Topologically massive QCD meets gravity

The Cotton Double Copy

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Based on 2202.10499 with Will Emond and 2212.04783 with Mariana Carrillo González, Will Emond, Justinas Rumbutis and Chris White.

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Topologically massive theories are gauge invariant theories of massive particles, with Lagrangians [Deser, Jackiw & Templeton, 1982]

$$\mathcal{L}_{Gauge} = -\frac{1}{2e^2}\mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right) - \frac{m}{2e^2}\varepsilon^{\mu\nu\alpha}\mathrm{Tr}\left(F_{\mu\nu}A_\alpha - \frac{2}{3}A_\mu A_\nu A_\alpha\right),$$

$$\mathcal{L}_{Gravity} = \frac{2R}{\kappa^2} - \frac{1}{4\kappa^2 m}\varepsilon^{\lambda\mu\nu}\Gamma^\rho_{\lambda\sigma}\left(\partial_\mu\Gamma^\sigma_{\nu\rho} + \frac{2}{3}\Gamma^\sigma_{\mu\tau}\Gamma^\tau_{\nu\rho}\right).$$

One d.o.f each, helicity determined by $\pm m$.

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Massive Gauge Theories

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One d.o.f each, helicity determined by $\pm m$.

Equations of motion:

$$D^\mu F^a_{\mu\nu} + \frac{m}{2}\epsilon_{\nu\rho\sigma}F^{a\rho\sigma} = J^a_\nu, \qquad G_{\mu\nu} + \frac{1}{m}C_{\mu\nu} = -\kappa^2 T_{\mu\nu}$$

Where $C_{\mu\nu} = \epsilon_\mu^{\ \alpha\rho}\nabla_\alpha\left(R_{\nu\rho} - \frac{1}{4}g_{\nu\rho}R\right)$ is the **Cotton tensor.**

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Motivation

Let's take the equation of motion...

$$D^\mu F^a_{\mu\nu} + \frac{m}{2}\epsilon_{\nu\rho\sigma} F^{a\rho\sigma} = J^a_\nu$$

and naively replace colour with kinematics.

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Taking $a \to \alpha\beta$ and $D^\mu \to \nabla^\mu$ [See talk by Mangan] , we find

$$\nabla^\mu R_{\mu\nu\alpha\beta} + \frac{m}{2}\epsilon_{\nu\rho\sigma} R^{\rho\sigma}{}_{\alpha\beta} = J_{\alpha\beta\nu}$$

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This is **topologically massive gravity** [NM, 2021]

$$G_{\mu\nu} + \frac{1}{m}C_{\mu\nu} = -\frac{1}{2m}\delta^\gamma_{(\mu}\delta^\lambda_{\nu)}\varepsilon_\lambda{}^{\alpha\beta}\left[\nabla^\tau R_{\tau\gamma\alpha\beta} + \frac{m}{2}\epsilon_{\gamma\rho\sigma}R^{\rho\sigma}{}_{\alpha\beta}\right]$$

Motivation for a double copy.... Let's review it in spinor form.

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Spinors

Tensors with s Lorentz indices can be replaced with **spinors** with $2s$ indices using *Infeld–Van der Waerden symbols* σ.

In $D = 4$, we use $\sigma^{\bar{\mu}}_{AA'} = (1, \sigma^i)_{AA'}$ such that e.g.

$$T_{\bar{\mu}\bar{\nu}\bar{\rho}}\sigma^{\bar{\mu}}_{AA'}\sigma^{\bar{\nu}}_{BB'}\sigma^{\bar{\rho}}_{CC'} = T_{AA'BB'CC'}.$$

No chirality in $D = 3$ — no primed indices!

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No chirality in $D = 3$ — no primed indices!

We can choose a real basis: $\sigma^{\mu}_{AB} = \{1, \sigma^1, \sigma^3\}$

$$T_{\mu\nu}\sigma^{\mu}_{AB}\sigma^{\nu}_{CD} = T_{ABCD} = \frac{1}{4}T_{(AB)(CD)}$$

We can use this to explore the Weyl double copy....

