Finite-temperature higher-dimensional operators affect GW predictions

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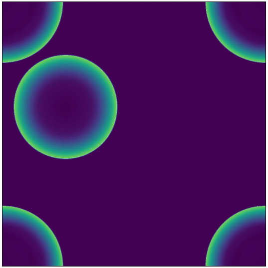
Based on collaboration of F. Bernardo, P. Klose, P. Schicho, and T. V. I. Tenkanen, *Higher-dimensional operators at finite-temperature affect gravitational-wave predictions*, [2503.18904], A. Ekstedt, P. Schicho, and T. V. I. Tenkanen, *Cosmological phase transitions at three loops: The final verdict on perturbation theory*, Phys. Rev. D **110** (2024) 096006 [2405.18349]. Supported by the SNSF under grant PZ00P2-215997.

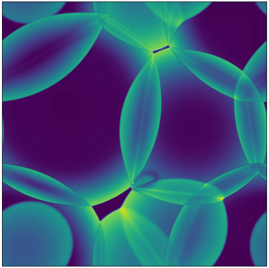
The thermal history of electroweak symmetry breaking

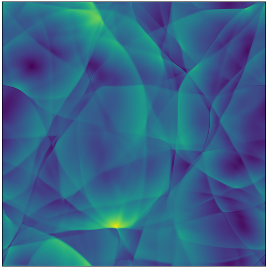
Strong first-order cosmic phase transition at EW scale $T_c \sim 100$ GeV:

- ▷ Baryogenesis — Baryon asymmetry of the universe
- ▷ Colliding bubbles — Gravitational wave (GW) production

In Standard Model, EWSB occurs via a smooth crossover but possible that it is first-order in Beyond the Standard Model (BSM) extensions.







figures by D. Cutting, M. Hindmarsh, and D. J. Weir, *Vorticity, kinetic energy, and suppressed gravitational wave production in strong first order phase transitions*, Phys. Rev. Lett. **125** (2020) 021302 [1906.00480]

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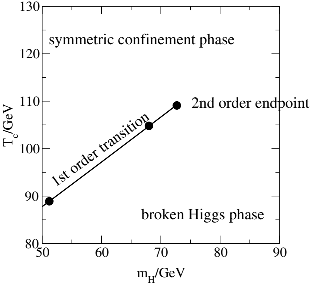

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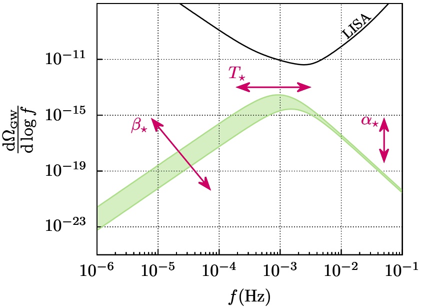
figure by M. Laine, *Electroweak phase transition beyond the standard model*, in *4th International Conference on Strong and Electroweak Matter*, pp. 58–69, 6, 2000 [hep-ph/0010275]

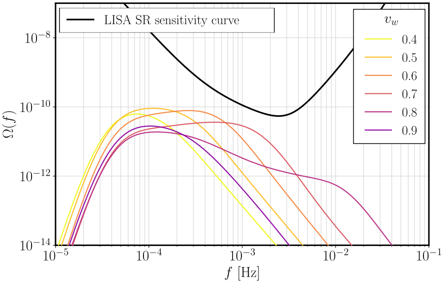
The effective potential V_{eff} in perturbation theory

encodes equilibrium thermodynamics as function of BSM parameters.

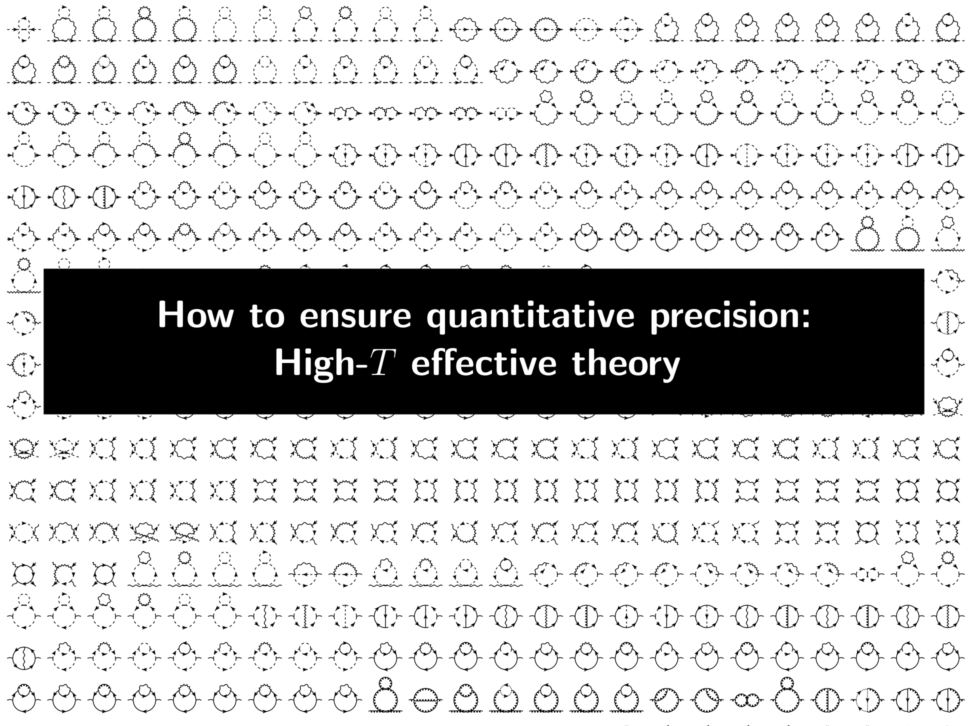
Origin of uncertainty.

