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# Black Holes and Massive Higher Spins

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Based on: 2107.14779, 2212.06120, 2301.xxxxx

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2107.14779 (M Chiodaroli, H Johansson, **PP**)

2212.06120, 2301.xxxxx (L Cangemi, M Chiodaroli, H Johansson, A Ochirov, **PP**, E Skvortsov)

## Review

## Higher Spin Theory

- Low Spin

- High-Energy Unitarity

- Massive Gauge Symmetry

## Conclusion

# Review: Three-point



Kerr energy-momentum tensor contracted into on-shell graviton [Vines (2017)]

$$\varepsilon_{\mu\nu}(k) T_{\text{BH}}^{\mu\nu}(k) = (\varepsilon_k \cdot p)^2 \exp(k_\mu S_{\text{BH}}^\mu), \quad S_{\text{BH}}^\mu = (0, 0, 0, ma)$$

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QFT  $\rightarrow$  Pauli-Lubanski vector

$$S^\mu(p) \equiv \left\langle \frac{1}{2m} \epsilon^{\mu\nu\rho\sigma} p_\nu M_{\rho\sigma} \right\rangle = \frac{1}{m} \langle \hat{W}^\mu \rangle$$

# Review: Three-point



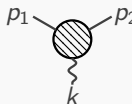
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Massive spinning three-point amplitude: [Arkani-Hamed, Huang, Huang (2017)]


$$= (\varepsilon_k \cdot p_1)^2 \left( \frac{[12]}{m} \right)^{2s}$$





# Review: Compton



Classical observables in terms of amplitudes:

[Bjerrum-Bohr,...; Guevara,...; Haddad,...; Huang,...; Kosower,...; Luna,...; Mogull,...; O'Connell,...; ...]

$$\Delta\mathcal{O} \sim \begin{array}{c} \diagup \\ \text{shaded circle} \\ \diagdown \\ \text{wavy line} \\ \diagup \\ \text{shaded circle} \\ \diagdown \end{array} + \begin{array}{c} \diagup \\ \text{shaded circle} \\ \text{horizontal line} \\ \text{shaded circle} \\ \diagdown \\ \text{wavy line} \\ \text{shaded oval} \\ \text{wavy line} \\ \diagup \\ \text{shaded circle} \\ \diagdown \end{array} + \dots$$





Classical observables in terms of amplitudes:

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$$\Delta\mathcal{O} \sim \text{[Diagram 1]} + \text{[Diagram 2]} + \dots$$

The diagram shows two terms in a sum. The first term is a tree-level diagram with two external lines on the left and two on the right, connected by a vertical wavy line. The second term is a loop-level diagram with two external lines on the left and two on the right, connected by a horizontal wavy line and a vertical wavy line forming a loop.

Compton amplitude needed at NLO. BCFW results: [Arkani-Hamed,...; Johansson,...]

$$\begin{aligned} \text{[Diagram 1]} &= \frac{[4|p_1|3\rangle^{4-2s}([41]\langle 32\rangle + [42]\langle 31\rangle)^{2s}}{s_{12}(s_{13} - m^2)(s_{14} - m^2)} \\ \text{[Diagram 2]} &= \frac{\langle 12\rangle^{2s}[34]^4}{m^{2s-4}s_{12}(s_{13} - m^2)(s_{14} - m^2)} \end{aligned}$$

The diagrams are Compton amplitudes. The first diagram has external lines labeled 1, 2, 3-, and 4+. The second diagram has external lines labeled 1, 2, 3+, and 4+.



Classical observables in terms of amplitudes:

[Bjerrum-Bohr,...; Guevara,...; Haddad,...; Huang,...; Kosower,...; Luna,...; Mogull,...; O'Connell,...; ...]

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The diagrams are labeled with external legs: 1 and 2 are incoming, 3 and 4 are outgoing. The wavy line is labeled with 3+ and 3- for the first diagram, and 3+ and 3- for the second diagram.

