Muonic Atom Spectroscopy and the Proton Radius Puzzle

Julian J. Krauth

on behalf of the CREMA collaboration

Special IPP Seminar, ETH Zurich, 2017

CREMA Collaboration

(Charge Radius Experiment with Muonic Atoms)

Johannes Gutenberg-Universität Mainz, Germany
(before: Max-Planck-Institut für Quantenoptik, Garching, Germany)

- M. Diepold, B. Franke, J. Götzfried, T. W. Hänsch, J. J. Krauth, F. Mulhauser, T. Nebel, R. Pohl

Institut für Teilchenphysik, ETH Zürich, Switzerland

- A. Antognini, K. Kirch, F. Kottmann, B. Naar, K. Schuhmann, D. Taqqu

Paul Scherrer Institut, Switzerland

- A. J. Dax, M. Hildebrandt, A. Knecht

LKB, École Supérieure, CNRS, and Université P. et M. Curie, France

- F. Biraben, S. Galtier, P. Indelicato, L. Julien, F. Nez, C. Szabo-Foster

LIBPhys, Physics Department, Universidade de Coimbra, Portugal

- F. D. Amaro, J.M.R. Cardoso, L.M.P. Fernandes, A. L. Gouvea, J.A.M. Lopez, C.M.B. Monteiro, J.M.F. dos Santos

i3N, Universidade de Aveiro, Campus de Santiago, Portugal

- D. S. Covita, J.F.C.A. Veloso

Institut für Strahlwerkzeuge, Universität Stuttgart, Germany

- M. Abdou-Ahmed, T. Graf, A. Voss, B. Weichelt

Physics Department, National Tsing Hua University, Taiwan

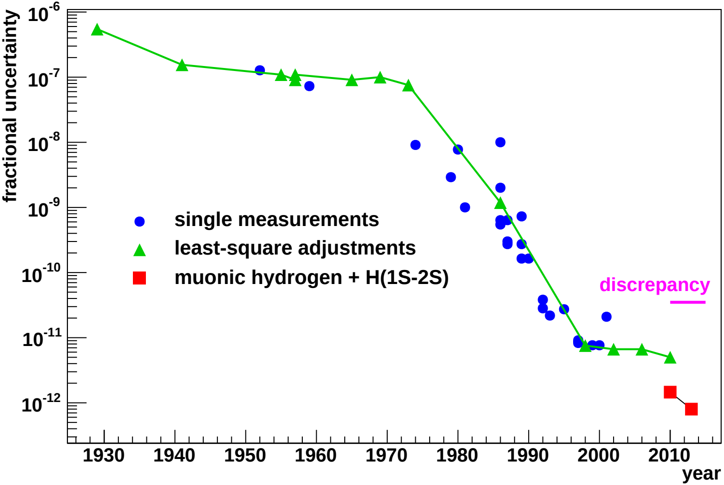
- T.-L. Chen, Y.-W. Liu

LIBPhys, Dep. Física, Universidade NOVA de Lisboa, Portugal

- P. Amaro, J.F.D.C. Machado, J. P. Santos

Motivation

Rydberg constant: Uncertainty in the last 80 years



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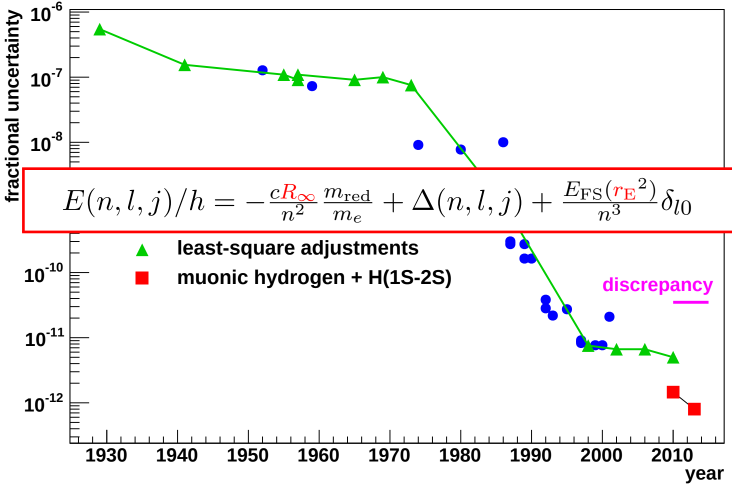

$$E(n,l,j)/h = -\frac{cR_\infty}{n^2}\frac{m_{\mathrm{red}}}{m_e} + \Delta(n,l,j) + \frac{E_{\mathrm{FS}}({r_{\mathrm{E}}}^2)}{n^3}\delta_{l0}$$

Motivation

Why measuring charge radii with muonic atoms is a good idea!