- $T_\star$ reference temperature of the transition ($T_\star = T_{\rm n}, T_{\rm p}$),
- α phase transition strength,
- β/H inverse duration of the transition,
- v_w Iteratively solve coupled fluid, scalar field, Boltzmann equations[1]





[1] Numerical package for v_w: Ekstedt, Gould, Hirvonen, Laurent, Niemi, Schicho, van de Vis: [2411.04970],
C. Gowling and M. Hindmarsh, *Observational prospects for phase transitions at LISA: Fisher matrix analysis*, JCAP **10** (2021) 039 [2106.05984]



How to ensure quantitative precision: High-T effective theory

Perturbative phase transitions need scale hierarchies

for quantum effects ΔV_{fluct} to influence the tree-level potential

$$V_{\text{eff}} = V_{\text{tree}} + \Delta V_{\text{fluct}} \ .$$

Assume particle χ couples to the SM via $g^2 \Phi^\dagger \Phi \chi^\dagger \chi$. If $M_\chi \gg m_\Phi$, integrating out χ introduces Higgs-mass corrections of the form:

$$(\Delta m_\Phi^2)\Phi^\dagger\Phi =$$

$$\sim g^2 M_\chi^2 \Phi^\dagger \Phi \,, \qquad \frac{(\Delta m_\Phi^2)}{m_\Phi^2} = g^2 \left[\frac{M_\chi}{m_\Phi}\right]^2 .$$

Relevant operators ($\sigma > 0$) in the IR get large UV contributions and

$$\frac{\Delta V_{\text{fluct}}}{V_{\text{tree}}} \sim g^2 \left[\frac{\Lambda_{\text{fluct}}}{\Lambda_{\text{tree}}}\right]^\sigma \overset{!}{\sim} 1 \quad \Rightarrow \begin{cases} \text{strong coupling} & g^2 \gtrsim 1 \\ \text{scale hierarchy} & \frac{\Lambda_{\text{fluct}}}{\Lambda_{\text{tree}}} \sim \left[\frac{1}{g^2}\right]^{\frac{1}{\sigma}} \gg 1 \end{cases}$$

Multi-scale hierarchy in hot classicalizing gauge theories

Evaluated **Matsubara sums** yield Bose(Fermi) distribution. Asymptotically high T and weak $g \ll 1$: effective expansion parameter

$$\epsilon_{\mathrm{B}} = g^2 n_{\mathrm{B}}(E) = \frac{g^2}{e^{E/T} - 1} \approx \frac{g^2 T}{E} \, .$$

Differs from weak coupling g^2. Fermions are IR-safe $g^2 n_{\mathrm{F}}(E) \sim g^2/2$.

$$E \sim \begin{cases} \pi T & \textit{hard}\text{ scale} \\ gT & \textit{soft}\text{ scale} \\ g^{3/2} T & \textit{supersoft}\text{ scale} \\ g^2 T/\pi & \textit{ultrasoft}\text{ scale} \end{cases}$$

quantum theory

symmetry breaking

Limit: Confinement-like behavior in ultrasoft sector $g^2 n_{\mathrm{B}}(g^2 T) \sim \mathcal{O}(1)$. Ultrasoft bosons are non-perturbative at finite T: Linde IR problem.[2]

[2] A. Linde, *Infrared problem in the thermodynamics of the Yang-Mills gas*, Phys. Lett. B **96** (1980) 289, O. Gould and T. V. I. Tenkanen, *Perturbative effective field theory expansions for cosmological phase transitions*, JHEP **01** (2024) 048 [2309.01672]

Effective Field Theory (EFT): Dimensional Reduction (DR)

Integrate out hard modes perturbatively → EFT for static modes.

Precision thermodynamics of non-Abelian gauge theories as QCD and (EW) phase transition[3] using e.g. `DRalgo`.[4] Two `step` procedure:

$\mathcal{L}_{\text{Full}}$, $(d+1)$-dim

hard — πT

`step1`: Dimensional Reduction

m_{D}

$\mathcal{L}_d$, d-dim

soft — gT

`step2`: Integrate out $m_{B_0} \sim m_{\text{D}}$ and $m_{B_i} \sim g\phi$

m_{nucl}

supersoft — $g^{3/2}T$

$\overline{\mathcal{L}}_d$, d-dim, HEFT-like

[3] K. Kajantie, M. Laine, K. Rummukainen, and M. E. Shaposhnikov, *Generic rules for high temperature dimensional reduction and their application to the standard model,* Nucl. Phys. B **458** (1996) 90 [`hep-ph/9508379`]

[4] A. Ekstedt, P. Schicho, and T. V. I. Tenkanen, `DRalgo`*: A package for effective field theory approach for thermal phase transitions,* Comput. Phys. Commun. **288** (2023) 108725 [`2205.08815`]

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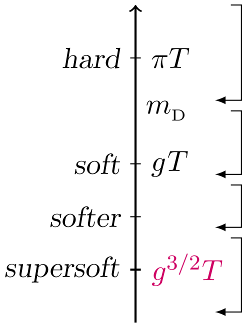

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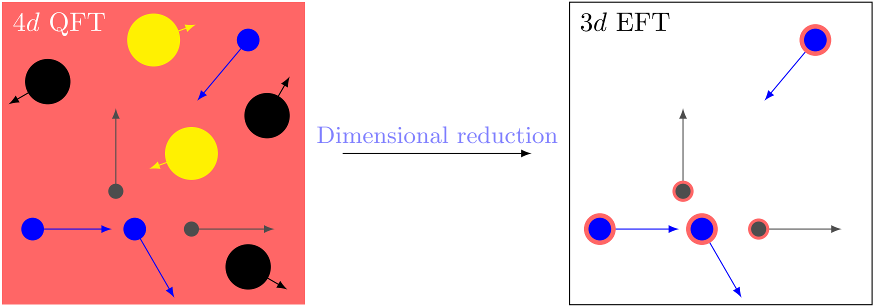