**Remarkable:**  $s \leq 2$  matches Kerr, **BUT**  $s > 2$  has spurious pole



Classical observables in terms of amplitudes:

[Bjerrum-Bohr,...; Guevara,...; Haddad,...; Huang,...; Kosower,...; Luna,...; Mogull,...; O'Connell,...; ...]

$$\Delta\mathcal{O} \sim \text{[Diagram 1]} + \text{[Diagram 2]} + \dots$$

The diagram shows two terms separated by a plus sign. The first term consists of two shaded circular vertices connected by a vertical wavy line. Each vertex has two external lines extending outwards. The second term consists of two shaded circular vertices connected by a horizontal wavy line. Each vertex has two external lines extending outwards. The entire expression is followed by a plus sign and an ellipsis.

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$$\begin{aligned} \text{[Diagram 1]} &= \frac{[4|p_1|3\rangle^{4-2s}([41]\langle 32\rangle + [42]\langle 31\rangle)^{2s}}{s_{12}(s_{13} - m^2)(s_{14} - m^2)} \\ \text{[Diagram 2]} &= \frac{\langle 12\rangle^{2s}[34]^4}{m^{2s-4}s_{12}(s_{13} - m^2)(s_{14} - m^2)} \end{aligned}$$

The diagrams are Feynman diagrams for Compton scattering. The first diagram shows a shaded circular vertex with two external lines labeled 1 and 2, and two wavy lines labeled 3 and 4. The second diagram is similar but with different line orientations.

**Remarkable:**  $s \leq 2$  matches Kerr, **BUT**  $s > 2$  has spurious pole

**Meaning:** BCFW does not work for higher spin.



## What we know: (potentials)

- ▶  $\mathcal{O}(G^2)$ : Hamiltonian up to  $\mathcal{O}(S^4)$ ,  $\mathcal{O}(S^{>5})$  conjectured

[Guevara, Ochirov, Vines; Luna, Kosmopoulos; Porto, Liu; Chen, Chung, Huang, Kim; Aoude, Haddad, Helset; Bern, Kosmopoulos, Luna, Roiban, Teng]

- ▶  $\mathcal{O}(G^3)$ : results at  $\mathcal{O}(S^2)$ ,  $\mathcal{O}(S^\infty)$  radiation-reaction

[Jakobsen, Mogull; Cordero, Kraus, Lin, Ruf, Zeng] [Alessio, Di Vecchia]

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## Open questions:



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## Open questions:

- ? Kerr black holes at  $\mathcal{O}(S^{\geq 5})$ ?

$$\lim_{s \rightarrow \infty} \mathcal{A}(\phi^s(p_1), \phi^s(p_2), h(p_3), \dots, h(p_n))$$



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- ? Higher-spin theories that give rise to the Kerr amplitudes?

$$m^{-2s} (\varepsilon_k \cdot p_1)^2 [\mathbf{12}]^{2s}$$



Generating function for root-Kerr amplitudes:

$$\sum_{s=0}^{\infty} m^{-2s} (\varepsilon_k \cdot p_1) [\mathbf{12}]^{2s} = A_{\phi\phi A} + \frac{A_{WWA} - (\varepsilon_1 \cdot \varepsilon_2)^2 A_{\phi\phi A}}{(1 + \varepsilon_1 \cdot \varepsilon_2)^2 + \frac{2}{m^2} \varepsilon_1 \cdot p_2 \varepsilon_2 \cdot p_1}$$





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Extract Lagrangians:

- ▶  $s = 0, 1/2$  minimally-coupled

$$\mathcal{L}^{s=0} = \overline{D_\mu \phi} D^\mu \phi - m^2 \bar{\phi} \phi$$

$$\mathcal{L}^{s=1/2} = \bar{\psi} (i \not{D} - m) \psi$$



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- ▶  $s = 1$  non-minimal

$$\mathcal{L}^{s=1} = 2D_{[\mu} \overline{W}_{\nu]} D^{[\mu} W^{\nu]} - m^2 \overline{W}_\mu W^\mu + ie F_{\mu\nu} \overline{W}^\mu W^\nu$$



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**!** **Lesson:** high-energy behaviour important



Generating function for Kerr amplitudes:

$$\sum_{s=0}^{\infty} m^{-2s} (\varepsilon_k \cdot p_1)^2 [\mathbf{12}]^{2s} = M_{\phi\phi h} + A_{WWA} \left( A_{\phi\phi A} + \frac{A_{WWA} - (\varepsilon_1 \cdot \varepsilon_2)^2 A_{\phi\phi A}}{(1 + \varepsilon_1 \cdot \varepsilon_2)^2 + \frac{2}{m^2} \varepsilon_1 \cdot p_2 \varepsilon_2 \cdot p_1} \right)$$



