On-shell Approach to Supersymmetric Massive Gravity

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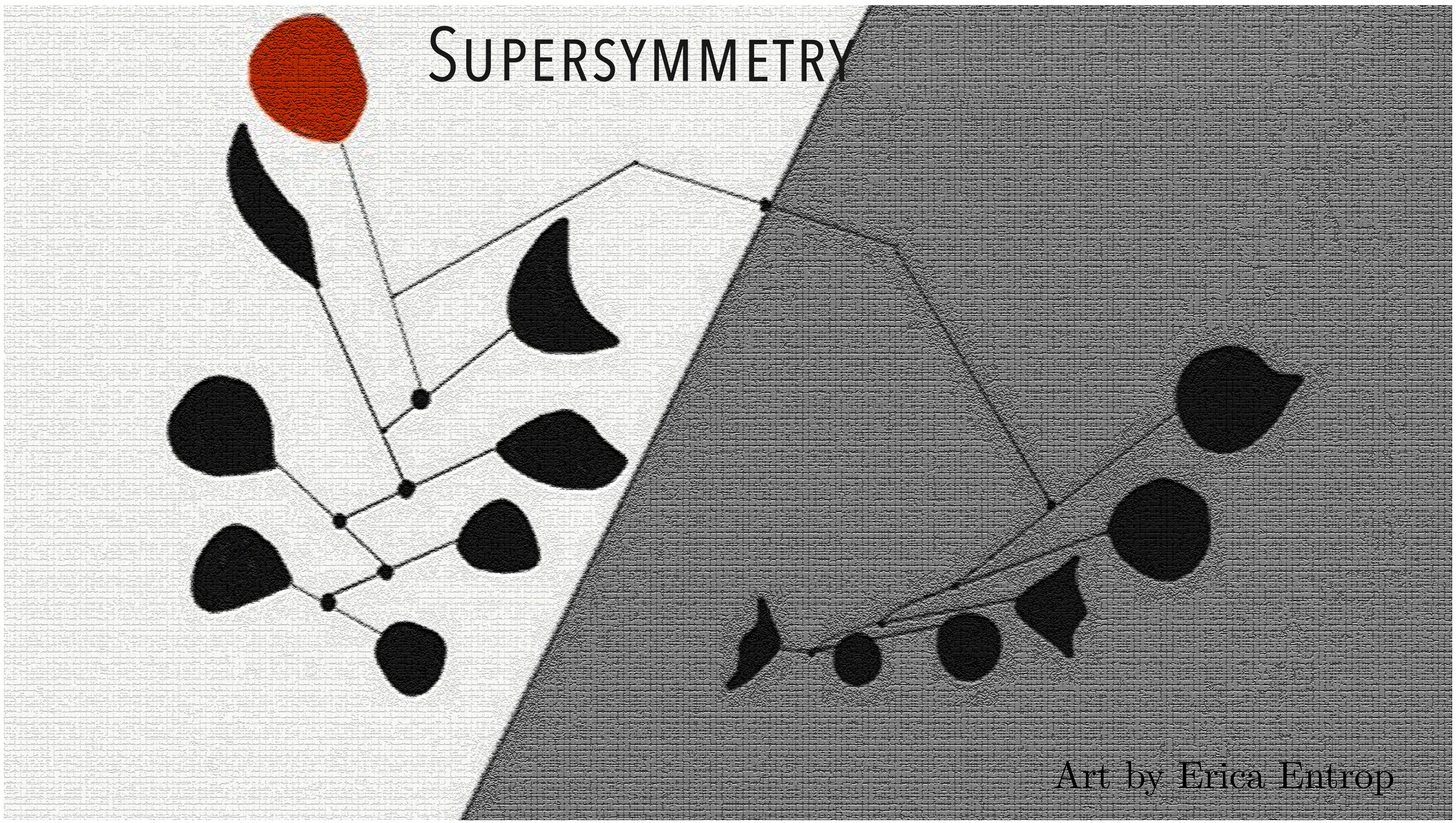


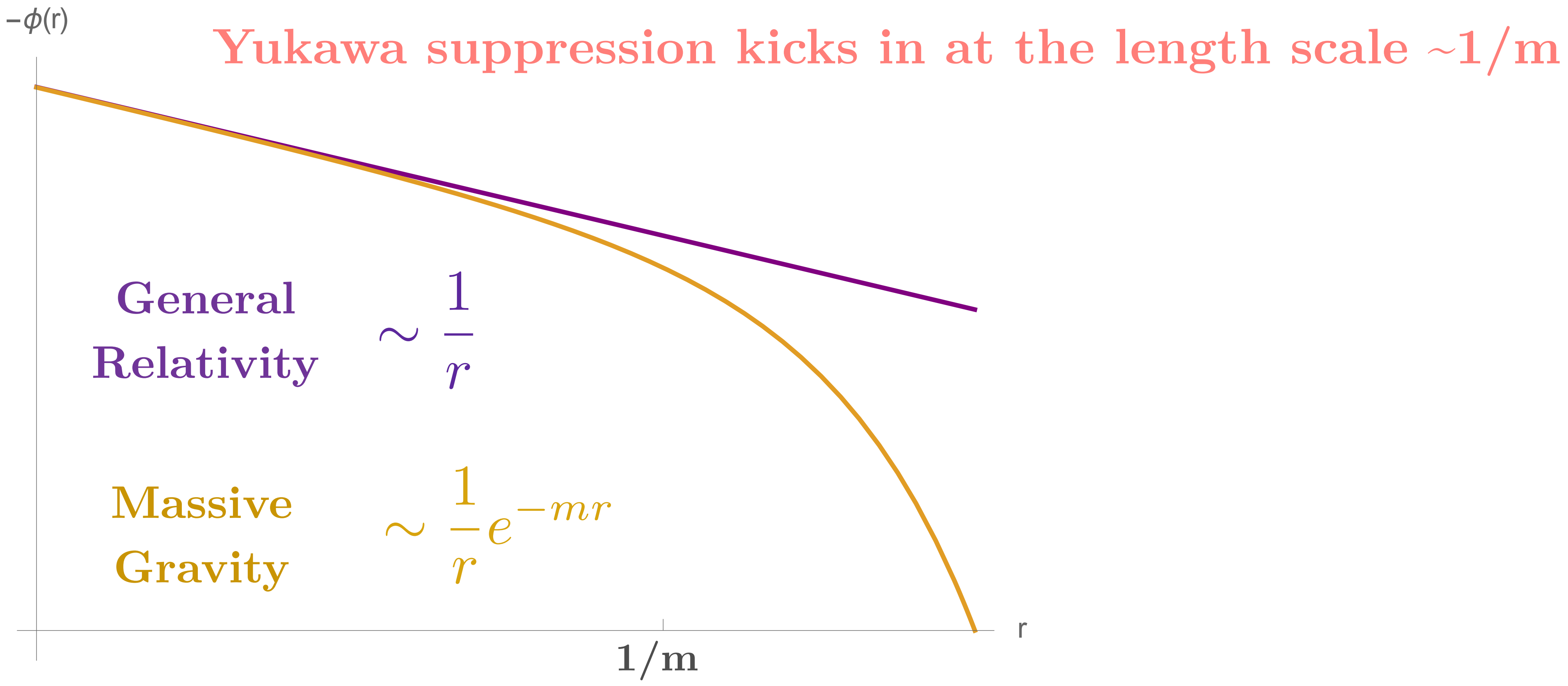
Collaborators: Callum Jones & Shruti Paranjape

Outline

- Introduction to massive gravity and the massive spin-2 cast of characters.
- Review of ingredients for constructing on-shell superamplitudes
- Procedure on how to build and constrain massive on-shell superamplitudes
- Application to massive spin-2 and the results for cubic amplitudes
- Double copy application
- Conclusions and current/future work

Why Massive Gravity?

- Can potentially give interesting alternatives with regards to understanding dark energy and the small value of the cosmological constant.



- Can describe composite states that arise, such as in nuclear resonances of QCD.

- May be necessary for "islands" to exist in information paradox resolution.

H. Geng and A. Karch, "Massive islands," JHEP 09 (2020) 121, arXiv:2006.02438 [hep-th]

Linearized Ghost-Free Massive Gravity

$$S = \int d^4x \Big[-\frac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} + \partial_\mu h_{\nu\lambda}\partial^\nu h^{\mu\lambda} - \partial_\mu h^{\mu\nu}\partial_\nu h + \frac{1}{2}\partial_\lambda h \partial^\lambda h$$

$$- \frac{1}{2}m^2(h_{\mu\nu}h^{\mu\nu} - h^2)\Big]$$

GR terms **mass terms**

- The linearized GR piece is invariant under

$$\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$$

- This is broken by the mass term

Helicity Analysis

Introduce helicity fields

$$h_{\mu\nu} \to h_{\mu\nu} + \frac{1}{m}(\nabla_\mu A_\nu + \nabla_\nu A_\mu) + \frac{2}{m^2}\nabla_\mu \nabla_\nu \phi$$

and take the relativistic limit

$$\left\{ \begin{array}{ccc} h_{\mu\nu} & \text{helicity } \pm 2 & 2 \text{ dof} \\ A_\mu & \text{helicity } \pm 1 & 2 \text{ dof} \\ \phi & \text{helicity } \quad 0 & 1 \text{ dof} \end{array} \right\}$$

giving a total of 5 degrees of freedom. (GR has 2 dof.)