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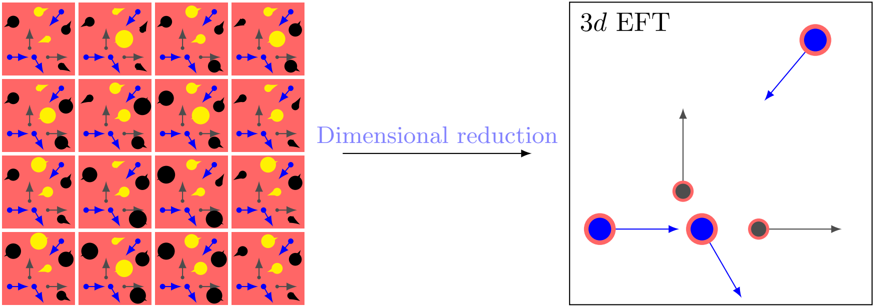

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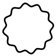
The SM-like $3d$ EFT $(\mathrm{U}(1)$+Higgs)

describes the thermodynamics[5] of several parent $4d$ theories:

$$\mathcal{L}_{\rm 3d}^{\rm soft} = \frac{1}{4}F_{ij}F_{ij} + (D_i\Phi)^\dagger(D_i\Phi) + (\partial_i B_0)^2 + V(\Phi)\;,$$
$$V(\Phi) = m_3^2\Phi^\dagger\Phi + m_{\rm D}^2 B_0 B_0 + \lambda_3(\Phi^\dagger\Phi)^2 + h_3(\Phi^\dagger\Phi)B_0B_0 + \dots\;.$$

If $m_{B_i} \sim g_3\phi \gg m_3$, integrating out vector boson introduces LO barrier

$$V_{\rm LO}(\Phi) = \quad \bullet \quad + $$  .

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$$V_{\rm LO}(\Phi) = m_3^2\Phi^\dagger\Phi + \lambda_3(\Phi^\dagger\Phi)^2 - \frac{g_3^3}{2\pi}\Big(\frac{\Phi^\dagger\Phi}{2}\Big)^{3/2} \; .$$

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$$V_{\text{LO}}(\Phi) \to y\Phi^\dagger\Phi + x(\Phi^\dagger\Phi)^2 - \frac{1}{2\pi}\Big(\frac{\Phi^\dagger\Phi}{2}\Big)^{3/2} \; .$$

Since $x \sim \frac{m_3^2}{m_{B_i}^2} \ll 1$, and at the phase transition $y \sim 1/x$, we strictly[6] x-pand the perturbative series using 3d EFT dimensionless couplings

$$x \equiv \frac{\lambda_3}{g_3^2} \, , \qquad\qquad y \equiv \frac{m_3^2}{g_3^4} \; .$$

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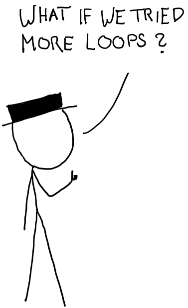
Limitations of GW predictions from cosmological phase transitions

Dimension-six operators in U(1) + Higgs[7]



So far truncated operators at high T at dimension 4:

$$S^{3\mathrm{d}}_{\mathrm{soft}} = \frac{1}{T}\int_{\mathbf{x}} \left\{ \mathcal{L}^{3\mathrm{d}}_{\mathrm{soft}} + \sum_{n\geq 5} \frac{\mathcal{O}_n}{(\pi T)^n} \right\},$$

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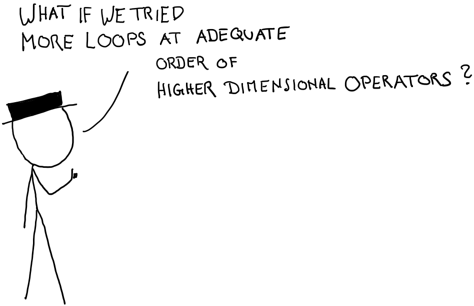

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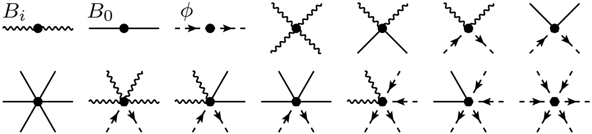
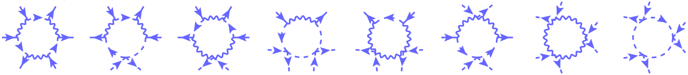
Vertex structures

$\mathcal{L}_{\text{soft}}$ is non-super-renormalizable.



Determine Wilson coefficients $\alpha_i(d)$ in d-dimensions:

- Evaluate (2–6)-point vertices at one-loop order



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Coefficients $\alpha_i(d)$ are gauge-parameter (ξ) independent order-by-order.[8]

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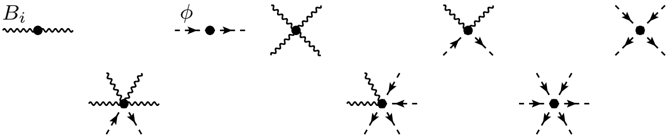

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Marginal operators at dimension six

Now focus on $c_6(\phi^\dagger\phi)^3$ effect.