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$$\mathcal{L}^{s=1} = 2 \nabla_{[\mu} \bar{W}_{\nu]} \nabla^{[\mu} W^{\nu]} - m^2 \bar{W}_{\mu} W^{\mu}$$

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$$\begin{aligned} \mathcal{L}^{s=2} = & \nabla_{\mu} \bar{H}_{\nu\rho} \nabla^{\mu} H^{\nu\rho} - 2 \nabla_{\nu} \bar{H}_{\mu}^{\nu} \nabla^{\rho} H_{\rho}^{\mu} - \bar{H}_{\rho}^{\rho} \nabla_{\mu} \nabla_{\nu} H^{\mu\nu} \\ & - H_{\rho}^{\rho} \nabla_{\mu} \nabla_{\nu} \bar{H}^{\mu\nu} - \nabla_{\mu} \bar{H}_{\nu}^{\nu} \nabla^{\mu} H_{\rho}^{\rho} - m^2 \bar{H}_{\mu\nu} H^{\mu\nu} \\ & + m^2 \bar{H}_{\mu}^{\mu} H_{\nu}^{\nu} - 2 R^{\mu\nu\rho\sigma} \bar{H}_{\mu\rho} H_{\nu\sigma} \end{aligned}$$

# High-Energy Unitarity

Example:  $s = 1$  gauge



Massive spin-1 field coupled to photon:

$$\mathcal{L} = 2D_{[\mu} \bar{W}_{\nu]} D^{[\mu} W^{\nu]} - m^2 \bar{W}_\mu W^\mu + ie\alpha F_{\mu\nu} \bar{W}^\mu W^\nu$$

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## Compton Ingredients

Three-point current ( $P$  off-shell,  $p_1$  and  $k$  on-shell;  $f_k \equiv k\varepsilon_k - \varepsilon_k k$ ):

$$J(P, p_1, k) = \varepsilon_P \cdot \varepsilon_1 \varepsilon_k \cdot (p_1 - P) - \varepsilon_1 \cdot \varepsilon_k \varepsilon_P \cdot p_1 + \varepsilon_k \cdot \varepsilon_P \varepsilon_1 \cdot P - \alpha f_k^{\mu\nu} \varepsilon_{1\mu} \varepsilon_{P\nu}$$



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Spin-1 propagator:

$$\Delta_{\mu\nu}(P) = \frac{1}{P^2 - m^2} \left( \eta_{\mu\nu} - \frac{P_\mu P_\nu}{m^2} \right)$$

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Compton amplitude **diverges** in the  $P/m \rightarrow \infty$  limit, unless

## Current constraint

$$P \cdot J = \mathcal{O}(m) \quad \Rightarrow \quad \alpha = 1$$

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root-Kerr three-point and Compton matched!

# High-Energy Unitarity

$s = 3/2$  gauge



Massive spin-3/2 field coupled to photon: [Deser,Waldron,Pascalutsa(2000)]

$$\mathcal{L} = \bar{\psi}^\mu \gamma_{\mu\nu\rho} \left( iD^\nu - \frac{1}{2} m \gamma^\nu \right) \psi^\rho - \frac{ie}{m} \left( l_1 \bar{\psi}_\mu F^{\mu\nu} \psi_\nu + l_2 \bar{\psi}_\mu F_{\rho\sigma} \gamma^\rho \gamma^\sigma \psi^\mu \right. \\ \left. + l_3 F^{\mu\nu} (\bar{\psi}_\mu \gamma_\nu \gamma \cdot \psi + \bar{\psi} \cdot \gamma \gamma_\mu \psi_\nu) + l_4 \bar{\psi} \cdot \gamma F_{\rho\sigma} \gamma^\rho \gamma^\sigma \gamma \cdot \psi + i l_5 F^{\mu\nu} (\bar{\psi}_\mu \gamma_\nu \gamma \cdot \psi - \bar{\psi} \cdot \gamma \gamma_\mu \psi_\nu) \right)$$

# High-Energy Unitarity

$s = 3/2$  gauge



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Spin-3/2 propagator:

$$\Delta^{\mu\nu}(P) \sim \left( \eta^{\mu\nu} - \frac{P^\mu P^\nu}{m^2} \right) (\not{P} + m) + \frac{1}{3} \left( \frac{P^\mu}{m} + \gamma^\mu \right) (\not{P} - m) \left( \frac{P^\nu}{m} + \gamma^\nu \right)$$