⇒ Atomic Physics: Tests of bound state QED in H-like atoms.

⇒ Nuclear Physics: Benchmark for ab initio calculations of few-nucleon nuclei.

⇒ Since 2010: the Proton Radius Puzzle

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Why measuring charge radii with muonic atoms is a good idea!

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- ⇒ Since 2010: the Proton Radius Puzzle

There are different ways to measure charge radii, but what is a charge radius?

About nuclear charge radii and form factors

The nuclear charge radius r_E is the

rms charge radius of a nucleus

which is hidden in the electric form factor of a nucleus and given by its slope at zero momentum transfer:

$$r_\mathrm{E}^2 = -6 \frac{dG_\mathrm{E}}{dQ^2}\Big|_{Q^2=0} \quad \left(\simeq \int r^2 \rho(r) d^3 r \right) \tag{2}$$

r_E is therefore a parameter of the charge distribution of a nucleus.

Its measurement is necessary for

- understanding nuclei
- testing higher order bound-state QED
- checking R_∞

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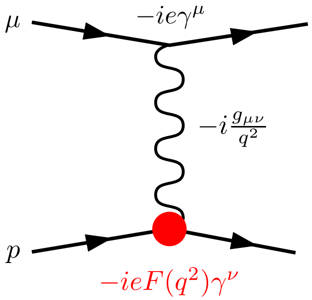
same contribution in all experiments



About nuclear charge radii and form factors

The nuclear charge radius r_{E} is the

For small $|\vec{q}|^2$ we can expand the form factor:

$$F(q^2) = \int e^{i\vec{q}\vec{x}} \rho(\vec{x}) \mathrm{d}^3 x \quad (3)$$

$$\approx \int \left(1 + i\vec{q}\vec{x} - \frac{(\vec{q}\vec{x})^2}{2} + ...\right) \rho(\vec{x}) \mathrm{d}^3 x \quad (4)$$

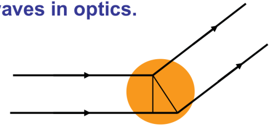
$$= 1 - \frac{1}{6}|\vec{q}|^2 \langle r^2 \rangle + \frac{1}{24}|\vec{q}|^4 \langle r^4 \rangle + ... \quad (5)$$

$$\Rightarrow \quad \langle r^2 \rangle = 6 \frac{\partial F(q^2)}{\partial q^2}\bigg|_{q^2=0} \quad (6)$$

- testing higher order bound-state QED
- checking R_∞

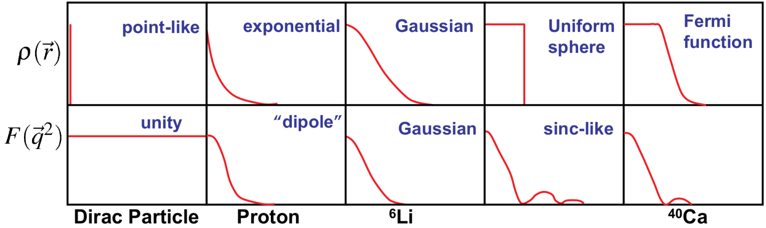
About nuclear charge radii and form factors

- **There is nothing mysterious about form factors – similar to diffraction of plane waves in optics.**



- **The finite size of the scattering centre introduces a phase difference between plane waves "scattered from different points in space". If wavelength is long compared to size all waves in phase and $F(\vec{q}^2) = 1$**

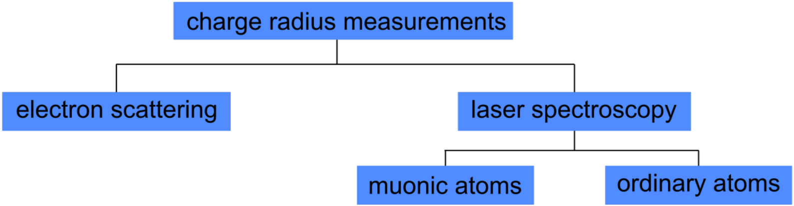
For example:



- **NOTE that for a point charge the form factor is unity.**

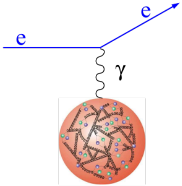
[from M. A. Thomson]