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The **Weyl double copy** is given by [Luna, Monteiro, Nicholson & O'Connell, 2018]

$$\Psi_{ABCD} = \frac{1}{S}\varphi_{(AB}\widetilde{\varphi}_{CD)},$$

which relates **Curvature spinors**

$$\varphi_{AB} = \frac{1}{2}F_{\bar{\mu}\bar{\nu}}\sigma^{\bar{\mu}}_{AA'}\bar{\sigma}^{\bar{\nu}A'}{}_{B}, \qquad \Psi_{ABCD} = \frac{1}{2}W_{\bar{\mu}\bar{\nu}\bar{\rho}\bar{\tau}}\sigma^{\bar{\mu}}_{AA'}\bar{\sigma}^{\bar{\nu}A'}{}_{B}\sigma^{\bar{\rho}}_{CC'}\bar{\sigma}^{\bar{\tau}C'}{}_{D}$$

Would like to define a 3D analogue – but $W^{\mu\nu\rho\sigma} = 0$!

The Cotton tensor seems like a good substitute... But first, let's explore some amplitudes.

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Spinor-Helicity

Amplitudes $\sim \mathcal{A}(p_i, \epsilon_j)$.

We can convert p_μ, ϵ_μ to **spinor-helicity** variables with σ^μ

$D = 4$	$p_{iAA'} = \|i\rangle_A [i\|_{A'}$	$p^2 = 0$
$D = 4$	$\epsilon^-_{iAA'} = \frac{\|i\rangle\|\eta\rangle}{[i\eta]}$	$\epsilon^- \cdot \epsilon^+ = 1$
$D = 3$	$p_{iAB} = \|i\rangle_{(A} \langle\bar{i}\|_{B)}$	$p^2 = -m^2 \propto \langle i\bar{i}\rangle^2$
$D = 3$	$\epsilon_{iAB} = \frac{\|i\rangle\|i\rangle}{m}$	$\epsilon \cdot \epsilon^* = 1$

Takehome: amplitudes are now functions of $|i\rangle$ and $[i|$ in $D = 4$, and $|i\rangle$ and $|\bar{i}\rangle$ in $D = 3$.

Can derive ϵ_{iAB} from the TMYM EOM [NM, 2020].

Note: Under $|i\rangle \to t\,|i\rangle$, $\epsilon \to t^2\epsilon$ and $p \to p$ – little group scaling.

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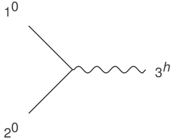
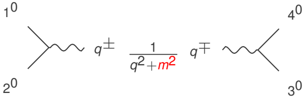
Matter coupled amplitudes [Arkani-Hamed, Huang & Huang, 2017] [NM, 2020] .



$$= \quad g(Mx)^h, \qquad x = \frac{\langle \bar{3}|u_1|\bar{3}\rangle}{m} \sim u_1 \cdot \epsilon_3$$

Factorisation gives us four-particle **anyon** amplitudes [NM, 2020] , e.g.



$$= \quad g^2 \frac{(M_1 M_2)^h}{q^2 + m^2} \left(\frac{x_1}{x_2}\right)^h$$

3pts and 4pts shown to DC [NM, 2020] [Burger, Emond & NM, 2021] including in the eikonal* [Carrillo González, Momeni & Rumbutis, 2021]

The amplitudes double copy - useful to express curvature spinors in terms of amplitudes using **KMOC** [Kosower, Maybee & O'Connell, 2018]

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Curvature Spinors from Amplitudes

We can mode expand and evolve

$$\mathbb{F}^{\mu\nu} = -i \sum_{\eta=\pm} \int_q \left(a_\eta(q) q^{[\mu} \varepsilon_\eta^{\nu]} e^{-iq\cdot x} - a_\eta^\dagger(q) q^{[\mu} \varepsilon_\eta^{\nu]} e^{iq\cdot x} \right)$$

and take an expectation value with a state $|\psi\rangle \sim \int_k \varphi(k) \, |k\rangle$

$$F^{\mu\nu} = \langle\psi| S^\dagger \mathbb{F}^{\mu\nu} S |\psi\rangle = 2\,\mathrm{Re}\, i \int_{k,k'} \langle k| \mathbb{F}^{\mu\nu} T |k'\rangle + \mathcal{O}(g^2).$$