Massive Gravity Non-Linear Interactions

$$S = \frac{1}{2} M_P^2 \int d^4x \sqrt{-g} \left[(R - 2\Lambda) - \frac{1}{4} m^2 V(g,h) \right]$$

$$V(g,h) = V_2(g,h) + V_3(g,h) + V_4(g,h) + V_5(g,h) + \cdots ,$$

$$\begin{aligned}
V_2(g,h) &= \langle h^2 \rangle - \langle h \rangle^2, \\
V_3(g,h) &= +c_1 \langle h^3 \rangle + c_2 \langle h^2 \rangle \langle h \rangle + c_3 \langle h \rangle^3, \\
V_4(g,h) &= +d_1 \langle h^4 \rangle + d_2 \langle h^3 \rangle \langle h \rangle + d_3 \langle h^2 \rangle^2 + d_4 \langle h^2 \rangle \langle h \rangle^2 + d_5 \langle h \rangle^4, \\
V_5(g,h) &= +f_1 \langle h^5 \rangle + f_2 \langle h^4 \rangle \langle h \rangle + f_3 \langle h^3 \rangle \langle h \rangle^2 + f_4 \langle h^3 \rangle \langle h^2 \rangle + f_5 \langle h^2 \rangle^2 \langle h \rangle \\
&\quad + f_6 \langle h^2 \rangle \langle h \rangle^3 + f_7 \langle h \rangle^5, \\
&\vdots
\end{aligned}$$

$\langle \cdots \rangle$ means trace with indices raised via $g^{\mu\nu}$

Scattering Amplitudes from Massive Gravity

- Generic massive gravity has a cutoff scale of Λ_5

$$\sim \frac{(\partial^2\phi)^3}{\Lambda_5^5}, \quad \Lambda_5 = (M_p m^4)^{1/5}$$

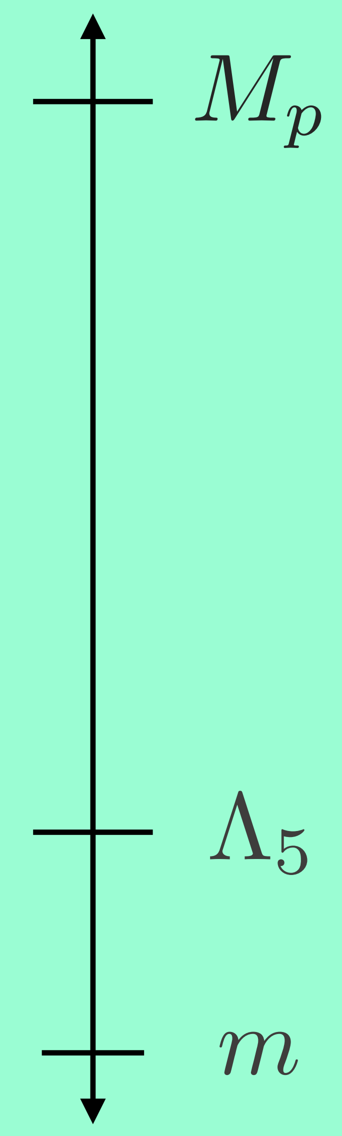
- If we pick $m_g \sim 10^{-32} eV$, the cutoff length is the size of the solar system

$$\frac{1}{\Lambda_5} \sim 10^{10} km$$

- In the high energy limit, generic massive gravity has four-graviton scattering amplitudes that grow with energy like

$$\mathcal{A}_4(hhhh) \sim E^{10} \qquad (\mathcal{A}_4^{GR} \sim E^2)$$

- Has an extra scalar degree of freedom with a wrong sign kinetic term, the Boulware-Deser ghost, giving the theory 6 degrees of freedom.



Scattering Amplitudes from tuned (dRGT) Massive Gravity

- Can tune the parameters to remove interactions coming in at scales:

$$\Lambda_5 = (M_p m^4)^{1/5} \quad \text{and} \quad \Lambda_4 = (M_p m^3)^{1/4}$$

- This raises the cutoff scale to Λ_3 and leaves 2 free parameters α_3, α_4

$$\sim \frac{h(\partial^2\phi)^n}{\Lambda_3^{3(n-1)}}, \frac{\partial A(\partial^2\phi)^n}{\Lambda_3^n}, \quad \Lambda_3 = (M_p m^2)^{1/3}$$

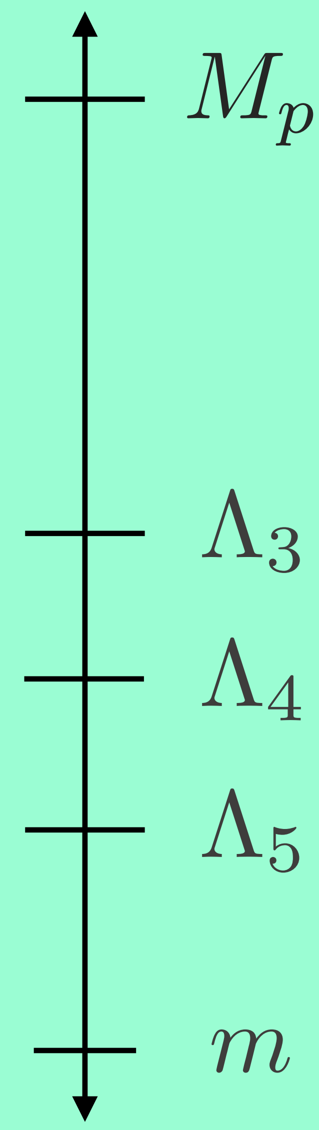
- For a graviton mass, $m_g \sim 10^{-32} eV$, the cutoff length is size of Texas.

$$\frac{1}{\Lambda_3} \sim 10^3 km$$

- In high energy limit, four-graviton amplitudes grow with energy like

$$\mathcal{A}_4(hhhh) \sim E^6$$

- The Boulware-Deser ghost is removed, giving the theory 5 DOF.



C. de Rham, G. Gabadadze, and A. J. Tolley, "Resummation of Massive Gravity," arXiv:1011.1232 [hep-th].

S. F. Hassan and R. A. Rosen, "Resolving the Ghost Problem in non-Linear Massive Gravity," arXiv:1106.3344 [hep-th].

Special Values of dRGT Massive Gravity

- Partially massless decoupling limit on de Sitter spacetime

$$\alpha_3 = -\frac{1}{2}, \quad \alpha_4 = \frac{1}{8}$$

C. De Rham, K. Hinterbichler, and L. A. Johnson, *On the (A)dS Decoupling Limits of Massive Gravity*, JHEP 09, 154 (2018), arXiv:1807.08754 [hep-th].

- Eikonal 4-point scattering

  -large impact parameter
  -small scattering angle
  -small momentum transfer

- Imposing no asymptotic superluminal propagation

$$\alpha_3 = -\frac{1}{2}$$

J. Bonifacio, K. Hinterbichler, A. Joyce, and R. A. Rosen, *Massive and Massless Spin-2 Scattering and Asymptotic Superluminality*, JHEP 06, 075, arXiv:1712.10020 [hepth].

Pseudo-Linear Massive Gravity

- Linearized Massive Gravity

$$S = \int d^4x \Big[-\frac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} + \partial_\mu h_{\nu\lambda}\partial^\nu h^{\mu\lambda} - \partial_\mu h^{\mu\nu}\partial_\nu h + \frac{1}{2}\partial_\lambda h \partial^\lambda h - \frac{1}{2}m^2(h_{\mu\nu}h^{\mu\nu} - h^2)\Big]$$

- with potential

$$V(h) = -\frac{\lambda_{0,3}}{2}\epsilon^{\mu_1\mu_2\mu_3\lambda}\epsilon^{\nu_1\nu_2\nu_3}{}_\lambda h_{\mu_1\nu_1}h_{\mu_2\nu_2}h_{\mu_3\nu_3} - \frac{\lambda_{0,4}}{2}\epsilon^{\mu_1\mu_2\mu_3\mu_4}\epsilon^{\nu_1\nu_2\nu_3\nu_4}h_{\mu_1\nu_1}h_{\mu_2\nu_2}h_{\mu_3\nu_3}h_{\mu_4\nu_4} - \frac{\lambda_{2,3}}{2}\epsilon^{\mu_1\mu_2\mu_3\mu_4}\epsilon^{\nu_1\nu_2\nu_3\nu_4}\partial_{\mu_1}\partial_{\nu_1}h_{\mu_2\nu_2}h_{\mu_3\nu_3}h_{\mu_4\nu_4}$$

- Also ghost-free and has a cutoff of Λ_3

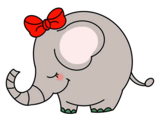
- In high energy limit, four-graviton amplitudes grow with energy like

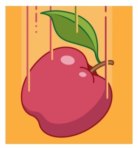
$$\mathcal{A}_4(hhhh) \sim E^6$$

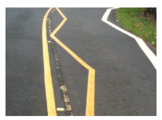
K. Hinterbichler, Ghost-free derivative interactions for a massive graviton, J. High Energy Phys. 10 (2013) 102.

James Bonifacio, Kurt Hinterbichler, Laura A. Johnson, "Pseudo-Linear Spin-2 Interactions," Phys.Rev. D99 (2019) no.2 024037, arXiv:1806.00483 [hep-th].

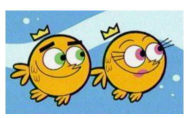
Massive Spin-2 Cast of Characters

 $$\mathcal{A}_1 = \frac{m^2}{M_P}(z_1\cdot z_2)(z_1\cdot z_3)(z_2\cdot z_3)$$

 $$\mathcal{A}_2 = \frac{1}{M_P}\left[(z_2\cdot z_3)(p_2\cdot z_1)+(z_1\cdot z_3)(p_3\cdot z_2)+(z_1\cdot z_2)(p_1\cdot z_3)\right]^2$$

 $$\mathcal{A}_3 = \frac{1}{M_P}\left[(z_2\cdot z_3)^2(p_2\cdot z_1)^2+(z_1\cdot z_3)^2(p_3\cdot z_2)^2+(z_1\cdot z_2)^2(p_1\cdot z_3)^2\right]$$

 $$\mathcal{A}_4 = \frac{1}{m^4M_P}(p_2\cdot z_1)^2(p_3\cdot z_2)^2(p_1\cdot z_3)^2$$

 $$\mathcal{B}_1 = \frac{1}{M_P}\left[(z_1\cdot z_3)(z_2\cdot z_3)\epsilon(p_1p_2z_1z_2)-(z_1\cdot z_2)(z_2\cdot z_3)\epsilon(p_1p_2z_1z_3)+(z_1\cdot z_2)(z_1\cdot z_3)\epsilon(p_1p_2z_2z_3)\right]$$

 $$\mathcal{B}_2 = \frac{1}{m^4M_P}(p_2\cdot z_1)(p_3\cdot z_2)(p_1\cdot z_3)\left[(p_1\cdot z_3)\epsilon(p_1p_2z_1z_2)-(p_3\cdot z_2)\epsilon(p_1p_2z_1z_3)+(p_2\cdot z_1)\epsilon(p_1p_2z_2z_3)\right]$$