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Impose current constraint

$$P \cdot J = \mathcal{O}(m) \Rightarrow l_1 = -2, l_2 = 1/2, l_3 = 1, l_5 = 0$$

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$$\mathcal{L} = \bar{\psi}^\mu \gamma_{\mu\nu\rho} \left( iD^\nu - \frac{1}{2} m \gamma^\nu \right) \psi^\rho - \frac{ie}{m} \left( l_1 \bar{\psi}_\mu F^{\mu\nu} \psi_\nu + l_2 \bar{\psi}_\mu F_{\rho\sigma} \gamma^\rho \gamma^\sigma \psi^\mu \right. \\ \left. + l_3 F^{\mu\nu} (\bar{\psi}_\mu \gamma_\nu \gamma \cdot \psi + \bar{\psi} \cdot \gamma \gamma_\mu \psi_\nu) + l_4 \bar{\psi} \cdot \gamma F_{\rho\sigma} \gamma^\rho \gamma^\sigma \gamma \cdot \psi + i l_5 F^{\mu\nu} (\bar{\psi}_\mu \gamma_\nu \gamma \cdot \psi - \bar{\psi} \cdot \gamma \gamma_\mu \psi_\nu) \right)$$

Spin-3/2 propagator:

$$\Delta^{\mu\nu}(P) \sim \left( \eta^{\mu\nu} - \frac{P^\mu P^\nu}{m^2} \right) (\not{P} + m) + \frac{1}{3} \left( \frac{P^\mu}{m} + \gamma^\mu \right) (\not{P} - m) \left( \frac{P^\nu}{m} + \gamma^\nu \right)$$

Impose current constraint

$$P \cdot J = \mathcal{O}(m) \Rightarrow l_1 = -2, l_2 = 1/2, l_3 = 1, l_5 = 0$$

root-Kerr three-point matched!



## Spin-3/2 Gauge Theory

$$\mathcal{L} = \mathcal{L}_{min} + \bar{\psi}_\mu \left( F^{\mu\nu} - \frac{i}{2} \gamma_5 \epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta} \right) \psi_\nu$$

$$\mathcal{A}_4 = \frac{[41]\langle 32 \rangle + [42]\langle 31 \rangle}{[4|p_1|3]} \left( \frac{([41]\langle 32 \rangle + [42]\langle 31 \rangle)^2}{(s_{13} - m^2)(s_{14} - m^2)} - \frac{[14][24]\langle 13 \rangle \langle 23 \rangle}{m^4} \right)$$





## Spin-3/2 Gauge Theory

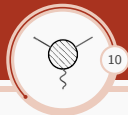
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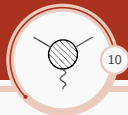
## Spin-5/2 Gravity

$$\mathcal{L} = \mathcal{L}_{min} + \bar{\psi}_{\mu\rho} \left( R^{\mu\nu\rho\sigma} - \frac{i}{2} \gamma_5 \epsilon^{\rho\sigma\alpha\beta} R^{\mu\nu}_{\alpha\beta} \right) \psi_{\nu\sigma}$$

$$\mathcal{M}_4 = \frac{[41]\langle 32 \rangle + [42]\langle 31 \rangle}{[4|p_1|3]} \left( \frac{([41]\langle 32 \rangle + [42]\langle 31 \rangle)^4}{s_{12}(s_{13} - m^2)(s_{14} - m^2)} - \frac{([14][24]\langle 13 \rangle \langle 23 \rangle)^2}{m^6} \right)$$



!  $P \cdot J = \mathcal{O}(m)$  not enough for  $s \geq 2$  gauge /  $s \geq 3$  gravity



!  $P \cdot J = \mathcal{O}(m)$  not enough for  $s \geq 2$  gauge /  $s \geq 3$  gravity

? What are we missing?

