Double Field Theory as the Double Copy of Yang-Mills Theory

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arXiv:2109.01153 [FDJ, O. Hohm, J. Plefka]
arXiv:2203.07397 [R. Bonezzi, FDJ, O. Hohm]
arXiv:2212.XXXX [R. Bonezzi, C. Chiaffrino, FDJ, O. Hohm]

QCD meets gravity

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Outline

Main goal: **Develop an off-shell, gauge invariant and local approach to double copy based on homotopy algebras that:**

- **allows to identify a "kinematic algebra"**
- **gives gravity in the form of double field theory**

up to and including quartic interactions.

The talk will be divided into:

- Framework: double copy, algebras and Yang-Mills.
- Algebraic double copy and double field theory.
- 4-point amplitudes.

Double copy (DC)

- BCJ DC: Color $\rightarrow$ Kinematics (YM $\rightarrow$ Gravity+B-field+Dilaton)

  [Bern, Carrasco, Johansson 2008]

- Loops? Kinematic algebra? General classical solutions?

$\longrightarrow$ Investigate the double copy at the level of gauge invariant action and gauge symmetries (including non-linear).

Otherwise stated: construct gravity using Yang-Mills building blocks.

[Bern, Dennen, Huang, Kiermaier 2010; Anastasiou, Borsten, Duff, Nagy, Zoccali 2020; Borsten, Jurco, Kim, Macrelli, Saemann, Wolf 2020,2021,2022;...]

L_∞-algebras and field theory

$$S_{\text{YM}} = \int A_a^\mu \Box A_\mu^a - \varphi_a \varphi^a + \varphi_a\, \partial^\mu A_\mu^a + \partial A^3 + A^4$$

An L_∞-algebra is a graded vector space $X_{\text{YM}} = \bigoplus_i X_{\text{YM}}^i$ equipped with a differential Q such that

$$\begin{array}{ccccccc} X_{\text{YM}}^1 & \xrightarrow{Q} & X_{\text{YM}}^0 & \xrightarrow{Q} & X_{\text{YM}}^{-1} & \xrightarrow{Q} & X_{\text{YM}}^{-2} \\ \lambda^a & & A_\mu^a & & E^a & & \\ & & \varphi^a & & E_\mu^a & & \mathcal{N}^a \end{array}$$

and multilinear maps $B_n : X_{\text{YM}}^{\otimes n} \to X_{\text{YM}}$ $(\mathcal{A} = (A_\mu^a, \varphi^a))$

- $S_{\text{YM}} = \frac{1}{2!}\langle \mathcal{A}, Q\mathcal{A}\rangle + \frac{1}{3!}\langle \mathcal{A}, B_2(\mathcal{A},\mathcal{A})\rangle + \frac{1}{4!}\langle \mathcal{A}, B_3(\mathcal{A},\mathcal{A},\mathcal{A})\rangle$
- $\delta_\lambda \mathcal{A} = Q\lambda + B_2(\mathcal{A}, \lambda)$ (gauge transformations)

that obey a set of relations (generalized Jacobi identities) encoding the consistency of the theory.

L_∞-algebras and field theory

Example: action of the differential

$$Q\mathcal{A} = \begin{pmatrix} \partial \cdot A^a - \varphi^a \\ \Box A^a_\mu - \partial_\mu \varphi^a \end{pmatrix} \in X^{\mathrm{YM}}_{-1}$$

$$Q\lambda = \begin{pmatrix} \partial_\mu \lambda^a \\ \Box \lambda^a \end{pmatrix} \in X^{\mathrm{YM}}_{0}$$

Stripping off color

There is a manifest factorization: $X_{\mathrm{YM}} = \mathcal{K} \otimes \mathfrak{g}$ [Zeitlin 2008]

$\mathcal{K}$ is a C_∞-algebra and $\mathfrak{g}$ is a Lie algebra.