Noting that $S = 1 + iT$ and

$$\langle k_1' \cdot\cdot k_m' | T | k_1 \cdot\cdot k_n \rangle = \mathcal{A}\left(k_1 \cdot\cdot k_n \to k_1' \cdot\cdot k_m'\right) \hat{\delta}^{(D)}\left(\sum k\right),$$

finding [Monteiro, O'Connell, Veiga & Sergola, 2020]

$$\varphi_{AB}(x) = 2\,\mathrm{Re}\, i \int_q |q\rangle_A \, |q\rangle_B \, \mathcal{A}_3(q) e^{-iqx} + \mathcal{O}(g^2).$$

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We can do exactly the same thing in gravity:

$$\Psi(x)_{ABCD} = 2\,\mathrm{Re}\, i \int_q |q\rangle_A \, |q\rangle_B \, |q\rangle_C \, |q\rangle_D \, \mathcal{M}_3(q) e^{-iqx}.$$

The Weyl double copy in **momentum space** is

$$|q\rangle_A \, |q\rangle_B \, |q\rangle_C \, |q\rangle_D \, \mathcal{M}_3^+ = \frac{1}{S}(|q\rangle_A \, |q\rangle_B \, \mathcal{A}_3^+)(|q\rangle_C \, |q\rangle_D \, \tilde{\mathcal{A}}_3^+),$$

where

$$S \sim \mathcal{A}_3^{scalar} \sim \lambda$$

This can be defined in 3D!

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Topologically Massive Gauge Theory

In 3D $\tilde{F}^{\mu a}(x) = \frac{1}{2}\epsilon^{\mu\nu\rho} F^a_{\nu\rho}(x) = \langle \tilde{\mathbb{F}}^{\mu a} \rangle$.

EOM is then

$$\varepsilon^{\mu\nu\rho} D_\nu \tilde{F}^a_\rho + m\tilde{F}^{\mu a} = 0$$

We can do the usual mode expansion for a **single** helicity

$$\tilde{\mathbb{F}}^\mu = \pm m \int_q \left[a(q)\epsilon^\mu_-(q) e^{-iq\cdot x} + a^\dagger(q)\epsilon^\mu_+(q) e^{iq\cdot x} \right]$$

We define $\varphi_{AB} = \tilde{F}^\mu \sigma_{\mu AB}$ and the same steps as in 4D lead to

$$\varphi_{AB}(x) = -\frac{1}{M} \mathrm{Re} \int \mathrm{d}\Phi(q)\delta(u\cdot q) \left[\mathcal{A}^{(3)}_+(q) \left|q\right\rangle_A \left|q\right\rangle_B e^{-iq\cdot x} \right],$$

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In topologically massive gravity, the free equations of motion are

$$G_{\mu\nu} + \frac{1}{m} C_{\mu\nu} = 0.$$

The **Cotton** tensor is a sensible candidate for the double copy

$$C^{\mu\nu} = -\frac{\kappa}{4}\,\partial_\lambda \partial^2 \varepsilon^{\lambda\sigma(\mu} h^{\nu)}{}_\sigma \; .$$

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Mode expand the graviton, follow the same steps as in gauge theory

$$\langle \mathbb{C}^{\mu\nu} \rangle = -\frac{\kappa m^2}{4M} \, \mathrm{Re} \int \mathrm{d}\Phi(q) \, \hat{\delta}(u \cdot q) \, \mathcal{M}^{(3)}_{-k}(q) \, q_\lambda \, \varepsilon^{\lambda\sigma(\mu} \epsilon^{\nu)}_k(q) \epsilon_{k\sigma}(q) \, e^{-iq\cdot x} \ .$$

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Plugging in $m\epsilon_{-}^{\mu} \sigma_{\mu AB} = |q\rangle_A \, |q\rangle_B$, much Schoutening,

$$\langle \Psi_{ABCD} \rangle = -\frac{\kappa}{2} \frac{m}{M} \, \mathrm{Re} \int \mathrm{d}\Phi(q) \, \hat{\delta}(u \cdot q) \Big[\, |q\rangle_A \, |q\rangle_B \, |q\rangle_C \, |q\rangle_D \, i\mathcal{M}^{(3)}_{+}(q) \Big] \, e^{-iq\cdot x} \; .$$

