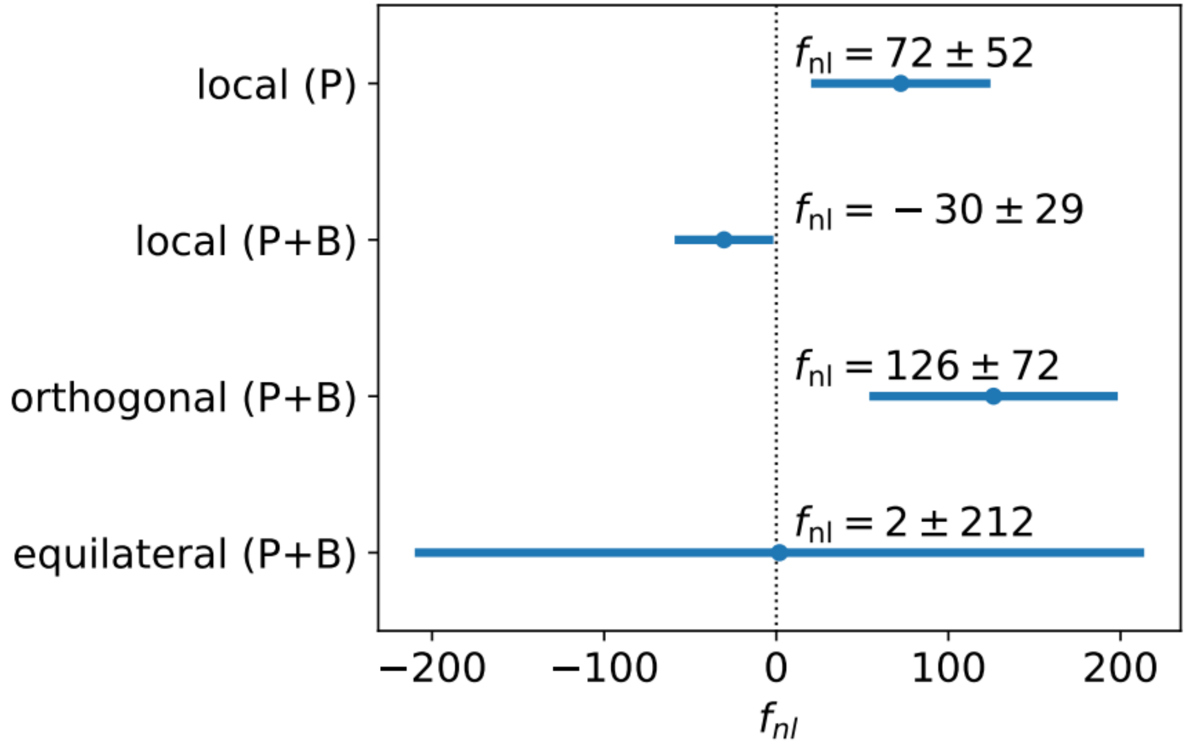
Inflationary parameter inference: f_{NL}