$$S_{\mathrm{YM}} = \frac{1}{2!}\langle \mathcal{A}^a, Q\mathcal{A}_a\rangle + \frac{1}{3!} f_{abc} \langle \mathcal{A}^a, m_2(\mathcal{A}^b, \mathcal{A}^c)\rangle + \frac{1}{4!} f^e{}_{ab} f_{ecd} \langle \mathcal{A}^a, m_3(\mathcal{A}^b, \mathcal{A}^c, \mathcal{A}^d)\rangle$$

e.g: $m_{2\,\mu}(\mathcal{A}_1, \mathcal{A}_2) = 2\, A_1 \cdot \partial A_{2\,\mu} + \partial_\mu A_1 \cdot A_2 + \partial \cdot A_1\, A_{2\,\mu} - (1 \leftrightarrow 2)$

There exists a map b such that $(\mathcal{K}, b, Q, \Box, m_2, m_3)$ forms an off-shell and gauge independent *kinematic algebra* up to tri-linear maps. ($BV_\infty^\Box$-algebra [Reiterer 2019])

$$b^2 = 0\,, \quad Qb + bQ = \Box,$$

$$b\mathcal{A} = 0 \text{ (gauge fixing)} \quad h = \frac{b}{\Box} \text{ (propagator)}$$

Action of b

b acts on X_{YM} as

$$b\mathcal{A} = \begin{pmatrix} \varphi^a \\ 0 \end{pmatrix} \in X^1_{\mathrm{YM}} \quad b\mathcal{E} = \begin{pmatrix} E^a_\mu \\ 0 \end{pmatrix} \in X^0_{\mathrm{YM}}$$

$$X^1_{\mathrm{YM}} \xrightarrow{Q} X^0_{\mathrm{YM}} \xrightarrow{Q} X^{-1}_{\mathrm{YM}} \xrightarrow{Q} X^{-2}_{\mathrm{YM}}$$

$$\begin{array}{ccccccc} \lambda^a & & A^a_\mu & & E^a & & \\ & \overset{b}{\nwarrow} & & \overset{b}{\nwarrow} & & \overset{b}{\nwarrow} & \\ & & \varphi^a & & E^a_\mu & & \mathcal{N}^a \end{array}$$

Kinematic algebra

Warm-up: Poisson algebra $\{F, G \cdot H\} - \{F, G\} \cdot H - G \cdot \{F, H\} = 0$

The bracket $\{\,,\,\}$ obeys Jacobi and the product $\cdot$ is associative.

One can generalize this to get a kinematic algebra $(\mathcal{K}, b, ...)$

$$
\begin{aligned}
\cdot &\to (m_2, m_3) \\
\{\,,\,\} &\to b_2(u_1, u_2) := b\, m_2(u_1, u_2) - m_2(bu_1, u_2) - m_2(u_1, bu_2)
\end{aligned}
$$

The compatibility relation generalizes to

$$
\begin{aligned}
b_2(u_1, m_2(u_2, u_3)) - m_2(b_2(u_1, u_2), u_3) - m_2(u_2, b_2(u_1, u_3)) \\
+ [Q, \theta_3](u_1, u_2, u_3) + d_{\Box} m_3(u_1, u_2, u_3) = 0
\end{aligned}
$$

A similar algebraic structure was found for Chern-Simons.

[Ben-Shahar, Johansson 2021]

Algebraic double copy

Following BCJ

$$X_{\text{YM}} = \mathcal{K} \otimes \mathfrak{g} \to X_{\text{DC}} = \mathcal{K} \otimes \bar{\mathcal{K}}$$
$$X_{\text{DFT}} = \left\{ \Psi \in \mathcal{K} \otimes \bar{\mathcal{K}} \mid b^- \Psi = 0 \right\}, \quad b^- = b - \bar{b}$$

For the maps to quartic order one has

$$B_1 = Q + \bar{Q}, \quad B_2 = b^- m_2 \otimes \bar{m}_2 = b_2 \otimes \bar{m}_2 - m_2 \otimes \bar{b}_2$$
$$B_3 = b^- \left\{ \theta_3 \otimes \bar{m}_2 \bar{m}_2 + m_2 b_2 \otimes \bar{m}_3 + d_\square m_3 \otimes \bar{m}_3 + \text{(un)-barred} \right\}$$