Methods to measure charge radii



Methods to measure charge radii

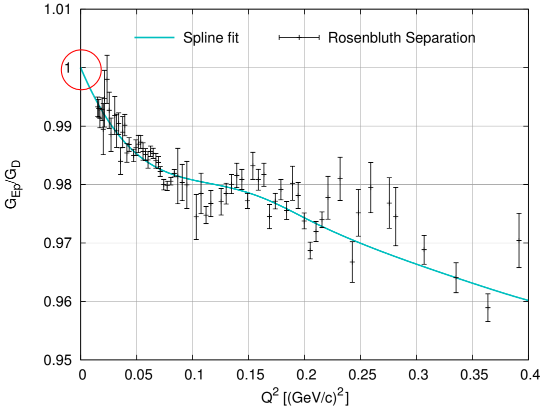
- charge radius measurements
  - electron scattering
  - laser spectroscopy
    - muonic atoms
    - ordinary atoms



Formfactor

Methods to measure charge radii

Electron scattering

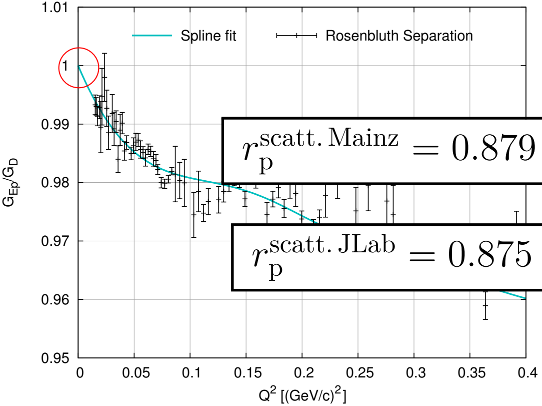


- measure cross sections at multiple momentum transfers
- extract electric and magnetic form factors (G_E, G_M) via Rosenbluth separation
- fit the electric form factor data versus momentum transfer (Q^2)
- $r_p^2 = -6 \frac{dG_E}{dQ^2}\Big|_{Q^2=0}$

[M. Vanderhaeghen, T. Walcher, arXiv:1008.4225v1 (2010)]

Methods to measure charge radii

Electron scattering



$$r_p^{\text{scatt. Mainz}} = 0.879\,(8)\ \text{fm}$$

$$r_p^{\text{scatt. JLab}} = 0.875\,(10)\ \text{fm}$$

- measure cross sections at multiple momentum transfers
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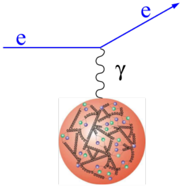

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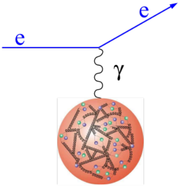
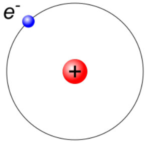


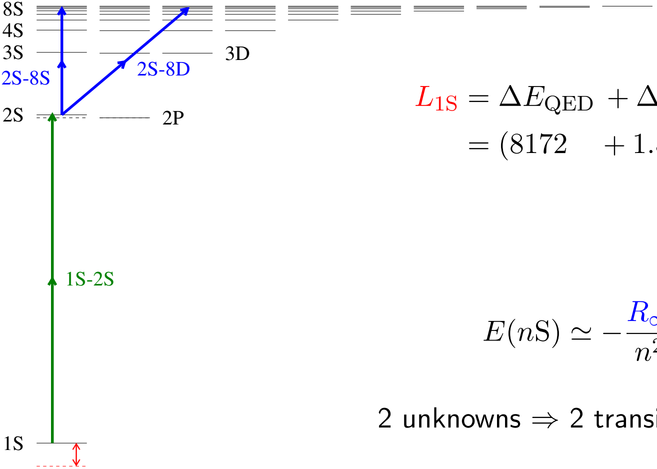
Formfactor



1S -> 2S & 2S -> nS/D
or
2S -> 2P

Methods to measure charge radii

Laser spectroscopy on ordinary atoms/ions (here in H)