# Massive Gauge Symmetry

## Spontaneous Symmetry Breaking



UV theory:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^i F^{i\mu\nu} + \frac{1}{2}(D_\mu\phi)_i(D^\mu\phi)_i + \frac{\mu^2}{2}\|\vec{\phi}\|^2 - \frac{\lambda}{4!}\|\vec{\phi}\|^4$$

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Spontaneous Symmetry Breaking (Goldstone  $\varphi$ ):

$$\begin{aligned}\mathcal{L} = & 2\overline{D_{[\mu}W_{\nu]}} D^{[\mu}W^{\nu]} + (m\overline{W}_\mu - \overline{D}_\mu\varphi)(mW^\mu - D^\mu\varphi) \\ & + ieF_{\mu\nu}\overline{W}^\mu W^\nu + \{\text{Higgs}, W^4, \dots\}\end{aligned}$$

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Massive (Stückelberg) gauge symmetry

$$W_\mu \rightarrow (W_\mu, \varphi) \quad \text{with} \quad \delta W_\mu = D_\mu\lambda, \quad \delta\varphi = m\lambda + \dots$$

Symmetry = no unphysical degrees of freedom

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Symmetry = no unphysical degrees of freedom

**?** Higher-spins?  $\Rightarrow$  no SSB, yes gauge symmetry!

# Massive Gauge Symmetry

Spin-2 Example



Fields  $\{H_{\mu\nu}, B_\mu, \varphi\}$ . Gauge symmetry:

$$\delta H_{\mu\nu} = D_\mu \xi_\nu + D_\nu \xi_\mu + mg_{\mu\nu} \lambda + \dots,$$

$$\delta B_\mu = D_\mu \lambda + m \xi_\mu + \dots, \delta \varphi = m \lambda + \dots$$



# Massive Gauge Symmetry

Spin-2 Example



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Non-minimal Lagrangian (lowest derivative):

$$\begin{aligned}& \frac{-ie}{m^2} F_{\mu\nu} \left( \overline{D_\mu H_{\alpha\beta}} D_\alpha H_{\beta\nu} - \overline{D_\alpha H_{\beta\nu}} D_\mu H_{\alpha\beta} + \overline{D_\alpha H_{\beta\mu}} D_\alpha H_{\beta\nu} \right. \\ & - 2 \overline{D_\alpha H_{\beta\mu}} D_\beta H_{\alpha\nu} - \overline{D_\mu H_{\alpha\beta}} D_\nu H_{\alpha\beta} + \overline{D_\mu H_{\nu\alpha}} D \cdot H_\alpha \\ & - \overline{D \cdot H_\alpha} D_\mu H_{\nu\alpha} + \overline{D \cdot H_\mu} D \cdot H_\nu - \overline{D \cdot H_\mu} D_\nu H \\ & \left. + \overline{D_\nu H D} \cdot H_\mu - \overline{D_\mu H_{\nu\alpha}} D_\alpha H + \overline{D_\alpha H D} \mu H_{\nu\alpha} + \overline{D_\mu H D} \nu H \right) \\ & \frac{-ie}{m^2} F_{\mu\nu} \left\{ m \left( \overline{D \cdot H_\mu} B_\nu - \overline{B_\nu} D \cdot H_\mu - \overline{D_\mu H} B_\nu + \overline{B_\nu} D_\mu H \right) \right\} \\ & \frac{-ie}{m^2} F_{\mu\nu} \left( \frac{m^2}{2} \overline{H_{\mu\alpha}} H_{\nu\alpha} + 2m^2 \overline{B_\mu} B_\mu \right)\end{aligned}$$

$\Rightarrow$  root-Kerr amplitude!

# Massive Gauge Symmetry

Massive Ward identities



Massive spin-1:

$$\delta W_\mu = \partial_\mu \lambda, \delta \varphi = m \lambda$$

$$\left( ip_1 \cdot \frac{\partial}{\partial \varepsilon_1} \mathcal{A}_3(WWA) + m \mathcal{A}_3(\varphi WA) \right) \Big|_{(2,3)} = 0, \quad (i, j) \text{ on-shell}$$