These maps form an L_∞-algebra encoding gauge invariant double field theory (DFT) to quartic order!

$$S_{\text{DFT}} = \frac{1}{2!} \big\langle \Psi, B_1(\Psi) \big\rangle + \frac{1}{3!} \big\langle \Psi, B_2(\Psi, \Psi) \big\rangle + \frac{1}{4!} \big\langle \Psi, B_3(\Psi, \Psi, \Psi) \big\rangle + \ldots$$

With $\Psi = (e_{\mu\bar{\nu}}, d), \quad e_{\mu\bar{\nu}}(x, \bar{x}) = A_\mu(x) \otimes \bar{A}_{\bar{\nu}}(\bar{x})$

Amplitudes and algebras

First Yang-Mills: $Q\mathcal{A} = 0,\ b\mathcal{A} = 0$. Equivalently $p_i^2 = 0,\ p_i \cdot \epsilon_i(p_i) = 0$ and the propagator is $h = \frac{b}{\Box}$

A 4-point amplitude can be written as

$$\begin{aligned}\mathcal{A}^{(4)}_{\text{Tree}} =& \Big\langle \epsilon_4, \{m_2(h\, m_2(\epsilon_1,\epsilon_2),\epsilon_3) + m_{3h}(\epsilon_1,\epsilon_2|\epsilon_3)\} \Big\rangle \operatorname{Tr}\big(t_4[[t_1,t_2],t_3]\big) + \ldots \\ =& \frac{n_s c_s}{s} + \frac{n_t c_t}{t} + \frac{n_u c_u}{u}\end{aligned}$$

with numerators $n_s = \langle \epsilon_4, \mathfrak{n}_s\rangle,\ \mathfrak{n}_s^\mu = m_2(b_2(\epsilon_1,\epsilon_2),\epsilon_3) + s\, m_{3h}(\epsilon_1,\epsilon_2,\epsilon_3)$

Recall that

$$b_2(\epsilon_1,\epsilon_2) = bm_2(\epsilon_1,\epsilon_2) - m_2(b\epsilon_1,\epsilon_2) - m_2(\epsilon_1, b\epsilon_2) = bm_2(\epsilon_1,\epsilon_2)$$

and $d_\Box m_3(\epsilon_1,\epsilon_2,\epsilon_3) = s\, m_3(\epsilon_1,\epsilon_2,\epsilon_3)$

4-point DFT amplitude

Similarly, for tensors in DFT $\varepsilon_{i\,\mu\bar{\nu}}(p_i,\bar{p}_i) = \epsilon_{i\,\mu}(p_i) \otimes \bar{\epsilon}_{i\,\bar{\nu}}(\bar{p}_i)$

$$\mathcal{M}^{(4)}_{\text{Tree}} = \Big\langle \varepsilon_4, B_2(\mathfrak{h}B_2(\varepsilon_1,\varepsilon_2),\varepsilon_3) + B_3(\varepsilon_1,\varepsilon_2,\varepsilon_3)\Big\rangle + \ldots$$
$$= \frac{n_s\bar{n}_s}{s} + \frac{n_t\bar{n}_t}{t} + \frac{n_u\bar{n}_u}{u}$$

The consistency of this amplitude relies on the Poisson relation

$$[b, m_2 m_2] - 3m_2 b_2 \pi - [Q,\theta_3] - m_{3h}(d_\square - 3\,d_s\,\pi) = 0$$

Taking the inner product of this relation with a polarization vector one obtains

$$n_s + n_t + n_u = 0$$

Summary and Outlook

Take-home messages:

- We have developed an algebraic approach to DC that is local, off-shell and gauge invariant.
- A new perspective on $n_s + n_t + n_u = 0$.

For the future

- Higher orders.
- Implementation to classical solutions.
- Applications to gravitational waves.

Thank you very much for your attention!