The soft-scale marginal operator is suppressed at $\mathcal{O}(g^6)$

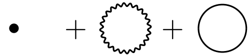
$$c_6 = \frac{\zeta_3}{32\pi^4}\Big(g^6 - \frac{31}{30}g^4\lambda + 5\,g^2\lambda^2 + \frac{20}{3}\lambda^3\Big) + \mathcal{O}(g^8)\,.$$

The softer-scale marginal operator is enhanced at $\mathcal{O}(g^3)$

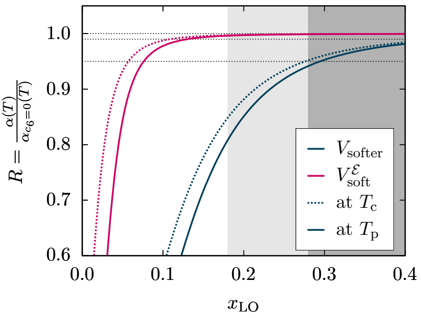
$$\bar{c}_6 = c_6 + \frac{\sqrt{3}\,g^3}{8\pi}\Big(\frac{m_{\mathrm{D}}^{\mathrm{LO}}}{m_{\mathrm{D}}}\Big)^3\big(1 - x_{\mathrm{LO}}\big) + \mathcal{O}(g^4)\,.$$

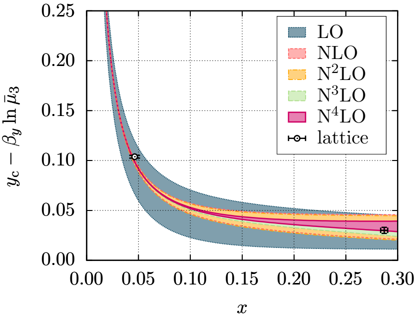
Validity of EFT

Leading-order effective potential given by

$$V^{\mathrm{LO}}_{\mathrm{soft}}(\Phi) =$$  .

Effect of c_6 shows broad window of high-T validity for soft EFT:[9]





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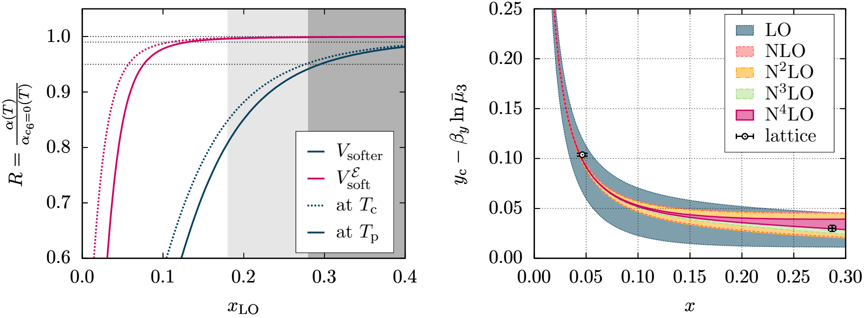

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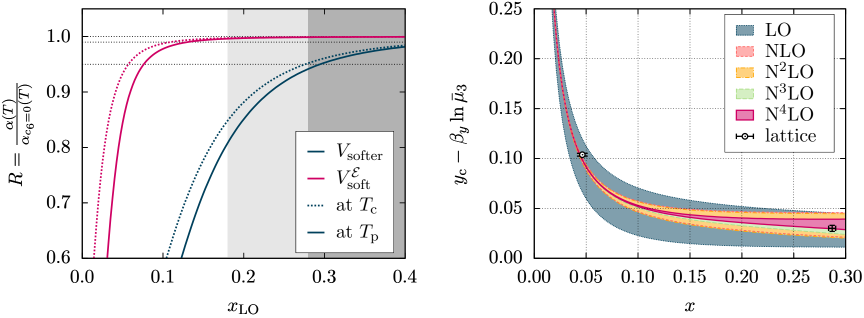

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Validity of EFT

Leading-order effective potential given by assuming $m_{\mathrm{D}}^2 \gg h_3 v_3^2$

$$V^{\mathrm{LO}}_{\mathrm{supersoft}}(\Phi) = \frac{1}{2}\bar{m}_3^2 v_3^2 + \frac{1}{4}\bar{\lambda}_3 v_3^4 + \frac{1}{8}\bar{c}_6 v_3^6 - \frac{1}{6\pi}\bar{g}_3^3 v_3^3 \,.$$

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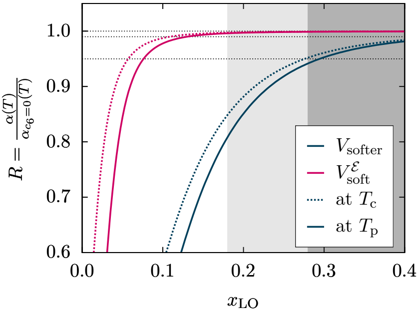

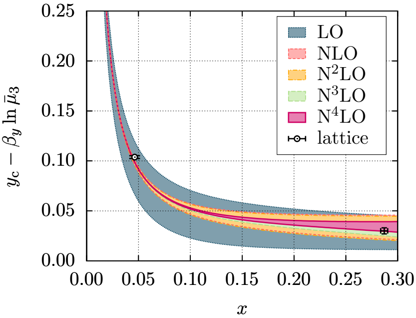

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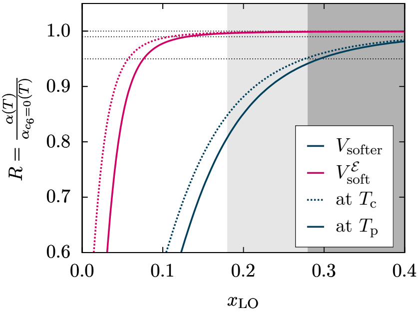
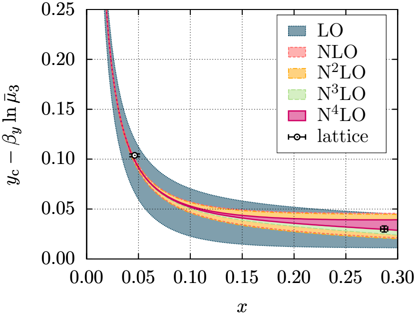
Validity of EFT

Leading-order effective potential given by assuming $m_{\mathrm{D}}^2 \ll h_3 v_3^2$

$$V^{\mathrm{LO}}_{\mathrm{supersoft}}(\Phi) = \frac{1}{2}m_3^2 v_3^2 + \frac{1}{4}\lambda_3 v_3^4 + \frac{1}{8}c_6 v_3^6 - \frac{v_3^3}{6\pi}\left(g_3^3 + \frac{1}{2}h_3^{\frac{3}{2}}\right).$$