This matches the field theory calculation for e.g. Anyons [Emond & **NM**, 2022]

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Momentum Space Cotton Double Copy

In momentum-space, we find [Emond & **NM**, 2022]

$$
\begin{aligned}
\Psi_{ABCD}(q) &= -im\, |q\rangle_A\, |q\rangle_B\, |q\rangle_C\, |q\rangle_D\, \mathcal{M}_+^{(3)}(q), \\
\Phi_{AB}(q) &= |q\rangle_A\, |q\rangle_B\, \mathcal{A}_+^{(3)}(q).
\end{aligned}
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$$\Psi_{ABCD}(q) = -im\,|q\rangle_A\,|q\rangle_B\,|q\rangle_C\,|q\rangle_D\,\mathcal{M}_+^{(3)}(q),$$
$$\Phi_{AB}(q) = |q\rangle_A\,|q\rangle_B\,\mathcal{A}_+^{(3)}(q).$$

The momentum space Cotton double copy is then

$$\Psi_{ABCD}(q) = \frac{m}{2}\frac{\Phi_{(AB}(q)\Phi_{CD)}(q)}{S}.$$

Shown to hold in position space for type-N [Emond & **NM**, 2022] [González, Momeni & Rumbutis, 2022] .

What about type-D? This question is best asked in **twistor space**, let's (very) quickly review it...

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Formally, twistor space is the set of solutions of

$$\nabla_A^{(A'}\Omega^{B')} = 0,$$

which has a solution on $\mathbb{M}_4$ of the form $\Omega^{A'} = \mu^{A'} - x^{A'A}\lambda_A$.

Twistors $Z^A = (\mu^{A'}, \lambda_A) \in \mathbb{PT}$ are related to $\mathbb{M}_4$ via the incidence relations

$$\mu^{A'} = x^{A'A}\lambda_A$$

These are *projective*, since this is invariant under

$$\mu^{A'} \to t\mu^{A'}, \qquad \lambda_A \to t\lambda_A, \quad \forall t \in \mathbb{C}.$$

The matrix $x^{A'A} = x^{\bar{\mu}}\sigma_{\bar{\mu}}^{A'A}$ therefore defines a line on $\mathbb{PT}$.

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We have seen that curvature tensors have a KMOC representation

$$\varphi_{A_1 A_2 \cdots A_{2s}}(x) = \operatorname{Re} i \int \mathrm{d}\Phi(q)\, \hat{\delta}(u \cdot q) \Big[|q\rangle_{A_1} |q\rangle_{A_2} \cdots |q\rangle_{A_{2s}} \mathcal{A}_{+}^{(3)}(q) \Big] e^{-iq\cdot x} \,.$$

They also have a Twistor representation via the Penrose transform

$$\varphi_{A_1 A_2 \cdots A_{2s}}(x) = \frac{1}{2\pi i} \oint \langle \lambda \mathrm{d}\lambda \rangle\, \lambda_{A_1} \lambda_{A_2} \cdots \lambda_{A_{2s}}\, \rho_x \left[f(Z^A) \right],$$

where $f(tZ^A) = t^{-2s-2} f(Z^A)$ and ρ_X reminds us to restrict to the line corresponding to point $x^{\bar{\mu}}$.

These are related by coordinate transformation, and in $D = 4$, we find

[Guevara 2021] [Luna, **NM** & White, 2022]

$$f(Z^A) \sim \int_0^\infty d\omega \omega^s e^{-\frac{1}{2}\omega \langle \lambda | u | \mu]} \mathcal{A}_3^{(s)}(\lambda)$$

What does the Weyl DC look like on twistor space?

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Twistor-Weyl Double Copy

The Weyl DC on Twistor space is [White, 2020] [Chacón, Nagy & White, 2021]

$$f_{grav}^{(-6)} = \frac{f_{EM}^{(-4)}\tilde{f}_{EM}^{(-4)}}{f_{scal.}^{(-2)}}, \qquad f^{(2\ell)} = \frac{(Q_{AB}Z^A Z^B)^{\ell}}{|\ell|!},$$

where Q_{AB} encodes the particular solution (e.g. Kerr).