$$L_{1S} = \Delta E_{QED} + \Delta E_{fin.s.} \quad (7)$$

$$= (8172 \quad + 1.56\, r_p^2)\ \text{MHz} \quad (8)$$

$$E(nS) \simeq -\frac{R_\infty}{n^2} + \frac{L_{1S}}{n^3} \quad (9)$$

2 unknowns ⇒ 2 transitions

Methods to measure charge radii

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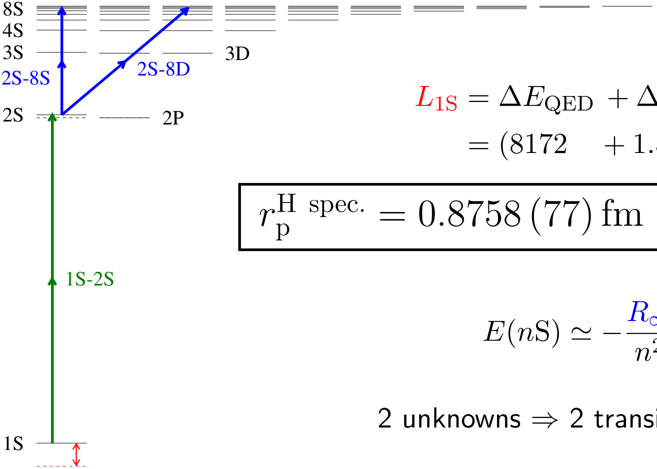

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$$\boxed{r_p^{H\ spec.} = 0.8758\,(77)\ \mathrm{fm}}$$

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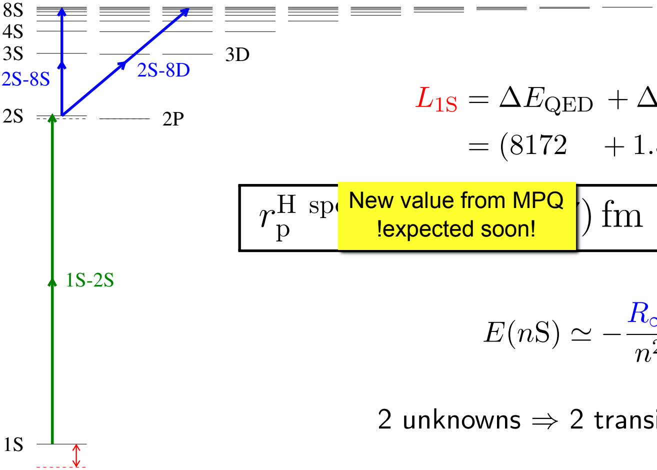

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$$= (8172 \quad + 1.56\, r_p^2)\ \text{MHz} \quad (8)$$

r_p^{H} sp...) fm

New value from MPQ
!expected soon!

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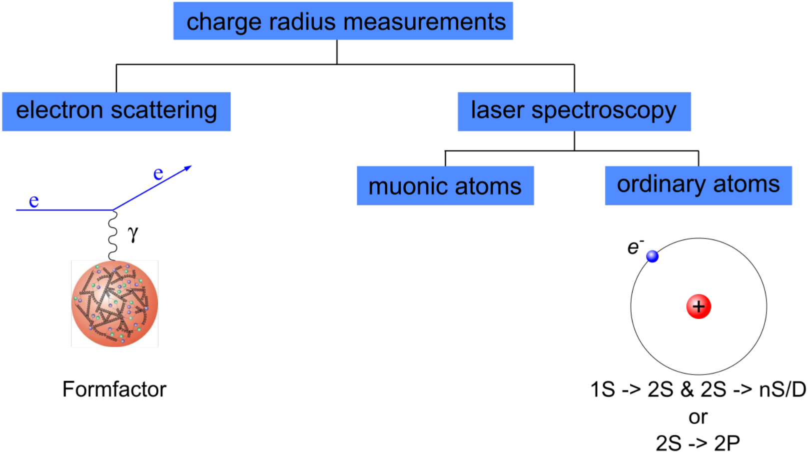
Methods to measure charge radii