Effect of c_6 shows broad window of high-T validity for soft EFT:[9]



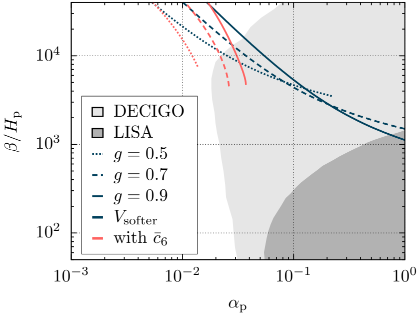


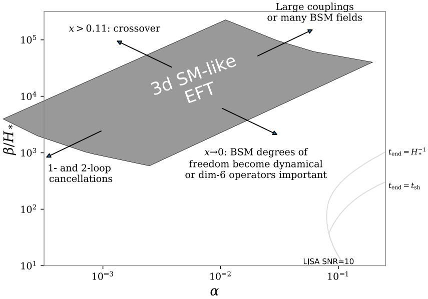
[9] F. Bernardo, P. Klose, P. Schicho, and T. V. I. Tenkanen, *Higher-dimensional operators at finite-temperature affect gravitational-wave predictions,* [2503.18904], A. Ekstedt, P. Schicho, and T. V. I. Tenkanen, *Cosmological phase transitions at three loops: The final verdict on perturbation theory,* Phys. Rev. D **110** (2024) 096006 [2405.18349]

Gravitational wave prospects

$\alpha/(\beta/H)$ rhombus of SM-like EFT with no prospect for large SNR.[10]

Soft scale enhances phase-transition strength, $\alpha(T)$.
High-T expansion is compromised for regime relevant for LISA:





Limitation: conventional lattice results of (dim-4) super-renormalizable 3d EFT do not describe hard/soft-scale driven transitions.[11]

[10] O. Gould, J. Kozaczuk, L. Niemi, M. J. Ramsey-Musolf, T. V. I. Tenkanen, and D. J. Weir, *Nonperturbative analysis of the gravitational waves from a first-order electroweak phase transition*, Phys. Rev. D **100** (2019) 115024 [1903.11604]

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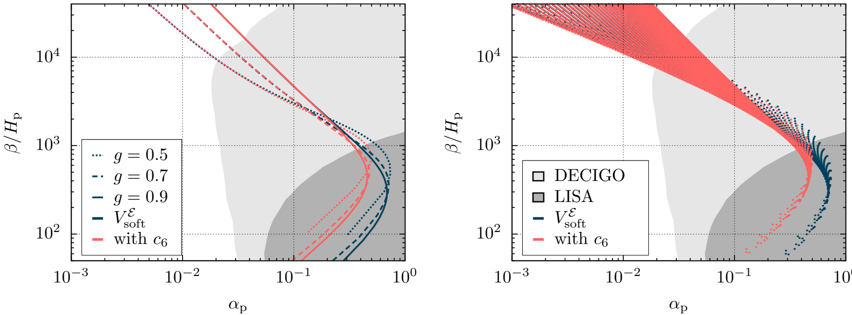

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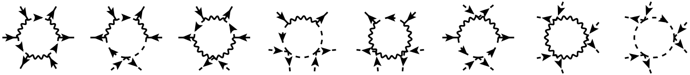
Conclusions

Precision thermodynamics of BSM theories:

- reliably describe cosmological FOPT and GW production,
- practical approach: **Effective Theories** + **universality**

Reaching perturbative limits and overcoming limitations:

- higher dimensional operators at $\mathcal{O}(g^6)$ in the $3d$ EFT,
- purely perturbative $\mathcal{O}(g^6)$ contributions from hard scale πT,



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Reaching perturbative limits and overcoming limitations:

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- purely perturbative $\mathcal{O}(g^6)$ contributions from hard scale πT,
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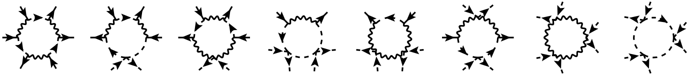
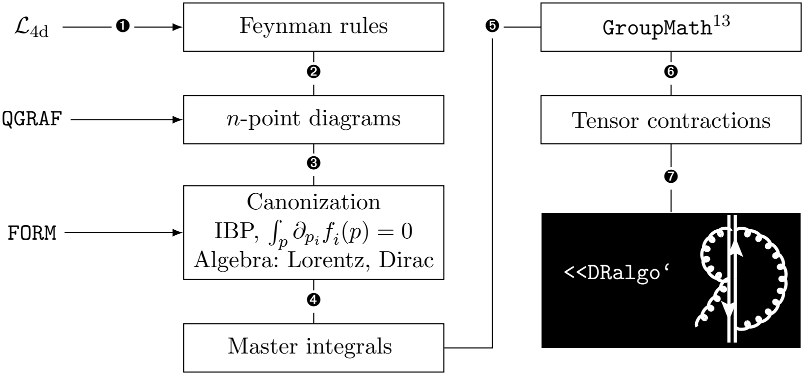


Backup

The Dimensional Reduction algorithm (DRalgo v1.3.0)

State-of-the-art dim-4 operator **Mathematica** package **DRalgo**.[12]



[12] github.com/DR-algo/DRalgo, A. Ekstedt, P. Schicho, and T. V. I. Tenkanen, DRalgo*: A package for effective field theory approach for thermal phase transitions,* Comput. Phys. Commun. **288** (2023) 108725 [2205.08815]

[13] R. M. Fonseca, *GroupMath: A Mathematica package for group theory calculations,* Comput. Phys. Commun. **267** (2021) 108085 [2011.01764]