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Massive spin-2:

$$\begin{aligned} \delta H_{\mu\nu} &= \partial_\mu \xi_\nu + \partial_\nu \xi_\mu + m g_{\mu\nu} \lambda, \\ \delta B_\mu &= \partial_\mu \lambda + m \xi_\mu, \delta \varphi = m \lambda \end{aligned}$$

$$\begin{aligned} \left( ip_1 \cdot \frac{\partial}{\partial \epsilon_1} \mathcal{A}_3(HHA) + m \mathcal{A}_3(BHA) \right) \Big|_{(2,3)} &= 0 \\ \left( ip_1 \cdot \frac{\partial}{\partial \epsilon_1} \mathcal{A}_3(BHA) + m \mathcal{A}_3(\varphi HA) + \frac{m}{2} \frac{\partial}{\partial \epsilon_1} \cdot \frac{\partial}{\partial \epsilon_1} \mathcal{A}_3(HHA) \right) \Big|_{(2,3)} &= 0 \end{aligned}$$

# Massive Gauge Symmetry

Arbitrary Spin



Arbitrary spin:

$$\begin{aligned} \langle \xi^k \bar{\Phi}^s A^h \rangle &\equiv \frac{1}{k+1} p_1 \cdot \frac{\partial}{\partial \epsilon_1} \mathcal{L}(\Phi^{k+1} \bar{\Phi}^s A^h) + m \alpha_k \mathcal{L}(\Phi^k \bar{\Phi}^s A^h) \\ &+ \frac{m}{2} \beta_{k+2} \left( \frac{\partial}{\partial \epsilon_1} \right)^2 \mathcal{L}(\Phi^{k+2} \bar{\Phi}^s A^h) \end{aligned}$$

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**Impose:**

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- ▶ Lowest-derivative solution:

$$\mathcal{L}(\Phi^{s_1} \bar{\Phi}^{s_2} A^h) \sim \partial^{s_1+s_2-h}, \quad \mathcal{L}(\Phi^{s_1} \bar{\Phi}^{s_2 < s_1-h} A^h) = 0$$





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**Outcome:** Kerr and root-Kerr 3pt amplitude **unique** for any spin!

Checked up to  $s = 6$



Four-point Ward identities (spin-2 example):

$$ip_1 \cdot \frac{\partial}{\partial \epsilon_1} \mathcal{A}_{\Phi \bar{\Phi} AA} + m \mathcal{A}_{B \bar{\Phi} AA} = 0$$

$$ip_1 \cdot \frac{\partial}{\partial \epsilon_1} \mathcal{A}_{B \bar{\Phi} AA} + m \mathcal{A}_{\varphi \bar{\Phi} AA} + \frac{m}{2} \left( \frac{\partial}{\partial \epsilon_1} \right)^2 \mathcal{A}_{\Phi \bar{\Phi} AA} = 0$$



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Gauge theory Compton

$$A(\Phi_1^s \Phi_2^s A_3^- A_4^+) = \frac{\langle 3|1|4 \rangle^2 (T + S)^{2s}}{m^{4s} t_{13} t_{14}} + \frac{\langle 3|1|4 \rangle \langle 13 \rangle [24] P_{2s}}{m^{4s} t_{13}} \\ + \langle 13 \rangle \langle 32 \rangle [14] [42] \frac{P_{2s-1}}{m^{4s}} + C_s,$$



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Contact term  $C_{s \leq 1} = 0$ ,

$$C_2 = \frac{\langle 13 \rangle \langle 32 \rangle [14] [42]}{m^6} \left\{ x(\langle \mathbf{12} \rangle + [\mathbf{12}])^2 + y(\langle \mathbf{12} \rangle - [\mathbf{12}])^2 \right\} + \mathcal{O}(\hbar)$$



- ! Complete understanding of root-Kerr and Kerr cubic theory
- ! On-shell realisation of massive gauge symmetry  $\Rightarrow$  4pt and higher!



- ! Complete understanding of root-Kerr and Kerr cubic theory
- ! On-shell realisation of massive gauge symmetry  $\Rightarrow$  4pt and higher!
- ? More constraints? e.g. 4pt current constraint, off-shell gauge invariance
- ? Higher-derivative terms  $\rightarrow$  neutron stars?
- ? Apply methods to other massive spinning QFTs?
- ? Simpler description of massive gauge invariance? (in progress)

The image features a complex, swirling pattern of concentric, overlapping bands in shades of light blue and grey, creating a sense of depth and motion, reminiscent of a galaxy or a nebula. At the center of this swirl is a bright yellow, egg-shaped core. The text "Thank you!" is written in a bold, black, sans-serif font, positioned centrally over the yellow core. The overall background is a light, hazy blue with scattered white specks, suggesting a starry sky or a digital space environment.

**Thank you!**