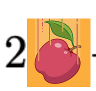
$$\epsilon_{\mu\nu}(p_i)\to z^i_\mu z^i_\nu$$

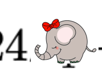
$$\epsilon_{\mu\nu\alpha\beta}v_1^\mu v_2^\nu v_3^\alpha v_4^\beta \equiv \epsilon(v_1v_2v_3v_4)$$

Lagrangian Operators

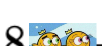
$$\mathcal{L}_1 = \frac{m^2}{3M_\mathrm{P}}h_\mu{}^\nu h_\nu{}^\lambda h_\lambda{}^\mu \quad\to\quad 2$$ 

$$\mathcal{L}_2 = \frac{M_\mathrm{P}^2}{2}\sqrt{-g}R|_{(3)} \quad\to\quad -6$$  $+2$ 

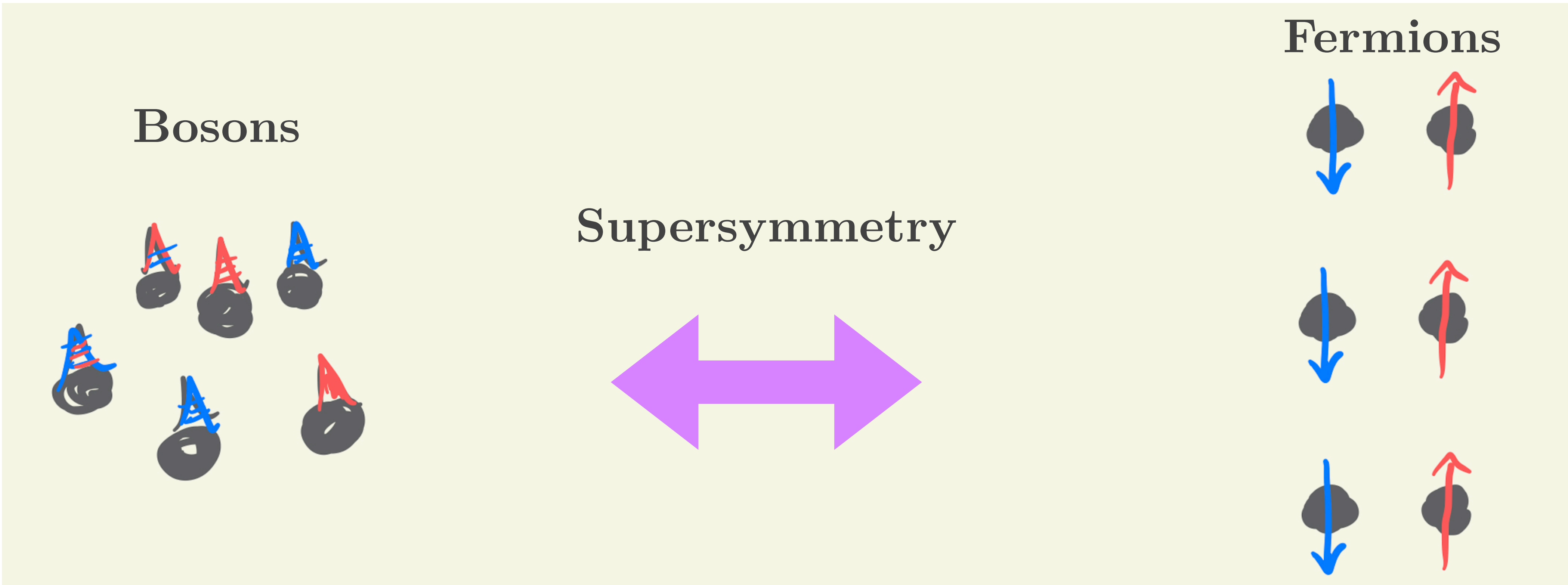
$$\mathcal{L}_3 = \frac{1}{M_\mathrm{P}}\epsilon^{\mu\nu\lambda\rho}\epsilon^{\alpha\beta\gamma\delta}\partial_\mu\partial_\alpha h_{\nu\beta}h_{\lambda\gamma}h_{\rho\delta} \quad\to\quad 12$$  -2  -2 

$$\mathcal{L}_4 = \frac{M_\mathrm{P}^2}{m^4}\sqrt{-g}R_{\mu\nu}{}^{\alpha\beta}R_{\alpha\beta}{}^{\lambda\rho}R_{\lambda\rho}{}^{\mu\nu}|_{(3)} \quad\to\quad 24$$  -12  $+4$  ,

$$\mathcal{L}_5 = \frac{1}{M_\mathrm{P}}\epsilon^{\mu\nu\lambda\rho}\partial_\mu h_{\nu\alpha}\partial_\lambda h_{\rho\beta}h^{\alpha\beta} \quad\to\quad 2$$ 

$$\mathcal{L}_6 = \frac{M_\mathrm{P}^2}{m^4}\sqrt{-g}\widetilde{R}_{\mu\nu}{}^{\alpha\beta}R_{\alpha\beta}{}^{\lambda\rho}R_{\lambda\rho}{}^{\mu\nu}|_{(3)} \quad\to\quad -8$$  -32 

Motivation for Supersymmetry



- UV completion of massive gravity is unknown
- Required in string theory to remove tachyonic modes and solves many other problems
- Can tame running parameters, provides non-renormalisation theorems
- Provides dark matter candidates, although strongly constrained
- Supersymmetry can often provide simpler toy models

Multiplets with Single Massive Spin-2 as the Highest Spin Particle

	spin-0	spin-1/2	spin-1	spin-3/2	spin-2
$\mathcal{N}=4$	42	48	27	8	1
$\mathcal{N}=3$	14	20	15	6	1
$\mathcal{N}=2$	1	4	6	4	1
$\mathcal{N}=1$	0	0	1	2	1

- We consider all particles to be massive and have the same mass.
- Unlike in GR where the maximal amount of supersymmetry is $\mathcal{N}=8$, for massive gravitons, it's $\mathcal{N}=4$.
- The multiplets have to be long. Otherwise the graviton would carry a charge.

Multiplets with Single Massive Spin-2

$\mathcal{N} = 1$

Field	Spin	$U(1)_R$	Dim.
ψ^{IJK}	$\frac{3}{2}$	1	$\mathbf{1}$
γ^{IJ}	1	0	$\mathbf{1}$
h^{IJKL}	2	0	$\mathbf{1}$
$\tilde{\psi}^{IJK}$	$\frac{3}{2}$	-1	$\mathbf{1}$

$\mathcal{N} = 2$

Field	Spin	$U(1)_R$	$SU(2)_R$	Dim.
γ^{IJ}	1	2	$\bullet$	$\mathbf{1}$
λ^{aI}	$\frac{1}{2}$	1	$\ydiagram{1}$	$\mathbf{2}$
ψ^{aIJK}	$\frac{3}{2}$	1	$\ydiagram{1}$	$\mathbf{2}$
h^{IJKL}	2	0	$\bullet$	$\mathbf{1}$
γ^{abIJ}	1	0	$\ydiagram{2}$	$\mathbf{3}$
V^{IJ}	1	0	$\bullet$	$\mathbf{1}$
ϕ	0	0	$\bullet$	$\mathbf{1}$
$\tilde{\lambda}^{aI}$	$\frac{1}{2}$	-1	$\ydiagram{1}$	$\mathbf{2}$
$\tilde{\psi}^{aIJK}$	$\frac{3}{2}$	-1	$\ydiagram{1}$	$\mathbf{2}$
$\tilde{\gamma}^{IJ}$	1	-2	$\bullet$	$\mathbf{1}$

$\mathcal{N} = 3$

Field	Spin	$U(1)_R$	$SU(3)_R$	Dim.
λ^{I}	$\frac{1}{2}$	3	$\bullet$	$\mathbf{1}$
ϕ^{a}	0	2	$\ydiagram{1}$	$\mathbf{3}$
γ^{aIJ}	1	2	$\ydiagram{1}$	$\mathbf{3}$
λ^{abI}	$\frac{1}{2}$	1	$\ydiagram{2}$	$\bar{\mathbf{6}}$
λ^{I}_{a}	$\frac{1}{2}$	1	$\ydiagram{1,1}$	$\bar{\mathbf{3}}$
ψ^{IJK}_{a}	$\frac{3}{2}$	1	$\ydiagram{1,1}$	$\bar{\mathbf{3}}$
ϕ^{abc}	0	0	$\ydiagram{2,1}$	$\mathbf{8}$
γ^{IJ}	1	0	$\bullet$	$\mathbf{1}$
γ^{abcIJ}	1	0	$\ydiagram{2,1}$	$\mathbf{8}$
h^{IJKL}	2	0	$\bullet$	$\mathbf{1}$
λ^{I}_{ab}	$\frac{1}{2}$	-1	$\ydiagram{2,2}$	$\mathbf{6}$
λ^{aI}	$\frac{1}{2}$	-1	$\ydiagram{1}$	$\mathbf{3}$
ψ^{aIJK}	$\frac{3}{2}$	-1	$\ydiagram{1}$	$\mathbf{3}$
γ^{IJ}_{a}	1	-2	$\ydiagram{1,1}$	$\bar{\mathbf{3}}$
ϕ_{a}	0	-2	$\ydiagram{1,1}$	$\bar{\mathbf{3}}$
$\tilde{\lambda}^{I}$	$\frac{1}{2}$	-3	$\bullet$	$\mathbf{1}$

$\mathcal{N} = 4$

Field	Spin	$U(1)_R$	$SU(4)_R$	Dim.
ϕ	0	4	$\bullet$	$\mathbf{1}$
λ^{aI}	$\frac{1}{2}$	3	$\ydiagram{1}$	$\mathbf{4}$
ϕ^{ab}	0	2	$\ydiagram{2}$	$\mathbf{10}$
γ^{IJ}_{ab}	1	2	$\ydiagram{1,1}$	$\mathbf{6}$
λ^{abcI}	$\frac{1}{2}$	1	$\ydiagram{2,1}$	$\overline{\mathbf{20}}$
ψ^{IJK}_{a}	$\frac{3}{2}$	1	$\ydiagram{1,1,1}$	$\bar{\mathbf{4}}$
ϕ^{abcd}	0	0	$\ydiagram{2,2}$	$\mathbf{20'}$
γ^{abcdIJ}	1	0	$\ydiagram{2,1,1}$	$\mathbf{15}$
h^{IJKL}	2	0	$\bullet$	$\mathbf{1}$
ψ^{aIJK}	$\frac{3}{2}$	-1	$\ydiagram{1}$	$\mathbf{4}$
λ^{abcdeI}	$\frac{1}{2}$	-1	$\ydiagram{2,2,1}$	$\mathbf{20}$
ϕ_{ab}	0	-2	$\ydiagram{2,2,2}$	$\overline{\mathbf{10}}$
γ^{abIJ}	1	-2	$\ydiagram{1,1}$	$\mathbf{6}$
λ^{I}_{a}	$\frac{1}{2}$	-3	$\ydiagram{1,1,1}$	$\bar{\mathbf{4}}$
$\tilde{\phi}$	0	-4	$\bullet$	$\mathbf{1}$

States are labelled with capital Latin indices I,J,... corresponding to $SU(2)_{\mathrm{LG}}$ and lower- case Latin indices a,b,... corresponding to $SU(N)_R$.