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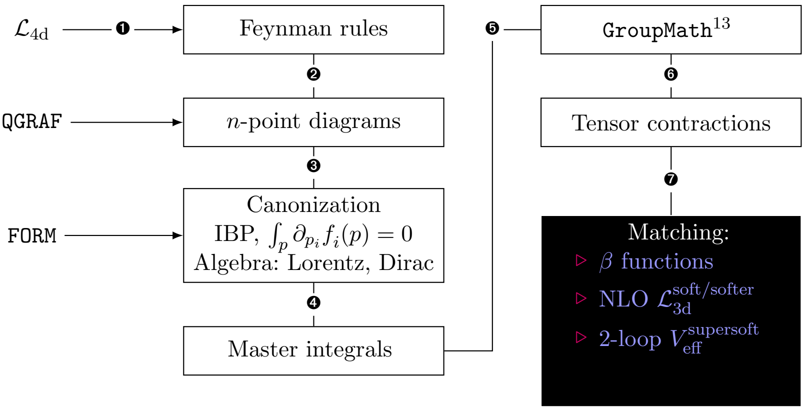

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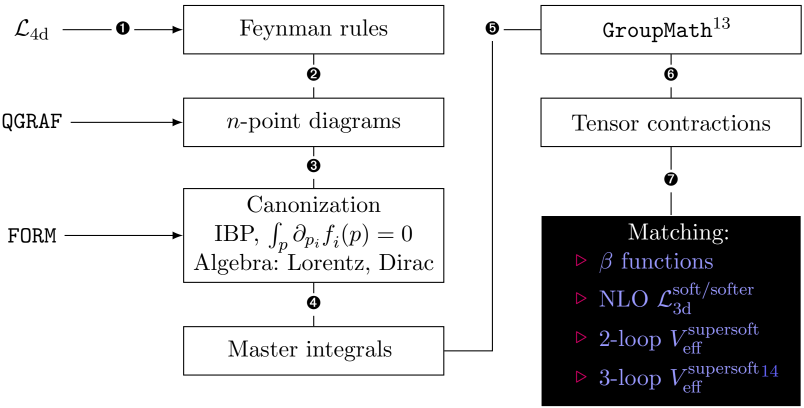

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Constructing the supersoft EFT

Two approaches to a *final* (nucleation) EFT

$$\mathcal{L}_{4\mathrm{d}} \to \mathcal{L}_{3\mathrm{d}}^{\mathrm{soft}} \to \mathcal{L}_{3\mathrm{d}}^{\mathrm{softer}} \to V_{\mathrm{eff}}^{\mathrm{supersoft}} \ ,$$

In the broken phase, utilize different hierarchies among Lorentz scalars, temporal scalars, and vectors:

$$m^2_{B_0}\big[\sim (gT)^2\big] \overset{\texttt{step2.1}}{\gg} m^2_{B_i}\big[\sim (g_3\phi)^2\big] \overset{\texttt{step2.2}}{\gg} m^2_3\big[\sim \lambda_3\phi^2\big]\ ,$$

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$$m_{B_0}^2\big[\sim (gT)^2\big] = m_{B_i}^2\big[\sim (g_3\phi)^2\big] \overset{\texttt{step2}}{\gg} m_3^2\big[\sim \lambda_3\phi^2\big] \,.$$

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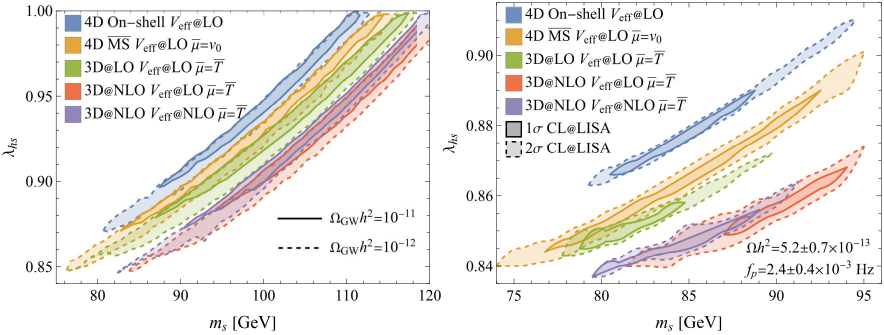
In the symmetric phase, the hierarchy is flipped

$$m^2_{B_0}\big[\sim (gT)^2\big] \gg m^2_3\big[\sim (g^{3/2}T)^2\big] \overset{\texttt{step2}}{\gg} m^2_{B_i}\big[\sim (g^2T)^2\big]\;.$$

❶ $\mathcal{L}_{\mathrm{3d}}$ (hard-to-soft matching)

Monitor Higgs (v) and **real singlet** (x) VEV after shift $s \to x + s$.
2-loop corrections move "Bananas": significant effect on GW signal.[15]

Parameters	m_s	λ_{hs}	$\lambda_s = 1$	$\alpha > 1$	$\mathrm{SNR}_{\mathrm{LISA}} > 10$



[15] M. Lewicki, M. Merchand, L. Sagunski, P. Schicho, and D. Schmitt, *Impact of theoretical uncertainties on model parameter reconstruction from GW signals sourced by cosmological phase transitions*, Phys. Rev. D **110** (2024) 023538 [2403.03769], L. Niemi, P. Schicho, and T. V. I. Tenkanen, *Singlet-assisted electroweak phase transition at two loops*, Phys. Rev. D **103** (2021) 115035 [2103.07467]

Does the banana move?