Closely resembles the momentum space form involving amplitudes....

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where Q_{AB} encodes the particular solution (e.g. Kerr).

Closely resembles the momentum space form involving amplitudes....

In fact... [Guevara 2021] [Luna, **NM** & White, 2022]

$$\int_0^{\infty} d\omega\omega^s e^{-\frac{1}{2}\omega\langle\lambda|u|\mu]} \mathcal{A}_3^{(s)}(\lambda) \propto \frac{1}{(Q_{AB}Z^A Z^B)^{s+1}}$$

Because 3pt amplitudes are simple on twistor space, we have only simple poles — gives rise to a local DC on position space.

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Mini-Twistor space

The relationship between $\mathbb{M}_3$ and **miniwistor space** $\mathbb{MT}$ is defined by

$$\langle \mu\lambda \rangle = u = x^{AB}\lambda_A\lambda_B, \qquad Z^A = (u, \lambda_A).$$

This is clearly invariant under $\lambda \to t\lambda$ and $\mu \to t\mu$ — we identify

$$(u, \lambda_A) \sim (t^2 u, t\lambda_A)$$

We can instead consider twistors of the form $Z^A = (\mu^A, \lambda_A)$, provided we make the identification [Tsai, 1996]

$$(\mu^A, \lambda_A) \sim (t\mu^A + tb\lambda^A, t\lambda_A).$$

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The Penrose transform is given by

$$\varphi_{A_1 A_2 \cdots A_{2s}}(x) = \frac{1}{2\pi i} \oint \langle \lambda \mathrm{d}\lambda \rangle \, \lambda_{A_1} \lambda_{A_2} \cdots \lambda_{A_{2s}} \; \rho_x \left[\check{f}(Z^A) \right],$$

however we now require that

$$\check{f}(Z^A) = e^{-m \frac{\langle a\mu \rangle}{\langle a\lambda \rangle}} g(u, \lambda_A), \qquad g(t^2 u, t\lambda_A) = t^{-2s-2} g(u, \lambda_A).$$

For f as above, these fields satisfy [Tsai, 1996]

$$\partial^B_{A_1} \varphi_{B A_2 \cdots A_{2s}}(x) = m \varphi_{A_1 A_2 \cdots A_{2s}}(x).$$

For $s = 1$ ($s = 2$), this is the spinor EOM for topologically massive EM (gravity).

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Twistor-Cotton Double Copy

We see then that the Cotton DC can be expressed as [Carrillo-Gonzalez, Emond, **NM**, Rumbutis & White]

$$\check{f}^{(-6)}_{grav} = \frac{\check{f}^{(-4)}_{EM}\,\tilde{\check{f}}^{(-4)}_{EM}}{\check{f}^{(-2)}_{scal.}} = e^{-m\frac{\langle a\mu\rangle}{\langle a\lambda\rangle}}\,\frac{g^{(-4)}_{EM}\,\tilde{g}^{(-4)}_{EM}}{g^{(-2)}_{scal.}} = e^{-m\frac{\langle a\mu\rangle}{\langle a\lambda\rangle}}\,g^{(-6)}_{grav}\,.$$

From the amplitude representation we find

$$\varphi_{A_1\cdots A_{2s}} \sim \oint \langle\lambda \mathrm{d}\lambda\rangle\, \lambda_{A_1}\cdots\lambda_{A_{2s}}\, \frac{e^{-m\frac{\langle\mu|u|\lambda\rangle}{\langle\lambda|u|\lambda\rangle}}}{\langle\lambda|u|\lambda\rangle^{s+1}}$$

This has an essential singularity, required to give the transcendental solutions to the EOM (Bessel functions).

No simple Cotton DC beyond type-N in position space!

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What's been done?