Massive On-shell Superspace

A. Herderschee, S. Koren, and T. Trott, "Massive On-Shell Supersymmetric Scattering Amplitudes," JHEP 10 (2019) 092, arXiv:1902.07204 [hep-th]

- On-shell superspace helps organize the states and amplitudes of a given supermultiplet
- We introduced Grassmann variables η_A^I which allow us to write the states as a superfield

$\underline{\mathcal{N}=1}$

$$\underbrace{\Psi^{IJK} = \psi^{IJK}}_{\text{gravitino}} + \underbrace{\frac{1}{2\sqrt{3}}\eta^{(I}\gamma^{JK)}}_{\text{gravi-photon}} + \underbrace{\eta_L h^{IJKL}}_{\text{graviton}} + \underbrace{\frac{1}{2}\eta_L\eta^L\tilde{\psi}^{IJK}}_{\text{gravitino}}$$

$\underline{\mathcal{N}=2}$

$$\begin{aligned}\Gamma^{IJ} = {}& \gamma^{IJ} + \frac{1}{\sqrt{6}}\eta_a^{(I}\lambda^{aJ)} + \eta_{aK}\psi^{aIJK} + \frac{1}{2}\epsilon^{ab}\eta_{aK}\eta_{bL}h^{IJKL} + \frac{1}{2}\eta_{aK}\eta_b^K\gamma^{abIJ} \\ & + \frac{1}{8}\epsilon^{ab}\eta_a^{(I}\eta_{bK}V^{J)K} + \frac{1}{2\sqrt{3}}\epsilon^{ab}\eta_a^I\eta_b^J\phi + \frac{1}{4}\eta_{bK}\eta^{bK}\eta_a^{(I}\tilde{\lambda}^{aJ)} + \frac{1}{\sqrt{2}}\eta_{bK}\eta^{bK}\eta_{aL}\tilde{\psi}^{aIJL} \\ & + \frac{1}{8}\eta_{bK}\eta^{bK}\eta_{cL}\eta^{cL}\tilde{\gamma}^{IJ}\end{aligned}$$

Massive On-shell Superspace

$\underline{\mathcal{N}=3}$

$$\Lambda^I = \lambda^I + \eta_a^I \phi^a + \eta_{aJ}\gamma^{aIJ} + \frac{1}{2}\eta_{aJ}\eta_b^J\lambda^{abI} + \frac{1}{2}\epsilon^{abc}\eta_a^I\eta_{bJ}\lambda_c^J + \frac{1}{2}\epsilon^{abc}\eta_{aK}\eta_{bJ}\psi_c^{IJK}$$
$$+\frac{1}{6}\eta_{aJ}\eta_b^{(J}\eta_c^{I)}\phi^{abc} + \frac{1}{2\sqrt{6}}\epsilon^{abc}\eta_a^I\eta_{bJ}\eta_{cK}\gamma^{JK} + \frac{1}{6\sqrt{2}}\eta_{aJ}\eta_b^{(J}\eta_{cK}\gamma^{abcK)I}$$
$$+\frac{1}{6}\epsilon^{abc}\eta_{aJ}\eta_{bK}\eta_{cL}h^{IJKL} + \frac{1}{2\sqrt{2}}\epsilon^{abc}\eta_a^I\eta_b^J\eta_c^K\eta_{dJ}\lambda_K^d + \frac{1}{12}\epsilon^{abc}\epsilon^{def}\eta_{aJ}\eta_{bK}\eta_d^J\eta_e^K\lambda_{cf}^I$$
$$+\frac{1}{8}\epsilon^{abc}\eta_{aJ}\eta_{bK}\eta_{cL}\eta_d^L\psi^{dIJK} + \frac{1}{16}\epsilon^{abc}\eta_{aJ}\eta_{bK}\eta_{cL}\epsilon^{def}\eta_d^I\eta_e^J\gamma_f^{KL}$$
$$+\frac{1}{24\sqrt{2}}\epsilon^{abc}\eta_a^I\eta_b^J\eta_c^K\epsilon^{def}\eta_{dJ}\eta_{eK}\phi_f + \frac{1}{144}\epsilon^{abc}\eta_a^J\eta_b^K\eta_c^L\epsilon^{def}\eta_{dJ}\eta_{eK}\eta_{fL}\tilde{\lambda}^I$$

$\underline{\mathcal{N}=4}$

$$\Phi = \phi + \eta_{aI}\lambda^{aI} + \frac{1}{2}\eta_{aI}\eta_b^I\phi^{ab} + \frac{1}{2\sqrt{2}}\epsilon^{abcd}\eta_{aI}\eta_{bJ}\gamma_{cd}^{IJ} + \frac{1}{3\sqrt{2}}\eta_{aI}\eta_b^{(I}\eta_{cJ}\lambda^{abcJ)}$$
$$+\frac{1}{6}\epsilon^{abcd}\eta_{aI}\eta_{bJ}\eta_{cK}\psi_d^{IJK} + \frac{1}{12}\eta_{aI}\eta_b^{(I}\eta_{cJ}\eta_d^{J)}\phi^{abcd} + \frac{1}{8\sqrt{6}}\eta_{aI}\eta_b^{(I}\eta_{cJ}\eta_{dK}\gamma^{abcdJK)}$$
$$+\frac{1}{24}\epsilon^{abcd}\eta_{aI}\eta_{bJ}\eta_{cK}\eta_{dL}h^{IJKL} + \frac{1}{30}\epsilon^{abcd}\eta_{aI}\eta_{bJ}\eta_{cK}\eta_{dL}\eta_e^L\psi^{eIJK}$$
$$+\frac{1}{24\sqrt{3}}\eta_{aI}\eta_b^{(I}\eta_{cJ}\eta_d^J\eta_{eK}\lambda^{K)abcde} + \frac{1}{144}\epsilon^{abcd}\epsilon^{efgh}\eta_{aI}\eta_{bJ}\eta_{cK}\eta_e^I\eta_f^J\eta_g^K\phi_{dh}$$
$$+\frac{1}{80}\epsilon^{abcd}\eta_{aI}\eta_{bJ}\eta_{cK}\eta_{dL}\eta_e^I\eta_f^J\gamma^{efKL} + \frac{1}{360}\epsilon^{abcd}\epsilon^{efgh}\eta_{aI}\eta_{bJ}\eta_{cK}\eta_{dL}\eta_e^I\eta_f^J\eta_g^K\lambda_h^L$$
$$+\frac{1}{2880}\epsilon^{abcd}\epsilon^{efgh}\eta_{aI}\eta_{bJ}\eta_{cK}\eta_{dL}\eta_e^I\eta_f^J\eta_g^K\eta_h^L\tilde{\phi}$$

On-shell Superspace

- Graviton field can be projected out from the superfield.

$$h^{IJKL}_{\mathcal{N}=1} \propto \frac{\partial}{\partial\eta_{(I}}\Psi^{JKL)}\Big|_{\eta=0}$$

$$h^{IJKL}_{\mathcal{N}=2} \propto \epsilon_{ab}\frac{\partial^2}{\partial\eta_{(I,a}\partial\eta_{J,b}}\Gamma^{KL)}\Big|_{\eta=0}$$

$$h^{IJKL}_{\mathcal{N}=3} \propto \epsilon_{abc}\frac{\partial^3}{\partial\eta_{(I,a}\partial\eta_{J,b}\partial\eta_{K,c}}\Lambda^{L)}\Big|_{\eta=0}$$

$$h^{IJKL}_{\mathcal{N}=4} \propto \epsilon_{abcd}\frac{\partial^4}{\partial\eta_{(I,a}\partial\eta_{J,b}\partial\eta_{K,c}\partial\eta_{L),d}}\Phi\Big|_{\eta=0}$$

Constraints on the Superamplitudes

- We construct superamplitudes which encode all of the scattering data.
- Supersymmetry implies superamplitudes must be annihilated by the supercharges,

$$Q^a_\alpha \cdot \mathcal{A}_n\left(\{\eta\}\right) = 0, \qquad\qquad Q^\dagger_{a\dot{\alpha}} \cdot \mathcal{A}_n\left(\{\eta\}\right) = 0$$

The most general form of a superamplitude with only massive external states is

$$\mathcal{A}_n\left(\{\eta_i\}\right) = \delta^{(2\mathcal{N})}\left(Q^\dagger\right) G\left(\{\eta_{in}\}\right),$$

where $G\left(\{\eta_{in}\}\right)$ is an arbitrary polynomial in the $2(n-2)\mathcal{N}$ Grassmann variables

$$\eta^{I_i}_{a,in} \equiv \eta^{I_i}_{a,i} - \frac{1}{m_n}[i^{I_i} n_{I_n}]\eta^{I_n}_{a,n}, \qquad i = 1, ..., n-2,$$

and the supersymmetric delta function is

$$\delta^{(2\mathcal{N})}\left(Q^\dagger\right) = \prod_{a=1}^{\mathcal{N}}\left(\sum_{i<j}\langle i_{I_i} j_{I_j}\rangle \eta^{I_i}_{a,i}\eta^{I_j}_{a,j} + \frac{1}{2}\sum_{i=1}^{n} m_i \eta_{a,iI_i}\eta^{I_i}_{a,i}\right).$$

Cubic Superamplitude Construction

- The 3-graviton contribution to the superamplitude takes the form

$$\mathcal{A}_3\left(\Phi^{\{I_1\}}\Phi^{\{I_2\}}\Phi^{\{I_3\}}\right)\Big|_{\text{R-singlet}} = \underbrace{\delta^{(2\mathcal{N})}}_{\text{Grassman delta-function}}\left(Q^\dagger\right)\underbrace{F^{\{I_1\}\{I_2\}\{I_3\}}}_{\text{function of massive spinor-helicity variables}}{}_{J_1\ldots J_\mathcal{N}}\epsilon^{a_1\ldots a_\mathcal{N}}\underbrace{\eta^{J_1}_{12,a_1}\ldots\eta^{J_\mathcal{N}}_{12,a_\mathcal{N}}}_{\text{Grassman polynomial}}$$

1. Write down the most general expression for F using spinor-helicity variables.
2. Remove redundancies due to Schouten identities.
3. Enforce Bose/Fermi symmetry for indistinguishable superfields.