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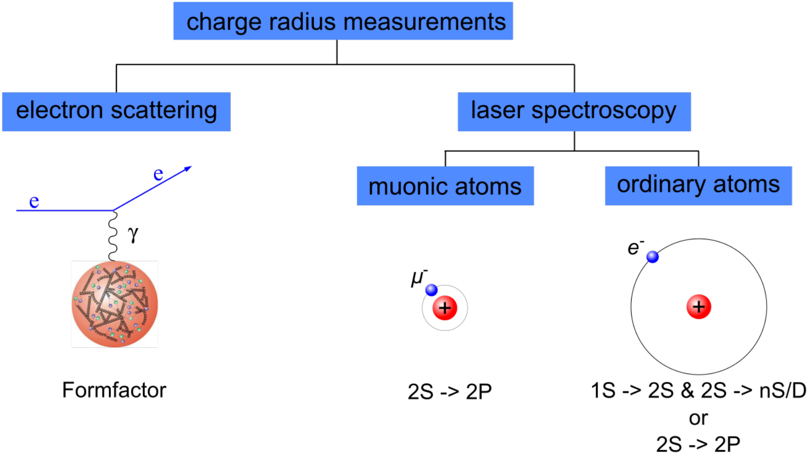
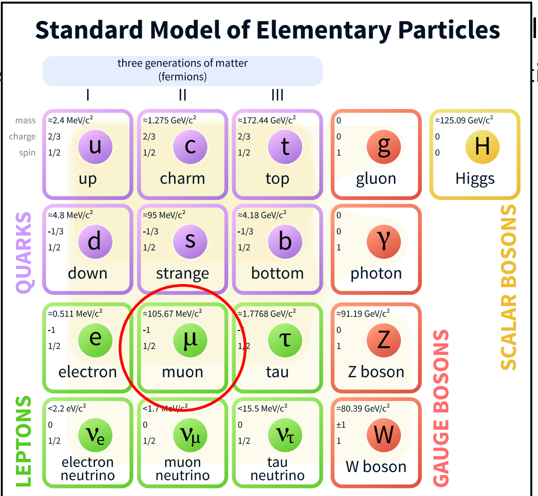


Motivation

Why measuring charge radii with muonic atoms is a good idea!

- ⇒ Atom like atoms.
- ⇒ Nucle tions of few-n
- ⇒ Since



[from Wikipedia]

Motivation

Why measuring charge radii with muonic atoms is a good idea!

- $\Rightarrow$ Atomic Physics: Tests of bound state QED in H-like atoms.
- $\Rightarrow$ Nuclear Physics: Benchmark for ab initio calculations of few-nucleon nuclei.
- $\Rightarrow$ Since 2010: the Proton Radius Puzzle

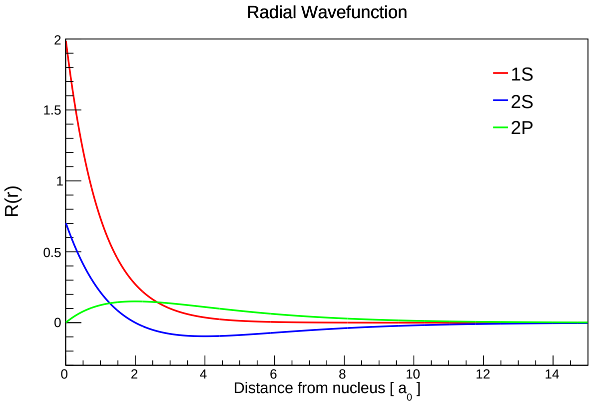
Muonic atoms are highly sensitive to nuclear parameters as e.g. the charge radius!

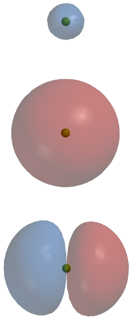
- bound system of a negative muon μ^- and a bare nucleus (p, d, he, α, ...).
- $m_\mu \approx 200 \times m_e$

$$\text{fin.size effect} \propto |\Psi_S|^2 \propto \frac{1}{a_0^3} \propto m_l^3 \quad \rightarrow \quad \text{factor: 10 Mio.!} \qquad (11)$$

Motivation

The Lamb shift (2S→2P) is correlated with the charge radius





- S states: non-zero probability to be inside the nucleus!
- S states: great probe for nuclear structure (r_C, r_M, TPE)

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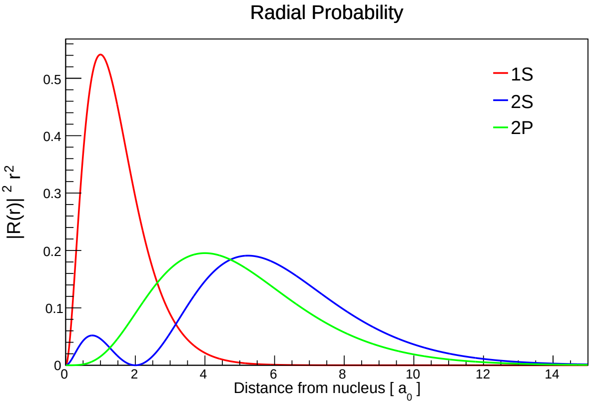