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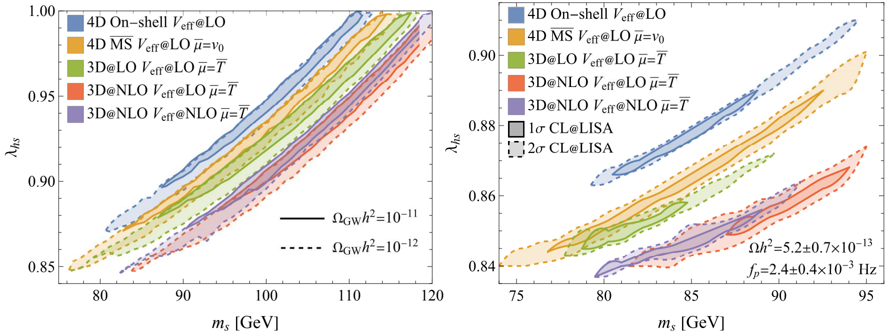

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Renormalization scale (in)dependence at finite T

At zero temperature

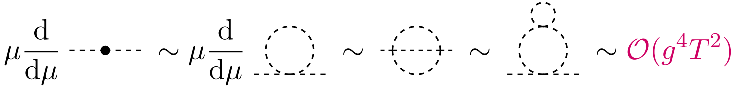
$$V_{\text{eff}}(\phi,\bar{\mu}) = \boxed{\underset{\mathcal{O}(g^2)}{V^{\text{tree}}_{\text{eff}}}} + \boxed{\underset{\mathcal{O}(g^4)}{V_{\text{CW},1\ell}}}\,, \quad \mu\frac{\mathrm{d}}{\mathrm{d}\mu}\Big(V^{\text{tree}}_{\text{eff}} + V_{\text{CW},1\ell}\Big) = 0\,.$$

At finite temperature[16]

$$V^{\text{res.}}_{\text{eff}}(\phi,T,\bar{\mu}) = \boxed{\underset{\mathcal{O}(g^2)}{V^{\text{tree}}_{\text{eff}}}} + \boxed{\underset{\mathcal{O}(g^3)}{V_{\text{res.,soft}}}} + \boxed{\underset{\mathcal{O}(g^2T^2)+\mathcal{O}(g^4)}{V_{\text{hard}}}}\,,$$

running of 1-loop thermal masses is of the same order as 2-loop thermal-mass logarithms.

Automatically included in dimensionally reduced 3d EFT:



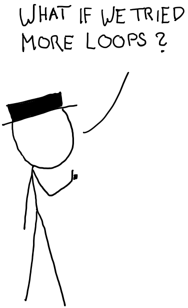
[16] O. Gould and T. V. I. Tenkanen, *On the perturbative expansion at high temperature and implications for cosmological phase transitions*, JHEP **06** (2021) 069 [2104.04399]

❷ $V_{\text{eff}}^{3\text{d}}$ (soft-to-supersoft matching)

Integrating out vector bosons in two steps up to 2-loops with `DRalgo`

$$m_{B_0}^2\left[\sim (gT)^2\right] \overset{\texttt{step2.1}}{\gg} m_{B_i}^2\left[\sim (g_3\phi)^2\right] \overset{\texttt{step2.2}}{\gg} m_3^2\left[\sim \lambda_3\phi^2\right] \, .$$

Focus on `step2.2` and add last perturbative orders N^3LO and N^4LO.[17]



[17] A. Ekstedt, O. Gould, and J. Löfgren, *Radiative first-order phase transitions to next-to-next-to-leading order,* Phys. Rev. D **106** (2022) 036012 [2205.07241], A. Ekstedt, P. Schicho, and T. V. I. Tenkanen, *Cosmological phase transitions at three loops: The final verdict on perturbation theory,* Phys. Rev. D **110** (2024) 096006 [2405.18349]

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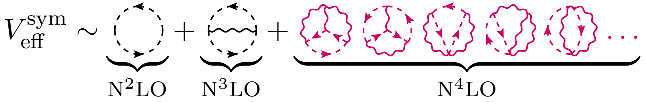

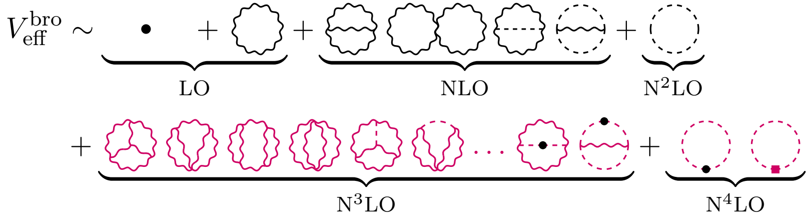

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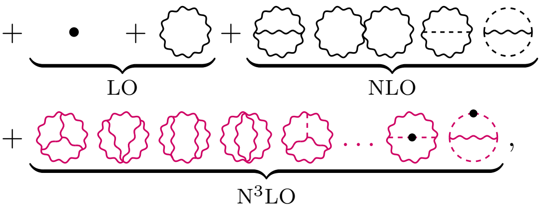
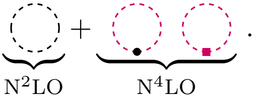
Focus on `step2.2` and add last perturbative orders N^{3}LO and N^{4}LO.[17]
In strict EFT expansion this organization can also be understood as:

$$S^{\text{tree}}_{\text{supersoft}} = \int_{\mathbf{x}} \frac{1}{2}(\partial_i s)^2 Z_s + \ldots$$



$$S^{\text{1-loop}}_{\text{supersoft}} = \ldots$$



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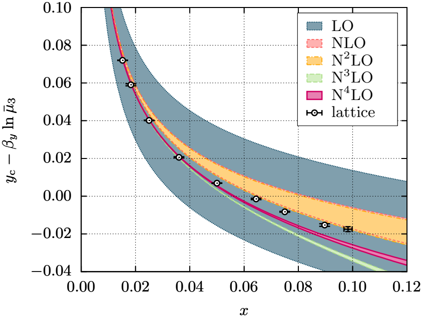
By using $F \sim V_{\text{eff}}(\phi_{\text{min}})$, determine the critical mass y_{c} (or T_{c})

$$\Delta F(y_{\text{c}}(x), x) = [F_{\text{bro}} - F_{\text{sym}}](y_{\text{c}}(x), x) = 0\,,$$

and the scalar *condensates*[18]

$$\Delta\langle\Phi^\dagger\Phi\rangle \equiv \frac{\partial}{\partial y}\Delta F\,, \qquad \Delta\langle(\Phi^\dagger\Phi)^2\rangle \equiv \frac{\partial}{\partial x}\Delta F\,.$$