Graviton Superamplitude Structure

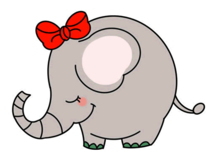
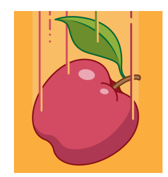
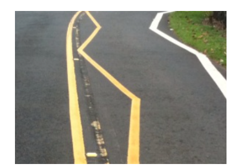
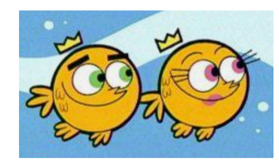
$$\mathcal{N} = 1$$

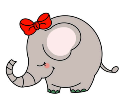
$$
\begin{aligned}
&F^L_{\mathcal{N}=1}\\
&= \beta_1\Big(\{\mathbf{33}\}(\{1^L\mathbf{3}\} - 2\{\mathbf{31}^L\})\{\mathbf{12}\}^3 + 3\{\mathbf{13}\}^2\{\mathbf{23}\}\{1^L\mathbf{2}\}\{\mathbf{12}\} + \{\mathbf{11}\}(\{\mathbf{12}\}((3\{\mathbf{31}^L\}\\
&\qquad - \{1^L\mathbf{3}\})\{\mathbf{23}\}^2 - 2(\{\mathbf{21}^L\}\{\mathbf{33}\} + \{\mathbf{32}\}\{\mathbf{31}^L\})\{\mathbf{23}\} + \{\mathbf{22}\}\{\mathbf{33}\}(2\{\mathbf{31}^L\} - \{1^L\mathbf{3}\}))\\
&\qquad + \{\mathbf{13}\}\{\mathbf{23}\}(\{\mathbf{23}\}(2\{\mathbf{21}^L\} - \{1^L\mathbf{2}\}) - \{\mathbf{22}\}(\{\mathbf{31}^L\} + \{1^L\mathbf{3}\})))\Big)\\
&+ \beta_2\Big(\{\mathbf{33}\}(2\{\mathbf{31}^L\} - \{1^L\mathbf{3}\})\{\mathbf{12}\}^3 + 3\{\mathbf{13}\}\{\mathbf{23}\}(\{1^L\mathbf{3}\} - 2\{\mathbf{31}^L\})\{\mathbf{12}\}^2\\
&\qquad + (3\{\mathbf{23}\}\{\mathbf{21}^L\}\{\mathbf{13}\}^2 + \{\mathbf{11}\}((6\{\mathbf{31}^L\} - 5\{1^L\mathbf{3}\})\{\mathbf{23}\}^2 - (\{\mathbf{21}^L\}\{\mathbf{33}\}\\
&\qquad + \{\mathbf{32}\}\{\mathbf{31}^L\})\{\mathbf{23}\} + \{\mathbf{22}\}\{\mathbf{33}\}(\{1^L\mathbf{3}\} - 2\{\mathbf{31}^L\})))\{\mathbf{12}\}\\
&\qquad + \{\mathbf{11}\}\{\mathbf{13}\}\{\mathbf{23}\}(\{\mathbf{23}\}(\{1^L\mathbf{2}\} - 2\{\mathbf{21}^L\}) + \{\mathbf{22}\}(\{\mathbf{31}^L\} + \{1^L\mathbf{3}\}))\Big)\\
&+ \beta_3\Big((\{\mathbf{13}\}(-\{\mathbf{21}^L\}\{\mathbf{33}\} + 2\{\mathbf{32}\}\{\mathbf{31}^L\} - 2\{\mathbf{23}\}(\{\mathbf{31}^L\} - \{1^L\mathbf{3}\})) + \{\mathbf{33}\}\{\mathbf{21}\}\{\mathbf{31}^L\}\\
&\qquad - \{\mathbf{12}\}\{1^L\mathbf{3}\}))\{\mathbf{12}\}^2 + \{\mathbf{11}\}\{\mathbf{23}\}(-\{\mathbf{11}^L\}\{\mathbf{22}\}\{\mathbf{33}\} + \{\mathbf{12}\}(\{\mathbf{21}^L\}\{\mathbf{33}\}\\
&\qquad - 2\{\mathbf{32}\}\{\mathbf{31}^L\} + 2\{\mathbf{23}\}(\{\mathbf{31}^L\} - \{1^L\mathbf{3}\})) + \{\mathbf{13}\}\{\mathbf{22}\}\{1^L\mathbf{3}\})\Big)\\
&+ \beta_4\Big(\{\mathbf{11}\}\{\mathbf{22}\}(-2\{\mathbf{11}^L\}\{\mathbf{23}\}\{\mathbf{33}\} + \{\mathbf{21}\}\{\mathbf{31}^L\} + \{\mathbf{12}\}(\{\mathbf{31}^L\} - \{1^L\mathbf{3}\}))\{\mathbf{33}\}\\
&\qquad + \{\mathbf{13}\}(\{\mathbf{32}\}\{\mathbf{31}^L\} + \{\mathbf{23}\}(\{1^L\mathbf{3}\} - \{\mathbf{31}^L\})))\Big)
\end{aligned}
$$

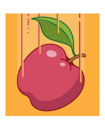
where we define $\{i^I j^J\} \equiv [i^I|\not{p}_1\not{p}_2|j^J]$

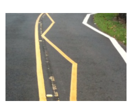
Graviton Superamplitude Structures

$$\mathcal{N} = 1$$

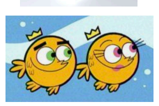
   

✓  $\mathcal{A}_1 = z_{12} z_{13} z_{23}$

✓  $\mathcal{A}_2 = (z_{23} zp_{12} + z_{13} zp_{23} + z_{12} zp_{31})^2$

✓  $\mathcal{A}_3 = z_{23}^2 zp_{12}^2 + z_{13}^2 zp_{23}^2 + z_{12}^2 zp_{31}^2$

X  $\mathcal{A}_4 = zp_{12}^2 zp_{23}^2 zp_{31}^2$

✓  $\mathcal{B}_1 = z_{13} z_{23} \epsilon(p_1 p_2 z_1 z_2) - z_{12} z_{23} \epsilon(p_1 p_2 z_1 z_3) + z_{12} z_{13} \epsilon(p_1 p_2 z_2 z_3)$

X  $\mathcal{B}_2 = zp_{12} zp_{23} zp_{31} (zp_{31} \epsilon(p_1 p_2 z_1 z_2) - zp_{23} \epsilon(p_1 p_2 z_1 z_3) + zp_{12} \epsilon(p_1 p_2 z_2 z_3))$

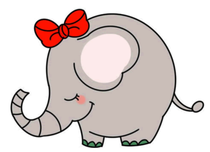
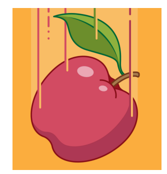
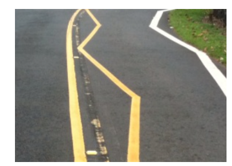
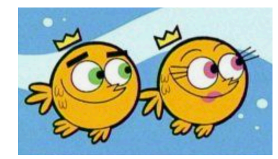
Graviton Superamplitude Structure