D'Amico, Lewandowski, Senatore, Zhang **arXiv:2201.11518**

Results using this method with BOSS

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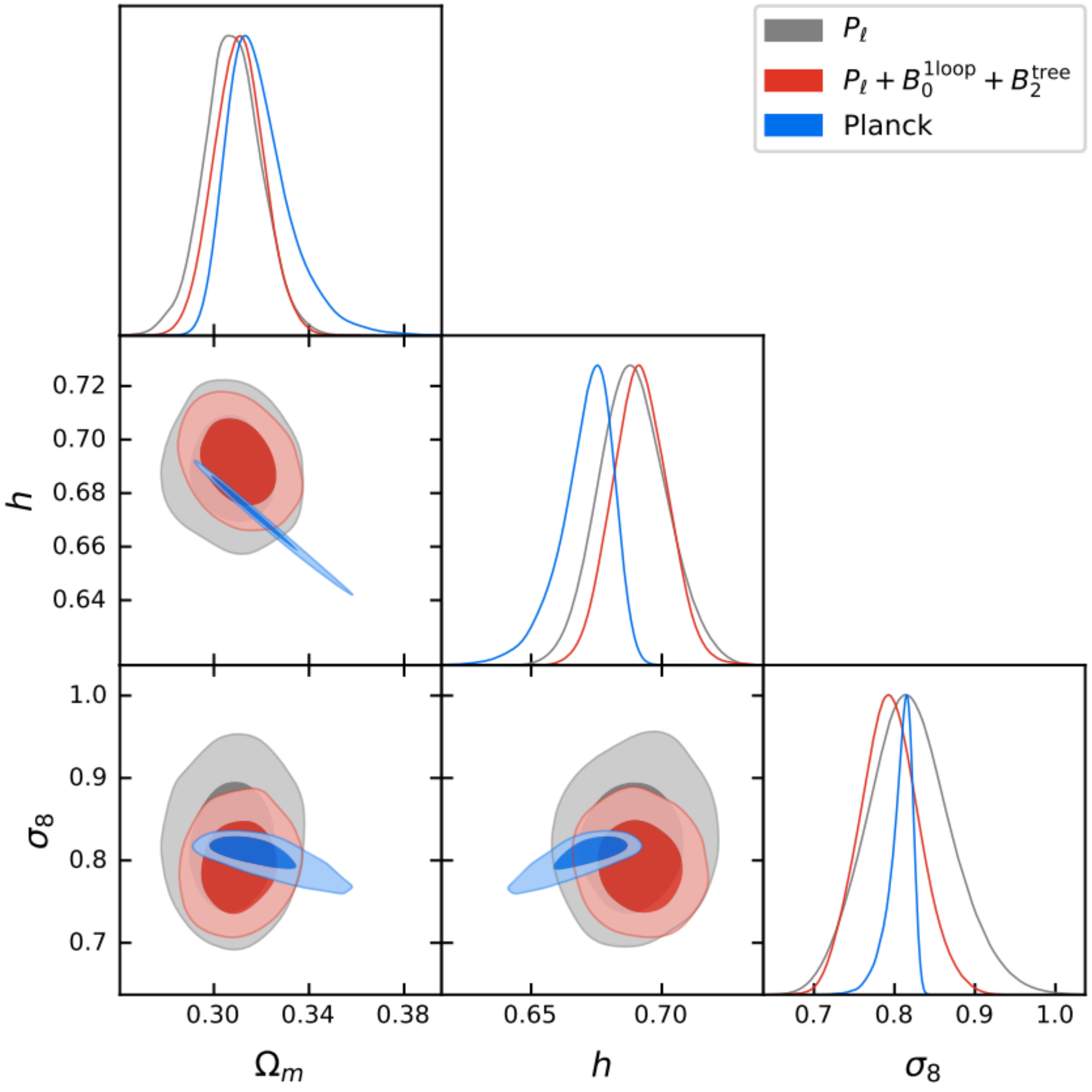

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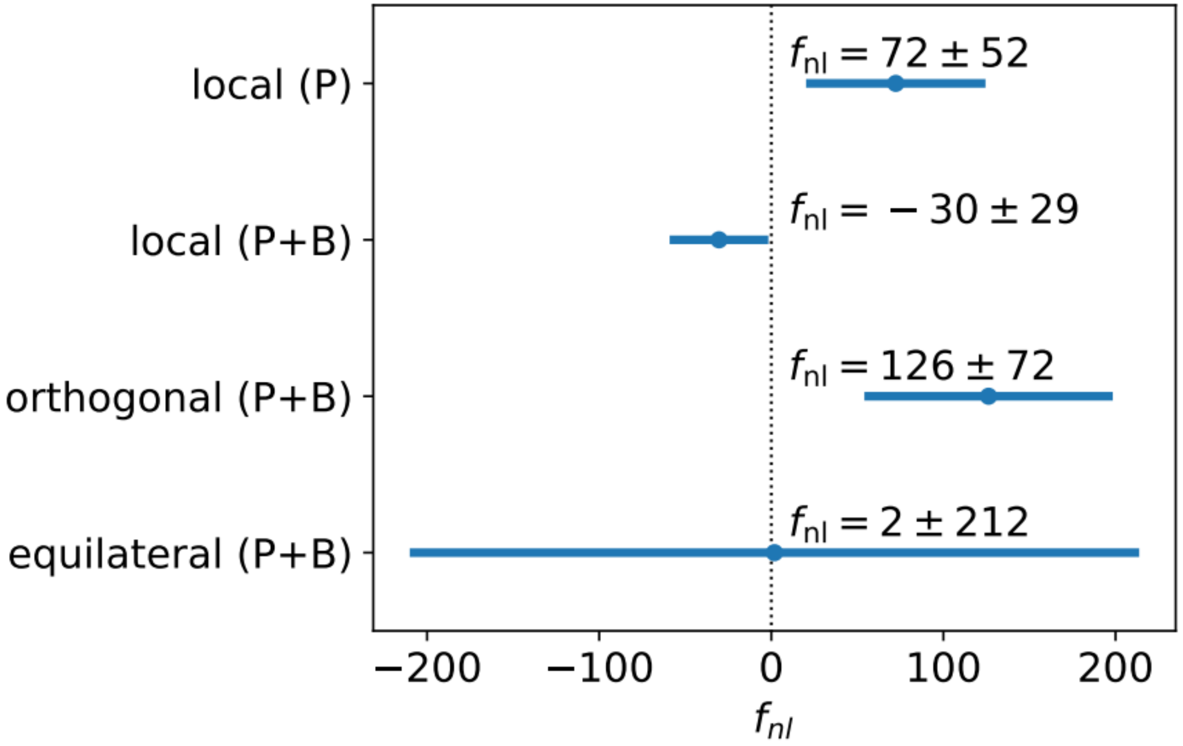