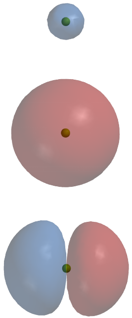

- S states: non-zero probability to be inside the nucleus!
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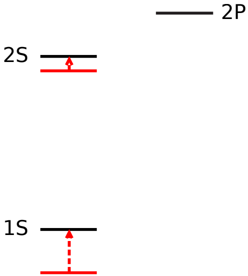
Motivation

The finite size effect contributes to the Lamb shift (2S→2P)

finite size effect



shift of S states in $\mu\mathrm{p}$:

$\Delta 2S \rightarrow$ 3.7 meV

$\Delta 1S \rightarrow$ 29.8 meV

(2S-2P difference is ~ 200 meV)

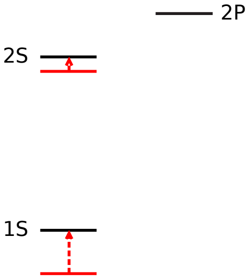
$\rightarrow \Delta 2S$ is therefore affecting the Lamb shift!

Lamb shift transition energy in light muonic atoms suited for laser spectroscopy!

Motivation

The finite size effect contributes to the Lamb shift (2S$\rightarrow$2P)

finite size effect



shift of S states in $\mu^3\mathrm{He}^+$:

$\Delta 2S \rightarrow$ 0.4 eV

$\Delta 1S \rightarrow$ 3.2 eV
(2S-2P difference is 1.3 to 1.5 eV)

$\rightarrow \Delta 2S$ is therefore affecting the Lamb shift!

Lamb shift transition energy in light muonic atoms suited for laser spectroscopy!

The Proton Radius Puzzle

shrinking the proton

$$r_p^{\text{CODATA}} = 0.8751(61)\,\text{fm}$$

$$\Downarrow$$

$$r_p^{\text{CREMA}} = 0.84087(39)\,\text{fm}$$

- 4% smaller
- > 10fold precision
- σ discrepant

[P. J. Mohr *et al.*, Rev. Mod. Phys. 88, 035009 (2016)]
[R. Pohl *et al.* (CREMA-coll.), Nature 466, 213 (2010)]
[A. Antognini *et al.* (CREMA-coll.), Science 339, 417 (2013)]

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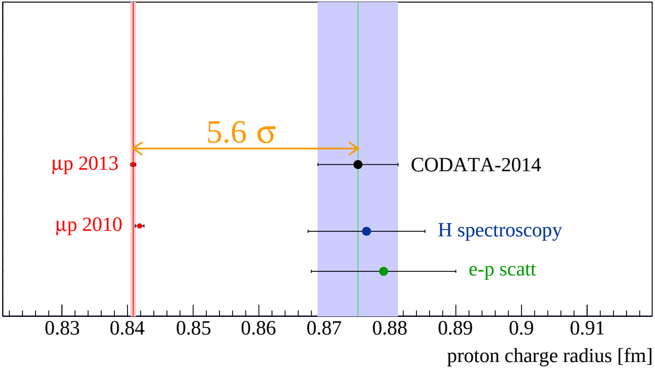

Summarizing all *electronic* measurements of r_p (spectroscopy and scattering) from hydrogen and deuterium data, yields a $5.6\,\sigma$ discrepancy to the CREMA measurement.

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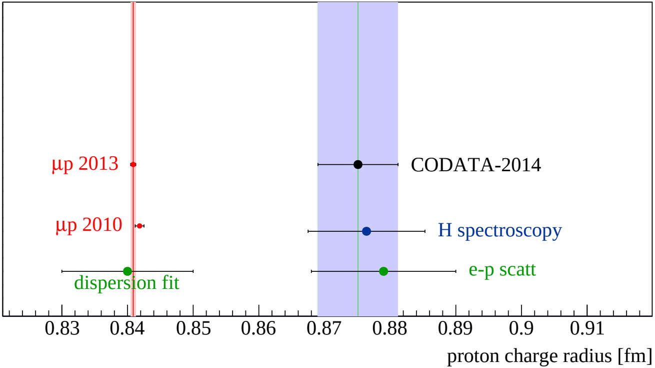