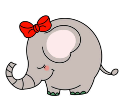
$$\mathcal{N} = 2$$

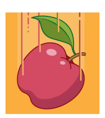
$$
\begin{aligned}
&F^{KL}_{\mathcal{N}=2} \\
&= \beta_1 \Big(\{\mathbf{11}\}(\{\mathbf{23}\}(\{\mathbf{2}1^L\}\{\mathbf{3}1^K\} + \{\mathbf{2}1^K\}\{\mathbf{3}1^L\} - \{\mathbf{32}\}\{1^K 1^L\}) + \{\mathbf{22}\}(-\{\mathbf{3}1^K\}\{\mathbf{3}1^L\} \\
&\qquad + \{\mathbf{33}\}\{1^K 1^L\})) + \{\mathbf{13}\}^2\{\mathbf{2}1^K\}(-\{\mathbf{2}1^L\} + \{1^L\mathbf{2}\}) + \frac{1}{2}\{\mathbf{12}\}\{\mathbf{23}\}(-\{\mathbf{1}1^L\}\{\mathbf{3}1^K\} \\
&\qquad + \{\mathbf{1}1^K\}(\{\mathbf{3}1^L\} - 2\{1^L\mathbf{3}\})) + \{\mathbf{12}\}^2\{\mathbf{3}1^K\}\{1^L\mathbf{3}\} + \frac{1}{2}\{\mathbf{12}\}\{\mathbf{13}\}(-2\{\mathbf{2}1^K\}\{1^L\mathbf{3}\} \\
&\qquad + \{\mathbf{23}\}(\{1^K 1^L\} + \{1^L 1^K\})) \Big) \\
&+ \beta_2 \Big(\{\mathbf{13}\}^2\{\mathbf{2}1^K\}(\{\mathbf{2}1^L\} - \{1^L\mathbf{2}\}) + \frac{1}{2}\{\mathbf{12}\}\{\mathbf{23}\}(\{\mathbf{1}1^L\}\{\mathbf{3}1^K\} - \{\mathbf{1}1^K\}\{\mathbf{3}1^L\}) \\
&\qquad - \{\mathbf{12}\}^2\{\mathbf{3}1^K\}\{1^L\mathbf{3}\} - \frac{1}{2}\{\mathbf{12}\}\{\mathbf{13}\}(-2\{\mathbf{2}1^K\}\{1^L\mathbf{3}\} + \{\mathbf{23}\}(\{1^K 1^L\} + \{1^L 1^K\})) \\
&\qquad + \{\mathbf{11}\}(-\{\mathbf{23}\}(\{\mathbf{2}1^L\}\{\mathbf{3}1^K\} + \{\mathbf{2}1^K\}\{\mathbf{3}1^L\} - \{\mathbf{32}\}\{1^K 1^L\}) + \{\mathbf{22}\}(\{\mathbf{3}1^K\}\{\mathbf{3}1^L\} \\
&\qquad + \{\mathbf{33}\}\{1^L 1^K\})) + \{\mathbf{12}\}\{\mathbf{23}\}\{\mathbf{1}1^K\}\{1^L\mathbf{3}\} \Big) \\
&+ \beta_3 \Big(-\{\mathbf{12}\}^2\{\mathbf{3}1^K\}(\{\mathbf{3}1^L\} - 2\{1^L\mathbf{3}\}) + \{\mathbf{11}\}\{\mathbf{23}\}(\{\mathbf{2}1^L\}\{\mathbf{3}1^K\} + \{\mathbf{23}\}\{1^K 1^L\} \\
&\qquad + \{\mathbf{2}1^K\}\{1^L\mathbf{3}\}) + \{\mathbf{12}\}(-2\{\mathbf{1}1^L\}\{\mathbf{23}\}\{\mathbf{3}1^K\} + \{\mathbf{13}\}\{\mathbf{2}1^K\}\{\mathbf{3}1^L\} \\
&\qquad - 2\{\mathbf{13}\}\{\mathbf{23}\}\{1^K 1^L\} + 2\{\mathbf{21}\}\{\mathbf{33}\}\{1^K 1^L\} + \{\mathbf{1}1^K\}\{\mathbf{23}\}(\{\mathbf{3}1^L\} - \{1^L\mathbf{3}\}) \\
&\qquad - 2\{\mathbf{13}\}\{\mathbf{2}1^K\}\{1^L\mathbf{3}\} + 2\{\mathbf{13}\}\{\mathbf{23}\}\{1^L 1^K\} + \{\mathbf{13}\}\{\mathbf{32}\}\{1^K 1^L\}) \Big) \\
&+ \beta_4 \Big(\{\mathbf{11}\}\{\mathbf{23}\}(\{\mathbf{23}\}\{1^L 1^K\} - \{\mathbf{2}1^L\}\{\mathbf{3}1^K\} - \{\mathbf{2}1^K\}\{1^L\mathbf{3}\}) + \frac{1}{2}\{\mathbf{12}\}(\{\mathbf{13}\}\{\mathbf{23}\}\{1^K 1^L\} \\
&\qquad + 3\{\mathbf{1}1^L\}\{\mathbf{23}\}\{\mathbf{3}1^K\} - 2\{\mathbf{13}\}\{\mathbf{2}1^K\}\{\mathbf{3}1^L\} - 2\{\mathbf{21}\}\{\mathbf{33}\}\{1^K 1^L\} \\
&\qquad + 4\{\mathbf{13}\}\{\mathbf{2}1^K\}\{1^L\mathbf{3}\} - 5\{\mathbf{13}\}\{\mathbf{23}\}\{1^L 1^K\} - \{\mathbf{1}1^K\}\{\mathbf{23}\}(\{\mathbf{3}1^L\} - 2\{1^L\mathbf{3}\})) \\
&\qquad + \{\mathbf{12}\}^2(\{\mathbf{3}1^K\}(\{\mathbf{3}1^L\} - 2\{1^L\mathbf{3}\}) + \{\mathbf{33}\}\{1^L 1^K\}) \Big) \\
&+ \beta_5 \Big(\frac{1}{2}\{\mathbf{12}\}(\{\mathbf{1}1^L\}\{\mathbf{23}\}\{\mathbf{3}1^K\} + 2\{\mathbf{12}\}\{\mathbf{33}\}\{1^K 1^L\} - 2\{\mathbf{13}\}\{\mathbf{32}\}\{1^K 1^L\} \\
&\qquad + \{\mathbf{13}\}\{\mathbf{23}\}\{1^K 1^L\} - \{\mathbf{1}1^K\}\{\mathbf{23}\}\{\mathbf{3}1^L\} - 2\{\mathbf{21}\}\{\mathbf{33}\}\{1^K 1^L\} - \{\mathbf{13}\}\{\mathbf{23}\}\{1^L 1^K\}) \Big)
\end{aligned}
$$

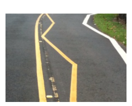
Graviton Superamplitude Structures

$$\mathcal{N} = 2$$

✓  $\mathcal{A}_1 = z_{12} z_{13} z_{23}$

✓  $\mathcal{A}_2 = (z_{23} zp_{12} + z_{13} zp_{23} + z_{12} zp_{31})^2$

✓  $\mathcal{A}_3 = z_{23}^2 zp_{12}^2 + z_{13}^2 zp_{23}^2 + z_{12}^2 zp_{31}^2$

X  $\mathcal{A}_4 = zp_{12}^2 zp_{23}^2 zp_{31}^2$

✓  $\mathcal{B}_1 = z_{13} z_{23} \epsilon(p_1 p_2 z_1 z_2) - z_{12} z_{23} \epsilon(p_1 p_2 z_1 z_3) + z_{12} z_{13} \epsilon(p_1 p_2 z_2 z_3)$

X  $\mathcal{B}_2 = zp_{12} zp_{23} zp_{31} (zp_{31} \epsilon(p_1 p_2 z_1 z_2) - zp_{23} \epsilon(p_1 p_2 z_1 z_3) + zp_{12} \epsilon(p_1 p_2 z_2 z_3))$

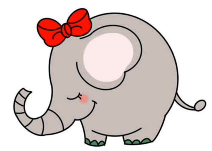
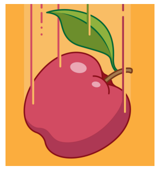
Graviton Superamplitude Structure

$$\mathcal{N} = 3$$

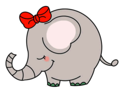
$$\begin{aligned}
&F^{JKL}_{\mathcal{N}=3} \\
&= \beta_1 \Big(-2\{\mathbf{1}1^L\}\{\mathbf{23}\}\{1^J1^K\} + \{\mathbf{13}\}\{\mathbf{2}1^L\}\{1^J1^K\} + \{\mathbf{1}1^K\}\{\mathbf{23}\}\{1^J1^L\} \\
&\qquad - 3\{\mathbf{13}\}\{\mathbf{2}1^K\}\{1^J1^L\} + 2\{\mathbf{12}\}\{\mathbf{3}1^K\}\{1^J1^L\} + \{\mathbf{13}\}\{\mathbf{2}1^J\}\{1^K1^L\} \\
&\qquad + 2\{\mathbf{21}\}\{\mathbf{3}1^J\}\{1^K1^L\} + 2\{\mathbf{13}\}\{1^J\mathbf{2}\}\{1^K1^L\} - 2\{\mathbf{12}\}\{1^J\mathbf{3}\}\{1^K1^L\} \\
&\qquad - \{\mathbf{1}1^K\}\{\mathbf{23}\}\{1^L1^J\} + \{\mathbf{13}\}\{\mathbf{2}1^K\}\{1^L1^J\} - \{\mathbf{12}\}\{\mathbf{3}1^K\}\{1^L1^J\} \\
&\qquad + \{\mathbf{12}\}\{\mathbf{3}1^J\}\{1^L1^K\} + \{\mathbf{1}1^J\}(3\{\mathbf{2}1^K\}\{\mathbf{3}1^L\} - \{\mathbf{23}\}(4\{1^K1^L\} + \{1^L1^K\})) \Big) \\
&+ \beta_2 \Big(-\{\mathbf{1}1^L\}\{\mathbf{23}\}\{1^J1^K\} + (-\{\mathbf{1}1^J\}\{\mathbf{23}\} + \{\mathbf{21}\}\{\mathbf{3}1^J\} + \{\mathbf{13}\}\{1^J\mathbf{2}\})\{1^K1^L\} \\
&\qquad + \{\mathbf{12}\}(\{\mathbf{3}1^L\}\{1^J1^K\} - \{1^J\mathbf{3}\}\{1^K1^L\}) \Big)
\end{aligned}$$

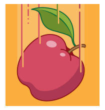
Graviton Superamplitude Structures

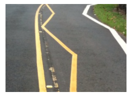
$$\mathcal{N} = 3$$

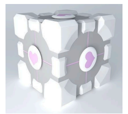
 

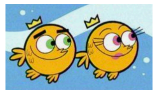
Only structures corresponding to dRGT massive gravity are allowed.

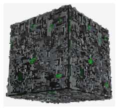
$\checkmark$  $\mathcal{A}_1 = z_{12} z_{13} z_{23}$

$\checkmark$  $\mathcal{A}_2 = (z_{23} zp_{12} + z_{13} zp_{23} + z_{12} zp_{31})^2$

X  $\mathcal{A}_3 = z_{23}^2 zp_{12}^2 + z_{13}^2 zp_{23}^2 + z_{12}^2 zp_{31}^2$

X  $\mathcal{A}_4 = zp_{12}^2 zp_{23}^2 zp_{31}^2$

X  $\mathcal{B}_1 = z_{13} z_{23} \epsilon(p_1 p_2 z_1 z_2) - z_{12} z_{23} \epsilon(p_1 p_2 z_1 z_3) + z_{12} z_{13} \epsilon(p_1 p_2 z_2 z_3)$

X  $\mathcal{B}_2 = zp_{12} zp_{23} zp_{31} (zp_{31} \epsilon(p_1 p_2 z_1 z_2) - zp_{23} \epsilon(p_1 p_2 z_1 z_3) + zp_{12} \epsilon(p_1 p_2 z_2 z_3))$

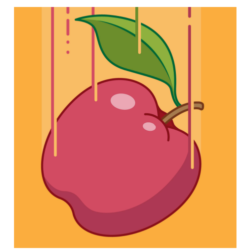
Graviton Superamplitude Structure

$$\mathcal{N} = 4$$

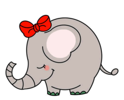
$$F_{\mathcal{N}=4}^{I_1 J_1 K_1 L_1} = \beta_1 \Big(\{1^{I_1} 1^{K_1}\}\{1^{L_1} 1^{J_1}\} + \{1^{J_1} 1^{I_1}\}\{1^{K_1} 1^{L_1}\} \Big)$$

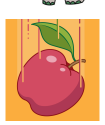
Graviton Superamplitude Structures

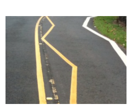
$$\mathcal{N} = 4$$



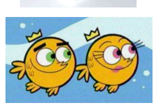
Only allowed structure corresponds to dRGT massive gravity with special tuning.