D'Amico, Lewandowski, Senatore, Zhang **arXiv:2201.11518**

The perspectives using this new method in future surveys are very optimistic!

All N-point functions at 1-loop

Using a result from van Neerven and Vermaseren (1984)

One-loop integrals for all N-point functions in the EFTofLSS

Example: one-loop box integral

$$I_4 \equiv \int d^D q \frac{1}{\mathcal{A}_1 \mathcal{A}_2 \mathcal{A}_3 \mathcal{A}_4} \qquad \mathcal{A}_i = (q + p_i)^2 + M_i \qquad p_i = \sum_{m=1}^{i} k_m$$

One can prove the following identity in 3d

$$\left[-2\rho_4 - \frac{1}{2} \sum_{i,j=1}^{3} (\rho_i - \rho_4) \, \Pi_{ij} \, (\rho_j - \rho_4) \right] \int d^D q \, \frac{1}{\mathcal{A}_1 \mathcal{A}_2 \mathcal{A}_3 \mathcal{A}_4} =$$

$$-\frac{1}{2} \int d^D q \, \frac{2\mathcal{A}_4 + \sum_{i,j=1}^{3} (\rho_i - \rho_4) \, \Pi_{ij} \, (\mathcal{A}_j - \mathcal{A}_4)}{\mathcal{A}_1 \mathcal{A}_2 \mathcal{A}_3 \mathcal{A}_4} + \mathcal{O}(\epsilon)$$

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LHS: box integral, RHS: 4 triangle integrals!
This procedure can be done for all N-point functions if N>3

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C

We have all the master integrals we need at 1-loop!

$$\left[-2\rho_4 - \frac{1}{2} \sum_{i,j=1}^{3} (\rho_i - \rho_4) \Pi_{ij} (\rho_j - \rho_4) \right] \int d^D q \frac{1}{\mathcal{A}_1 \mathcal{A}_2 \mathcal{A}_3 \mathcal{A}_4} =$$