- (Top. Massive Gauge)^2 $\sim$ Top. Massive Gravity [NM, 2020] [González, Momeni & Rumbutis, 2021] [Hang, He & Shen, 2021] [NM, 2021]
- BCJ Duality up to 5pt [González, Momeni & Rumbutis, 2021]
- Non-Perturbative [NM, 2021]
- Matter (Anyons) [Burger, Emond & NM, 2021] , Eikonal [González, Momeni & Rumbutis, 2021]
- Aharonov-Bohm [Burger, Emond & NM, 2021] [Emond, NM & Wei, 2021]
- Cotton DC [Emond & NM, 2022] [González, Momeni & Rumbutis, 2022] inc AdS/Shockwaves [González, Momeni & Rumbutis, 2022]

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Discussion

Plenty left to do:

- Stringy origin? Likely arises from Green-Schwarz mechanism, $m = f(\alpha')$ – See talk by **Markou**
- Kinematic algebra – See talks by **Ben-Shahar, Chen**
- Supersymmetry easily added to topologically massive theories [Agarwal, Lipstein & Young, 2014] — Double copy? See talk by **Engelbrecht**
- Hints that there should be a DC of the form NLSM $\otimes$ YM $\sim$ top. massive Born-Infeld [NM, 2021]
- Hints that this generalises to curved space [González, Momeni & Rumbutis, 2022]
- Lagrangian double copy? Double field theory? See talk by **Diaz-Jaramillo**

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Thank you for listening.

Any Questions?

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Position Space Example

In Kerr-Schild coordinates, the plane-wave metric is

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa\delta(\ell \cdot x)H(x^{\mu})\ell_{\mu}\ell_{\nu},$$

where $\ell^2 = 0$, $\ell_{\mu} = \partial_{\mu} u$ and u is a retarded coordinate.

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In vacuum, the EOM become

$$G_{\mu\nu} + \frac{1}{m}C_{\mu\nu} = -\frac{\kappa}{2}\delta(\ell \cdot x)\ell_\mu\ell_\nu\left[\frac{1}{m}H'''(x) + H''(x)\right] = 0.$$

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$$G_{\mu\nu} + \frac{1}{m}C_{\mu\nu} = -\frac{\kappa}{2}\delta(\ell \cdot x)\ell_\mu\ell_\nu \left[\frac{1}{m}H'''(x) + H''(x)\right] = 0.$$

Assuming $x > 0$, we find

$$H(x) = \kappa e^{-m|x|} + c_1|x| + c_2.$$

Note: delta function sets $|x| = x$ or $|x| = y$, depending on choice of u.
We drop c_1 and c_2 terms.

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On the gauge theory side, we need to solve

$$(\partial^2 - m^2)\tilde{F}^\mu = 0.$$

Solved by $\tilde{F}^\mu = e\delta(\ell \cdot x)e^{-m|x|}\ell^\mu$.

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Solved by $\tilde{F}^\mu = e\delta(\ell \cdot x)e^{-m|x|}\ell^\mu$.

Kerr-Schild double copy for topologically massive plane waves

$$h_{\mu\nu}(x) = \kappa\delta(\ell \cdot x)e^{-m\sqrt{x^2}}\ell_\mu\ell_\nu, \quad A_\mu(x) = e\delta(\ell \cdot x)e^{-m\sqrt{x^2}}\ell_\mu,$$
$$\phi(x) = \lambda\delta(\ell \cdot x)e^{-m\sqrt{x^2}}.$$

Let's examine the spinor form.

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The Maxwell spinor is given simply by

$$\Phi_{AB} = -2m\delta(\langle\ell|x|\ell\rangle)e^{-m\sqrt{x^2}}\,|\ell\rangle_A\,|\ell\rangle_B\,.$$

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The Maxwell spinor is given simply by

$$\Phi_{AB} = -2m\delta(\langle\ell|x|\ell\rangle)e^{-m\sqrt{x^2}}\,|\ell\rangle_A\,|\ell\rangle_B\,.$$

and the Cotton spinor by

$$\begin{aligned}\Psi_{ABCD} &= -\frac{\kappa}{2}\delta(\ell\cdot x)\varepsilon^{\lambda\sigma}{}_{\mu}\sigma^{\mu}_{AB}\partial_\lambda\partial^2 H(x)\ell_\sigma\ell_{CD}\\ &= \kappa^2 m^3\delta(\langle\ell|x|\ell\rangle)e^{-m\sqrt{x^2}}\,|\ell\rangle_A\,|\ell\rangle_B\,|\ell\rangle_C\,|\ell\rangle_D\,,\end{aligned}$$

using $\partial_\lambda H(x) = -m\hat{x}_\lambda H(x)$.