X  $\mathcal{A}_1 = z_{12}z_{13}z_{23}$

✓  $\mathcal{A}_2 = (z_{23}zp_{12} + z_{13}zp_{23} + z_{12}zp_{31})^2$

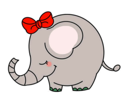
X  $\mathcal{A}_3 = z_{23}^2 zp_{12}^2 + z_{13}^2 zp_{23}^2 + z_{12}^2 zp_{31}^2$

X  $\mathcal{A}_4 = zp_{12}^2 zp_{23}^2 zp_{31}^2$

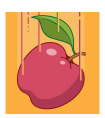
X  $\mathcal{B}_1 = z_{13}z_{23}\epsilon(p_1p_2z_1z_2) - z_{12}z_{23}\epsilon(p_1p_2z_1z_3) + z_{12}z_{13}\epsilon(p_1p_2z_2z_3)$

X  $\mathcal{B}_2 = zp_{12}zp_{23}zp_{31}(zp_{31}\epsilon(p_1p_2z_1z_2) - zp_{23}\epsilon(p_1p_2z_1z_3) + zp_{12}\epsilon(p_1p_2z_2z_3))$

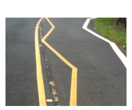
Cubic Interactions in the High Energy Limit



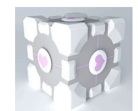
$$\xrightarrow{\text{HE}} \begin{cases} (h^- v^- \phi): & -\frac{1}{M_P m}\frac{\langle 12\rangle^3\langle 13\rangle}{\langle 23\rangle} \\ (h^+ v^+ \phi): & -\frac{1}{M_P m}\frac{[12]^3[13]}{[23]} \end{cases}$$



$$\xrightarrow{\text{HE}} \begin{cases} (h^- h^- h^+): & \frac{1}{M_P}\frac{\langle 12\rangle^6}{\langle 13\rangle^2\langle 23\rangle^2} \\ (h^- v^- v^+): & -\frac{2}{M_P}\frac{\langle 12\rangle^4}{\langle 23\rangle^2} \\ (h^- \phi\phi): & \frac{3}{M_P}\frac{\langle 12\rangle^2\langle 13\rangle^2}{\langle 23\rangle^2} \\ (h^+ \phi\phi): & \frac{3}{M_P}\frac{[12]^2[13]^2}{[23]^2} \\ (h^+ v^+ v^-): & -\frac{2}{M_P}\frac{[12]^4}{[23]^2} \\ (h^+ h^+ h^-): & \frac{1}{M_P}\frac{[12]^6}{[13]^2[23]^2} \end{cases}$$

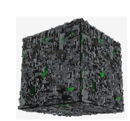


$$\xrightarrow{\text{HE}} \begin{cases} (h^- h^- \phi): & \frac{2}{M_P m^2}\langle 12\rangle^4 \\ (h^- v^- v^-): & -\frac{2}{M_P m^2}\langle 12\rangle^2\langle 13\rangle^2 \\ (h^+ v^+ v^+): & -\frac{2}{M_P m^2}[12]^2[13]^2 \\ (h^+ h^+ \phi): & \frac{2}{M_P m^2}[12]^4 \end{cases}$$



$$\xrightarrow{\text{HE}} \begin{cases} (h^- h^- h^-): & \frac{1}{M_P m^4}\langle 12\rangle^2\langle 13\rangle^2\langle 23\rangle^2 \\ (h^+ h^+ h^+): & \frac{1}{M_P m^4}[12]^2[13]^2[23]^2 \end{cases}$$

$$\xrightarrow{\text{HE}} \begin{cases} (h^- h^- \phi): & -\frac{1}{M_P m^2}\langle 12\rangle^4 \\ (h^- v^- v^-): & \frac{1}{M_P m^2}\langle 12\rangle^2\langle 13\rangle^2 \\ (h^+ v^+ v^+): & -\frac{1}{M_P m^2}[12]^2[13]^2 \\ (h^+ h^+ \phi): & \frac{1}{M_P m^2}[12]^4 \end{cases}$$

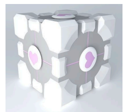
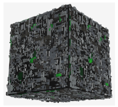


$$\xrightarrow{\text{HE}} \begin{cases} (h^- h^- h^-): & \frac{3}{M_P m^4}\langle 12\rangle^2\langle 13\rangle^2\langle 23\rangle^2 \\ (h^+ h^+ h^+): & -\frac{3}{M_P m^4}[12]^2[13]^2[23]^2 \end{cases}$$

Use Massless Susy Ward Identities

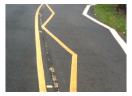
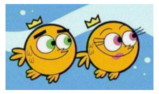
$\underline{\mathcal{N} = 1, 2, 3, 4}$

$$Q \cdot A_3\left(h^-, h^-, \psi^-\right) = |3]A_3\left(h^-, h^-, h^-\right) = 0$$

For any supersymmetric theory, this rules out X  X 

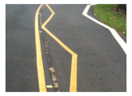
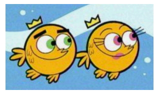
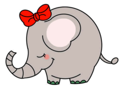
$\underline{\mathcal{N} = 3}$

$$Q_b \cdot A_3\left(h^-, h^-, \psi^+\right) = |3]A_3\left(h^-, h^-, \phi_b\right) = 0\,,$$
$$Q_b \cdot A_3\left(h^-, h^-, \psi_a^+\right) = |3]A_3\left(h^-, h^-, \phi_{ab}\right) = 0\,.$$

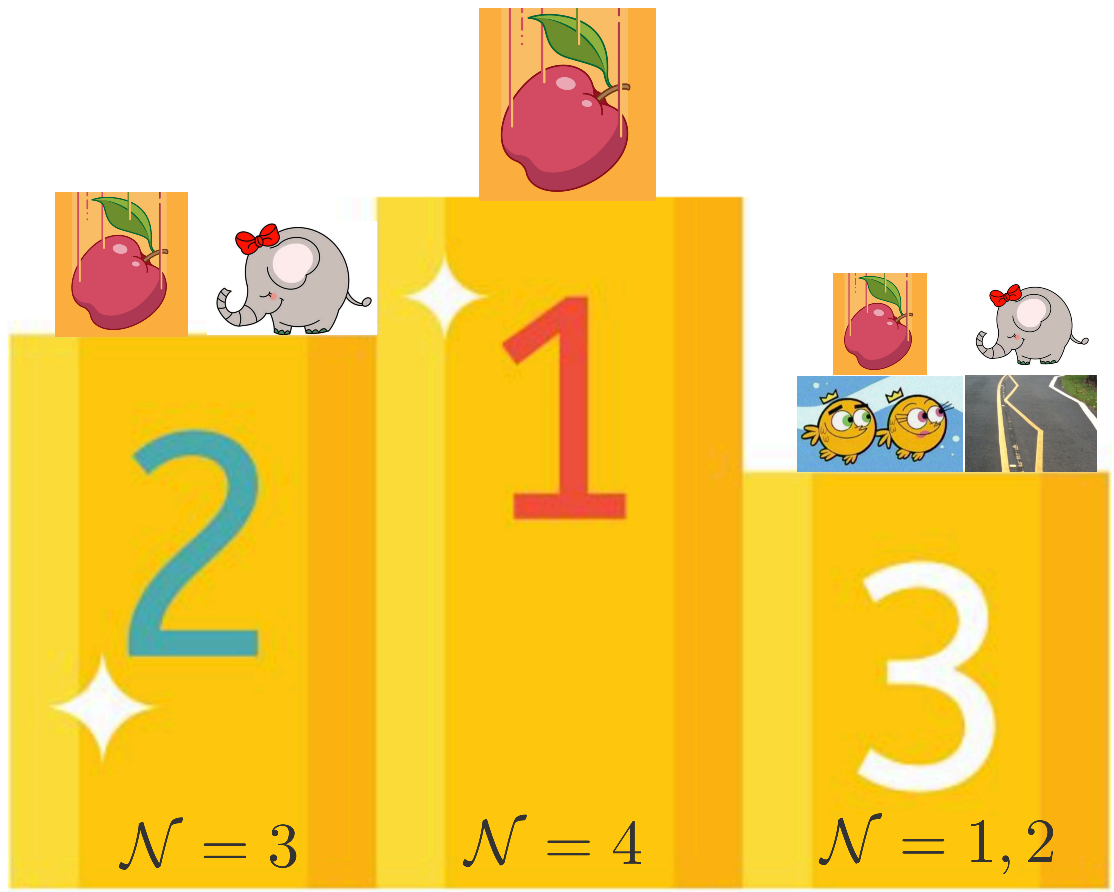
This rules out X  X 

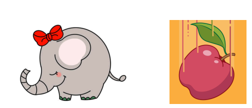
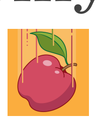
$\underline{\mathcal{N} = 4}$

$$Q_b \cdot A_3\left(h^-, h^-, \psi_a^+\right) = |3]A_3\left(h^-, h^-, \phi_{ab}\right) = 0$$
$$Q_b \cdot A_3\left(h^-, v_{abc}^-, \psi_a^+\right) = |3]A_3\left(h^-, v_{abc}^-, \phi_{ab}\right) = 0$$
$$Q_a \cdot A_3\left(h^-, v_{abc}^-, \psi_d^+\right) = |3]A_3\left(h^-, v_{abc}^-, \phi_{ad}\right) = 0$$

This rules out X  X  X 

Results



- **At $\mathcal{N} = 1, 2$, dRGT and pseudo-linear massive gravity as well as the parity-odd cubic vertex B1.** 
- **At $\mathcal{N} = 3$, only dRGT massive gravity is allowed.** 
- **At $\mathcal{N} = 4$, only dRGT massive gravity with the special parameter value is allowed.** 

Purely Massive Double Copy

- Cubic and quartic amplitudes of massive Yang-Mills have successfully been found to double copy to amplitudes of dRGT massive gravity.