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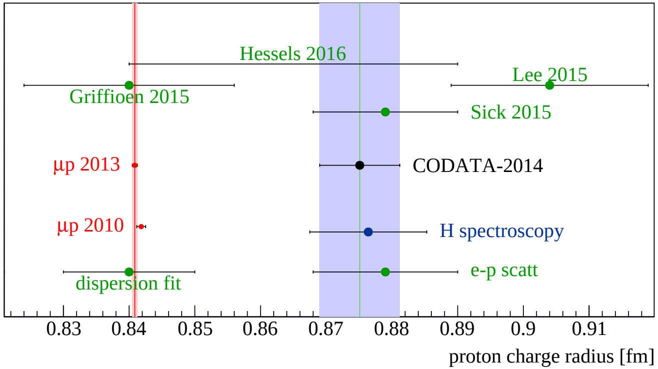

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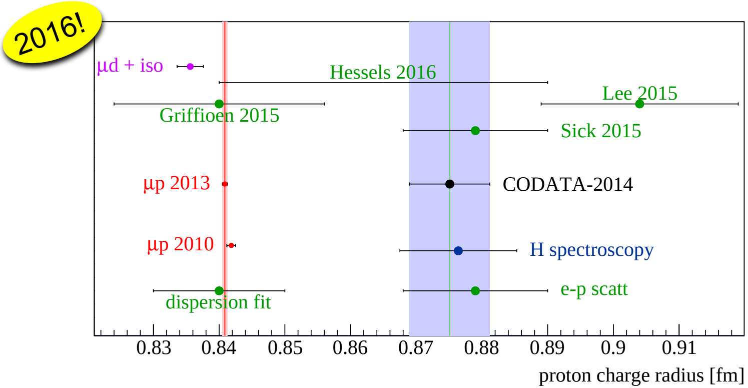

Summarizing all *electronic* measurements of r_p (spectroscopy and scattering) from hydrogen and deuterium data, yields a $5.6\,\sigma$ discrepancy to the CREMA measurement.

Possible solutions

$$L_{\mu p}^{\text{theo}}(r_{\text{p}}^{\text{CODATA}}) - L_{\mu p}^{\text{exp}} = \begin{cases} 75\,\text{GHz} \\ 0.33\,\text{meV} \\ 0.15\,\% \end{cases} \tag{12}$$

Possible solutions

$$\underline{L_{\mu p}^{\text{theo}}}(\underline{r_p^{\text{CODATA}}}) - \underline{L_{\mu p}^{\text{exp}}} = \begin{cases} 75\,\text{GHz} \\ 0.33\,\text{meV} \\ 0.15\,\% \end{cases} \tag{12}$$

- μp experiment wrong?
  - good statistics ($\sigma = 0.76$GHz)
  - linewidth ~ 19GHz
  - several methods for frequency calibration

Possible solutions

$$\underline{L_{\mu\mathrm{p}}^{\mathrm{theo}}}(\underline{r_{\mathrm{p}}^{\mathrm{CODATA}}}) - \underline{L_{\mu\mathrm{p}}^{\mathrm{exp}}} = \begin{cases} 75\,\mathrm{GHz} \\ 0.33\,\mathrm{meV} \\ 0.15\,\% \end{cases} \quad (12)$$

- CODATA wrong?
  - e-p scattering wrong?
  - H spectroscopy wrong?
  - H theory wrong?

Possible solutions

$$L_{\mu p}^{\text{theo}}(r_p^{\text{CODATA}}) - L_{\mu p}^{\text{exp}} = \begin{cases} 75\,\text{GHz} \\ 0.33\,\text{meV} \\ 0.15\,\% \end{cases} \tag{12}$$

- μp theory wrong?
    - mainly pure QED
    - hadronic terms small
    - pol. term 0.015(4)meV

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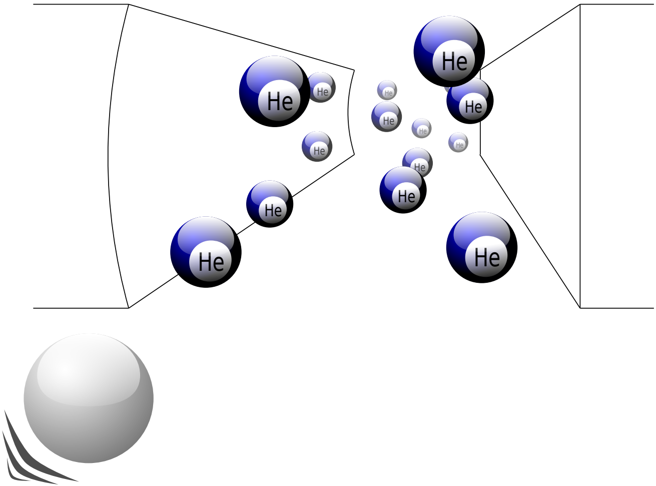
or even new physics???