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The Maxwell spinor is given simply by

$$\Phi_{AB} = -2m\delta(\langle\ell|x|\ell\rangle)e^{-m\sqrt{x^2}}\,|\ell\rangle_A\,|\ell\rangle_B\,.$$

and the Cotton spinor by

$$\begin{aligned}\Psi_{ABCD} &= -\frac{\kappa}{2}\delta(\ell\cdot x)\varepsilon^{\lambda\sigma}{}_{\mu}\sigma^{\mu}_{AB}\partial_\lambda\partial^2 H(x)\ell_\sigma\ell_{CD}\\ &= \kappa^2 m^3\delta(\langle\ell|x|\ell\rangle)e^{-m\sqrt{x^2}}\,|\ell\rangle_A\,|\ell\rangle_B\,|\ell\rangle_C\,|\ell\rangle_D\,,\end{aligned}$$

using $\partial_\lambda H(x) = -m\hat{x}_\lambda H(x)$.

The type-N Cotton double copy holds in position space

$$\Psi_{ABCD}(x) = \frac{m}{2}\frac{\Phi_{AB}(x)\Phi_{CD}(x)}{\phi(x)}.$$

Type-D more complicated... Let's turn to twistor space.

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Aharonov-Bohm Effect

Famously, anyons give rise to a non-zero AB phase. Defined as

$$\alpha = ie \oint \mathbf{A} \cdot d\mathbf{x} = e \int \mathbf{B} \cdot \hat{n} \, \mathrm{d}Area,$$

where **A** is the vector potential. Can we derive this on-shell?

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In the Born approximation, we can derive the relation

$$\alpha = \frac{ie}{4m_2} \oint d\mathbf{r} \int \frac{d^{D-1}q}{(2\pi)^{D-1}} e^{i\mathbf{q}\cdot\mathbf{r}} \frac{\partial \mathcal{A}}{\partial \mathbf{p}_1}\bigg|_{\mathbf{p}_1=0}$$

In 3D

$$\alpha_{D=3} = \frac{ie}{4m_2} \int_0^{2\pi} \int_0^{R} dr d\theta \epsilon^{ij} \partial_i \int \frac{d^2q}{(2\pi)^2} e^{i\mathbf{q}\cdot\mathbf{r}} \frac{\partial \mathcal{A}}{\partial p_1^j}\bigg|_{\mathbf{p}_1=0}$$

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Plugging the $\mathcal{A}$ at large r with $e \to \infty$ and $m \to \infty$ then gives

$$\alpha_{D=3} = \frac{e^2}{m}.$$

This agrees with the literature [Deser & McCarthy, 1990, Ortiz, 1990] .

Gravitational anyons also have a phase, given by taking $e \to \frac{1}{2}\kappa m_2$

$$\alpha^{Gravity}_{D=3} = \frac{\kappa^2 m_2^2}{4m} = 8\pi G \frac{m_2^2}{m}$$

Aharonov-Bohm phase also double copies!

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Twistor-Weyl Double Copy

For a spinless source $\sim \delta(u \cdot k)$, $Q_{AB} = \begin{pmatrix} 0 & u^A{}_{B'} \\ u^B{}_{A'} & 0 \end{pmatrix}$.

Spinning (dual) sources via Janis-Newman (duality) shift

$$\mathcal{A}_3^{(s)} \to \mathcal{A}_3^{(s)} e^{s(ik \cdot a + \theta)}$$

We find the so-called *kinematic twistor* [Penrose & MacCallum, 1972] [Luna, **NM** & White, 2022]

$$\tilde{Q}_{AB} = e^{-\theta} \begin{pmatrix} 0 & u^A{}_{B'} \\ u^B{}_{A'} & -u^{(A}{}_{A'} a^{B)A'} \end{pmatrix}$$

Spinning-Dyon for $s = 1$ and Kerr-Taub-NUT for $s = 2$.

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