  A. Momeni, J. Rumbutis and A. J. Tolley, Massive Gravity from Double Copy, JHEP 12 (2020) 030 [2004.07853].

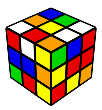
  L. A. Johnson, C. R. T. Jones and S. Paranjape, Constraints on a Massive Double-Copy and Applications to Massive Gravity, JHEP 02 (2021) 148 [2004.12948].

- Unfortunately a KLT-based massive double copy fails at 5-point due to spurious poles.

  L. A. Johnson, C. R. T. Jones and S. Paranjape, Constraints on a Massive Double-Copy and Applications to Massive Gravity, JHEP 02 (2021) 148 [2004.12948].

- It is unknown whether some alternate double copy prescription for massive particles could remove the pathologies, but we can look at the double copy for the superfields and cubic superamplitudes.

Massive Spin-1 Cast of Characters



$$\mathcal{C}_1 = \frac{1}{m^2} (z_1 \cdot p_2)(z_2 \cdot p_3)(z_3 \cdot p_1) \ ,$$

$$\mathcal{C}_2 = (z_2 \cdot z_3)(z_1 \cdot p_2) + (z_1 \cdot z_3)(z_2 \cdot p_3) + (z_1 \cdot z_2)(z_3 \cdot p_1) \ ,$$

$$\mathcal{C}_3 = \frac{1}{m^2} \left[(z_1 \cdot p_2)\, \epsilon(p_1 p_3 z_2 z_3) + (z_2 \cdot p_3)\, \epsilon(p_2 p_1 z_3 z_1) + (z_3 \cdot p_1)\, \epsilon(p_3 p_2 z_1 z_2) \right]$$

Lagrangian Operators

$$\hat{\mathcal{L}}_1 = \frac{f_{abc}}{m} F^{a\mu\rho} F_{\rho}^{b\nu} F_{\mu\nu}^{c} |_{(3)} \qquad \rightarrow \qquad \mathcal{C}_1$$

$$\hat{\mathcal{L}}_2 = F_{\mu\nu}^{a} F_{a}^{\mu\nu} |_{(3)} \qquad \rightarrow \qquad \mathcal{C}_2$$

$$\hat{\mathcal{L}}_3 = \frac{f_{abc}}{m} \epsilon_{\mu\nu\alpha\beta} F^{a\mu\rho} F_{\rho}^{b\nu} F^{c\alpha\beta} |_{(3)} \qquad \rightarrow \qquad \mathcal{C}_3.$$

Multiplets with Single Massive Spin-1

	spin-0	spin-1/2	spin-1
$\mathcal{N}=2$	5	4	1
$\mathcal{N}=1$	1	2	1

Massive Vector Superfields

$\mathcal{N}=1$

$$\Pi^I = \lambda^I + \eta^I H + \eta_J g^{IJ} + \frac{1}{2}\eta_J\eta^J\tilde{\lambda}^I$$

$\mathcal{N}=2$

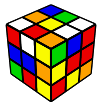
$$\Theta = \phi + \eta_{Ia}\psi^{Ia} + \frac{1}{2}\eta_{Ia}\eta^I_b H^{ab} + \frac{1}{2}\epsilon^{ab}\eta_{Ia}\eta_{Jb}g^{IJ} + \frac{1}{3}\eta_{Ia}\eta^a_J\eta^I_b\tilde{\psi}^{Jb} + \epsilon^{ab}\epsilon^{cd}\eta^I_a\eta^J_b\eta_{cI}\eta_{dJ}\tilde{\phi}$$

Massive Gluon Superamplitude Structures

$$\mathcal{N}=1 \qquad \mathcal{A}_3[\Pi,\Pi,\Pi] = \delta^{(2)}\left(Q^\dagger\right)\eta_{12,L_1}\beta_1\left[-2\{\mathbf{1}1^{L_1}\}\{\mathbf{23}\}+\{\mathbf{21}\}\{\mathbf{3}1^{L_1}\}+\{\mathbf{13}\}\{1^{L_1}\mathbf{2}\}\right.$$
$$\left.+\{\mathbf{12}\}\{\mathbf{3}1^{L_1}\}-\{\mathbf{12}\}\{1^{L_1}\mathbf{3}\}\right].$$

$$\mathcal{N}=2 \qquad \mathcal{A}_3[\Theta,\Theta,\Theta] = \delta^{(4)}\left(Q^\dagger\right)\epsilon^{ab}\eta_{12,a,K_1}\eta_{12,b,L_1}\beta_1\left[\{1^{K_1}1^{L_1}\}+\{1^{L_1}1^{K_1}\}\right]$$

- Projecting to get the 3-gluon component amplitudes gives

X  $$\mathcal{C}_1 = \frac{1}{m^2}\left(z_1\cdot p_2\right)\left(z_2\cdot p_3\right)\left(z_3\cdot p_1\right),$$

✓  $$\mathcal{C}_2 = \left(z_2\cdot z_3\right)\left(z_1\cdot p_2\right)+\left(z_1\cdot z_3\right)\left(z_2\cdot p_3\right)+\left(z_1\cdot z_2\right)\left(z_3\cdot p_1\right),$$

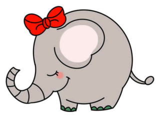
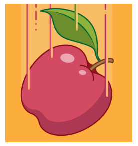
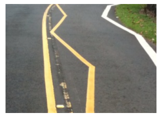
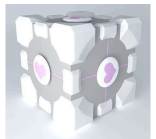
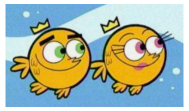
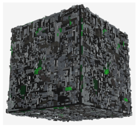
X  $$\mathcal{C}_3 = \frac{1}{m^2}\left[\left(z_1\cdot p_2\right)\epsilon\left(p_1p_3z_2z_3\right)+\left(z_2\cdot p_3\right)\epsilon\left(p_2p_1z_3z_1\right)+\left(z_3\cdot p_1\right)\epsilon\left(p_3p_2z_1z_2\right)\right]$$

Double Copy for Superfields

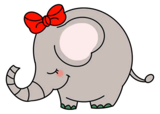
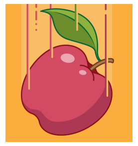
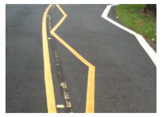
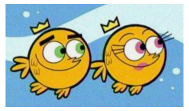
	Double Copy	Fields Generated	
$\mathcal{N}=0+0$	$g^{(IJ}\otimes g^{KL)}$ $\epsilon^{KL}g_K^{(I}\otimes g_L^{J)}$ $g^{IJ}\otimes g_{IJ}$	Graviton Massive 2-form Dilaton	h^{IJKL} B^{IJ} D
$\mathcal{N}=1+0$	$\Pi^{(I}\otimes g^{JK)}$ $\Pi_J\otimes g^{IJ}$	Graviton superfield Vector superfield	Ψ^{IJK} V^I
$\mathcal{N}=2+0$	$\Theta\otimes g^{IJ}$	Graviton superfield	Γ^{IJ}
$\mathcal{N}=1+1$	$\Pi^{(I}\otimes\Pi^{J)}$ $\epsilon_{IJ}\Pi^I\otimes\Pi^J$	Graviton superfield Vector superfield	Γ^{IJ} W
$\mathcal{N}=2+1$	$\Theta\otimes\Pi^I$	Graviton superfield	Λ^I
$\mathcal{N}=2+2$	$\Theta\otimes\Theta$	Graviton superfield	Φ

Graviton Vertices from Cubic Double Copy

$$\mathcal{N}=0\otimes\mathcal{N}=0$$

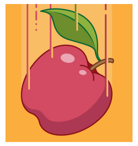
     

$$\mathcal{N}=1\otimes\mathcal{N}=0, \quad \mathcal{N}=2\otimes\mathcal{N}=0$$

Only 3 independent combinations

$$\mathcal{N}=1\otimes\mathcal{N}=1, \quad \mathcal{N}=2\otimes\mathcal{N}=1, \quad \mathcal{N}=2\otimes\mathcal{N}=2$$

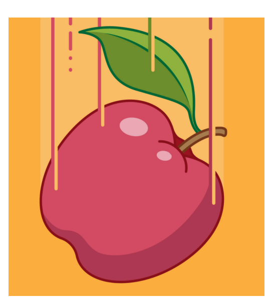


Summary/Awards Ceremony

- Sufficient amounts of supersymmetry selects out cubic amplitudes for ghost-free dRGT massive gravity.

- This can be seen by a brute force construction of the amplitudes, but can also be seen by taking the high energy limit and seeing which amplitudes are ruled out by massless supersymmetry Ward identities.

- A fully supersymmetric double copy selects the special cubic dRGT amplitude.



- Award for maximal supersymmetry compatibility.
- Award for double copy compatibility.
- Award for lack of asymptotic superluminal propagation in eikonal scattering.
- Award for partially massless symmetry and the highest cutoff in de Sitter spacetime.

Progress on 4-point Amplitudes

- This was **all** of the cubic amplitudes. At quartic order there are an infinite number of amplitudes that could be considered, which can be characterized by the number of derivatives.
- We allowed for 4-point graviton vertices with up to 6 derivatives.
- In spinor-helicity formalism this could require about 30,000 terms.
- Looking for a more clever way: restrict to amplitudes that give better high energy growth, restricting to parity even amplitudes, etc.
- Maybe there is a BCFW-like recursion scheme.

Future Work

- See how adding supersymmetry to massive gravity affects quantum corrections.
- Our results suggest for $\mathcal{N} \geq 3$ the dRGT potential will not get detuned by quantum corrections.

- See what sort of massive gravity is allowed for the shortened $\mathcal{N} = 8$ multiplet.

- Apply these methods to bigravity theories. It would be interesting to compare to cubic amplitudes from string theory.

D. Lust, C. Markou, P. Mazloumi, and S. Stieberger, "Extracting bigravity from string theory,"JHEP 12 (2021) 220, arXiv:2106.04614 [hep-th].

- Is there a massive double copy prescription that would hold beyond quartic order?