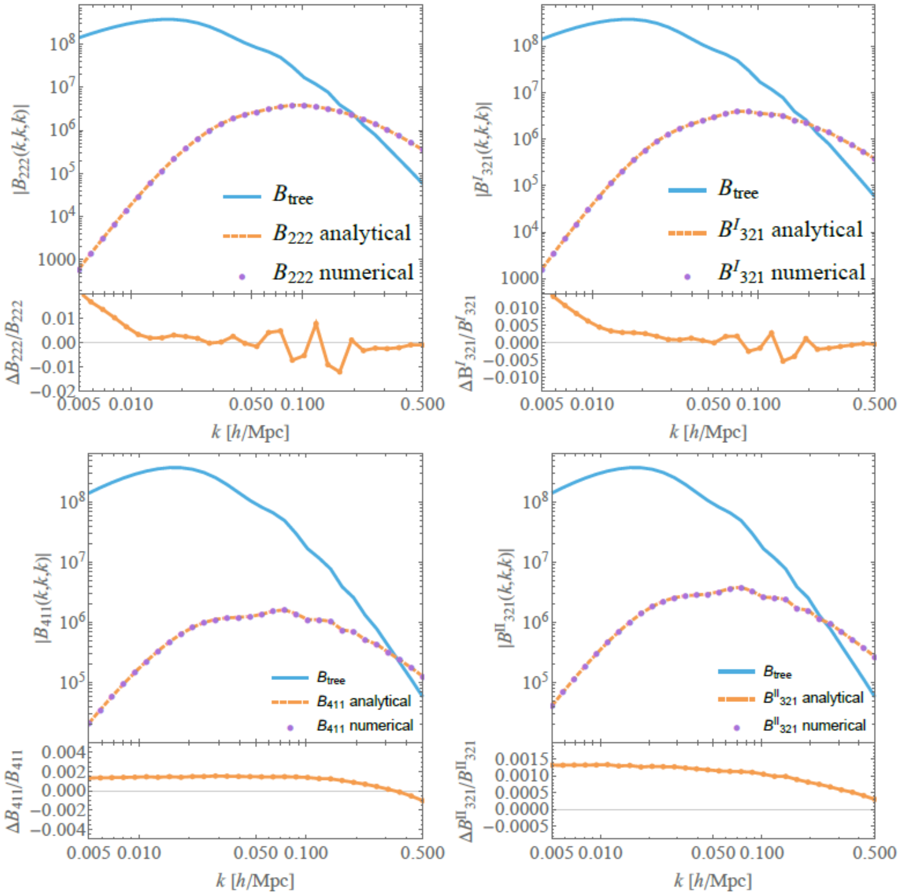
$$-\frac{1}{2} \int d^D q \frac{2\mathcal{A}_4 + \sum_{i,j=1}^{3} (\rho_i - \rho_4) \Pi_{ij} (\mathcal{A}_j - \mathcal{A}_4)}{\mathcal{A}_1 \mathcal{A}_2 \mathcal{A}_3 \mathcal{A}_4} + \mathcal{O}(\epsilon)$$

LHS: box integral, RHS: 4 triangle integrals!
This procedure can be done for all N-point functions if N>3

Conclusions

- A new fast method to calculate 1-loop corrections in the EFTofLSS was found
  - Uses QFT-like integrals with massive propagators
  - Overcomes problems of previous FFTLog method
  - Was already used in real data with good results
  - Developed just in time for larger surveys data analysis

- Open roads
  - Extend formalism to 2-loops (e.g., using DE formalism)
  - Include higher-order N-point functions in the analysis – we have the technique!



Thank you!

Happy to take questions!

UV correction

- To match numerical integration with enough precision, one needs to compensate for the part of the integral outside the limit of integration

$$m^{(13)}_{\mathrm{UV},i} \equiv \int_{\Omega_2} d\Omega_2 \lim_{q\to\infty} q^2\, 6F_3(\boldsymbol{q}, -\boldsymbol{q}, \boldsymbol{k}) f_i(q^2) \leq \mathcal{O}\left(\frac{k^2}{q^2}\right)$$

$$M^{(13)}_{\mathrm{UV},i} \equiv \int_{q_{\mathrm{UV}}}^{\infty} \frac{dq}{(2\pi)^3}\, m^{(13)}_{\mathrm{UV},i} \,.$$

$$\bar{P}^{\mathrm{UV}}_{13} = P_{\mathrm{lin}}\, M^{(13)}_{\mathrm{UV}} \cdot \alpha$$

- Then this is subtracted from the estimate of P_{13}

Proof of discontinuities of bubble master

- The important lemma is the following:

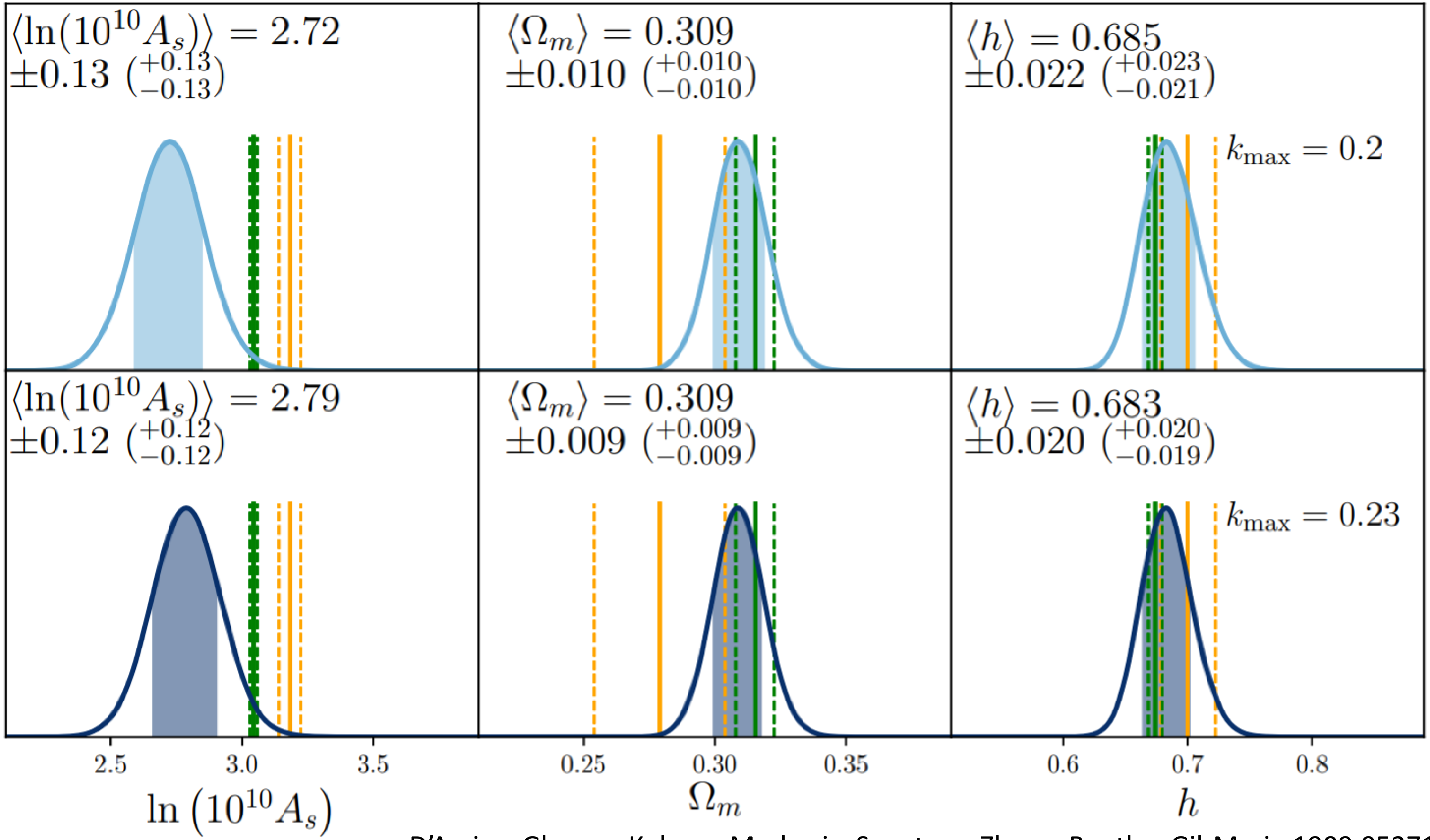
$$\begin{aligned}
\frac{dA}{dx} &= \frac{m_1 - m_2 - 2x + 1}{\sqrt{x(m_1 - m_2 - x + 1) + m_2}} - 2i \\
&= \frac{m_1 - m_2 - 2x + 1 - 2i\sqrt{x(m_1 - m_2 - x + 1) + m_2}}{\sqrt{x(m_1 - m_2 - x + 1) + m_2}} \\
&= -i\frac{A(x, m_1, m_2)}{\sqrt{x(m_1 - m_2 - x + 1) + m_2}} \\
&= \frac{it}{\sqrt{x(m_1 - m_2 - x + 1) + m_2}} .
\end{aligned}$$

Bubble master: opposite imaginary part sign

$$B_{\text{master}}(k^2, M_1, M_2) = \sqrt{\pi}\left(\int_0^{\frac{1}{2}-\frac{1}{\epsilon}} \frac{d\hat{x}}{\sqrt{\hat{x}(1-\hat{x})k^2 + M_1\hat{x} + M_2(1-\hat{x})}} + \int_{\frac{1}{2}+\frac{1}{\epsilon}}^{1} \frac{d\hat{x}}{\sqrt{\hat{x}(1-\hat{x})k^2 + M_1\hat{x} + M_2(1-\hat{x})}} + \frac{\pi}{k}\right)$$

- The result can then be shown to be equal to the case where the masses have the same imaginary part sign

Measurement of cosmological parameters using 1-loop power spectrum



D'Amico, Gleyzes, Kokron, Markovic, Senatore, Zhang, Beutler, Gil-Marin 1909.05271