[18] Lattice data: K. Kajantie, M. Laine, K. Rummukainen, and M. E. Shaposhnikov, *The Electroweak phase transition: A Nonperturbative analysis,* Nucl. Phys. B **466** (1996) 189 [hep-lat/9510020], O. Gould, S. Güyer, and K. Rummukainen, *First-order electroweak phase transitions: a nonperturbative update,* [2205.07238]

❸ Thermodynamics (for SU(2) + Higgs)

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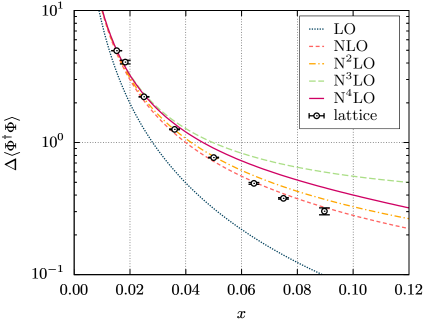

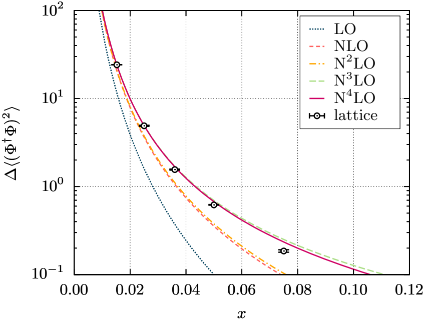

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❸ Thermodynamics (for U(1) + Higgs)

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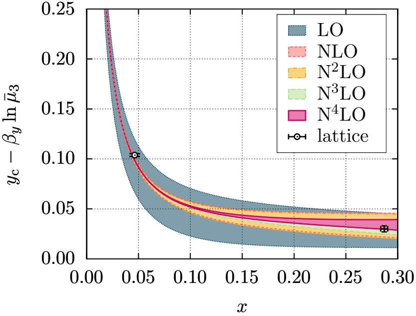


[18] Lattice data: K. Kajantie, M. Karjalainen, M. Laine, and J. Peisa, *Three-dimensional U(1) gauge + Higgs theory as an effective theory for finite temperature phase transitions,* Nucl. Phys. B **520** (1998) 345 [hep-lat/9711048], S. Mo, J. Hove, and A. Sudbo, *The Order of the metal to superconductor transition,* Phys. Rev. B **65** (2002) 104501 [cond-mat/0109260]

Predicting gravitational waves

Thermodynamics enters the GW spectrum through the strength and inverse duration of the transition, $h^2\Omega_{\mathrm{GW}}(f; H_\star, \alpha, \beta, v_w)$:

$$\alpha \sim \frac{\mathrm{d}\,\Delta F(y_{\mathrm{c}}, x)}{\mathrm{d}\ln T} = \left(\frac{\mathrm{d}\,y_{\mathrm{c}}}{\mathrm{d}\ln T}\right)\Delta\langle\Phi^\dagger\Phi\rangle + \left(\frac{\mathrm{d}\,x}{\mathrm{d}\ln T}\right)\Delta\langle(\Phi^\dagger\Phi)^2\rangle\,,$$

$$\frac{\beta}{H} = -\frac{\mathrm{d}\ln\Gamma}{\mathrm{d}\ln T}\,.$$

$\implies$ [UV] $\times$ [IR] factorization.[19]

Completed perturbative predictions for: α at $\mathrm{N^4LO}$, β/H at $\mathrm{N^2LO}$.

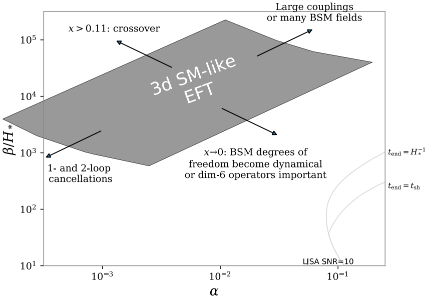
Limit: last perturbative order is $\mathrm{N^4LO}$.
Todo: final perturbative correction for thermal bubble nucleation rate.

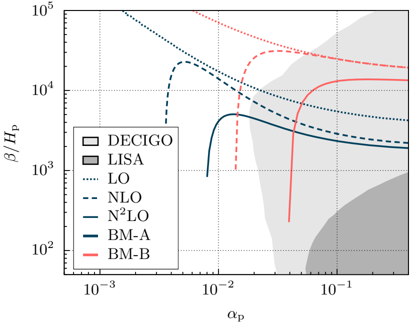
[19] O. Gould, J. Kozaczuk, L. Niemi, M. J. Ramsey-Musolf, T. V. I. Tenkanen, and D. J. Weir, *Nonperturbative analysis of the gravitational waves from a first-order electroweak phase transition*, Phys. Rev. D **100** (2019) 115024 [1903.11604]

Impact on gravitational waves

$\alpha/(\beta/H)$ rhombus of SM-like EFT with no prospect for large SNR.[20]
Better access interesting LISA SNR by increasing loop order:

BM-A xSM with weakly portal-coupled singlet (decoupled)
BM-B xSM with strongly portal-coupled singlet





[20] O. Gould, J. Kozaczuk, L. Niemi, M. J. Ramsey-Musolf, T. V. I. Tenkanen, and D. J. Weir, *Nonperturbative analysis of the gravitational waves from a first-order electroweak phase transition*, Phys. Rev. D **100** (2019) 115024 [1903.11604]