The muonic helium measurement!
(similar to μp and μd)

Basic principle

How can we create a muonic atom?



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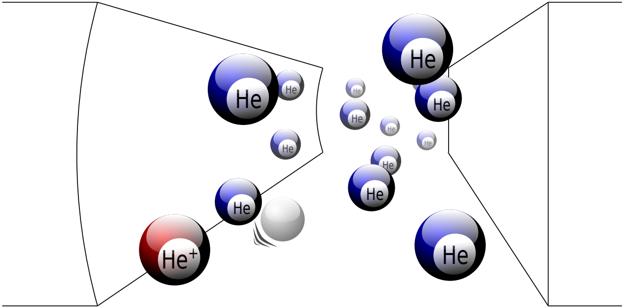

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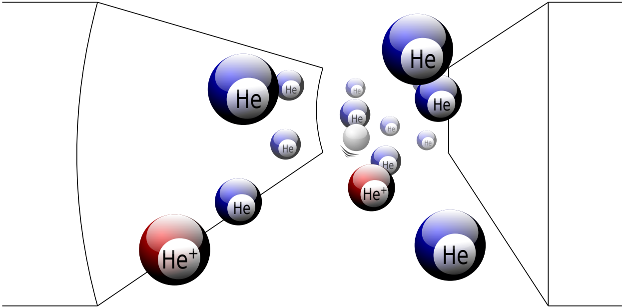

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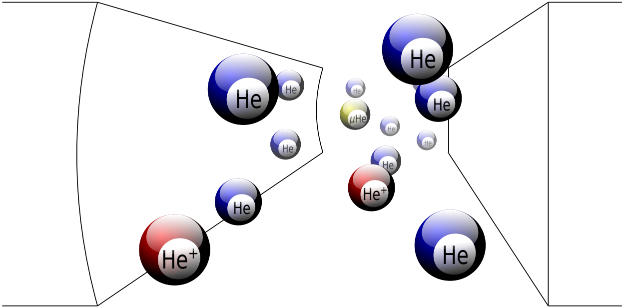

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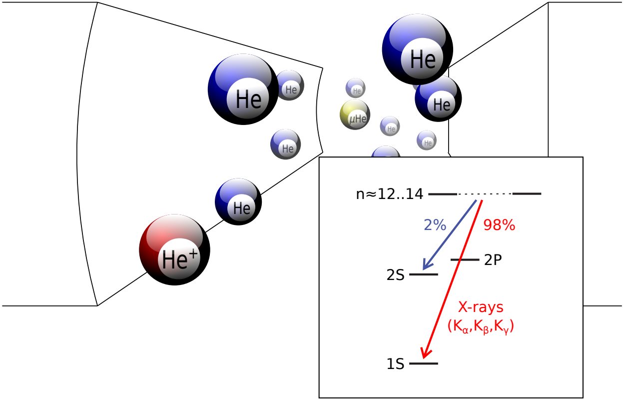


Experimental setup

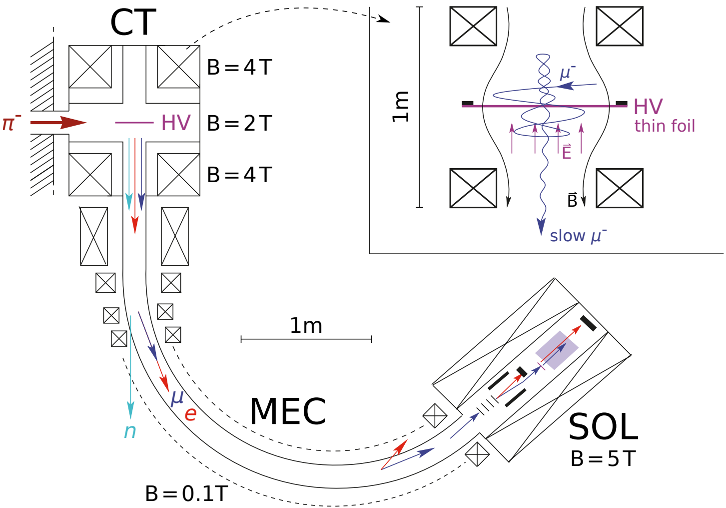
Paul Scherrer Institute (PSI)

Experimental setup

Paul Scherrer Institute (PSI)

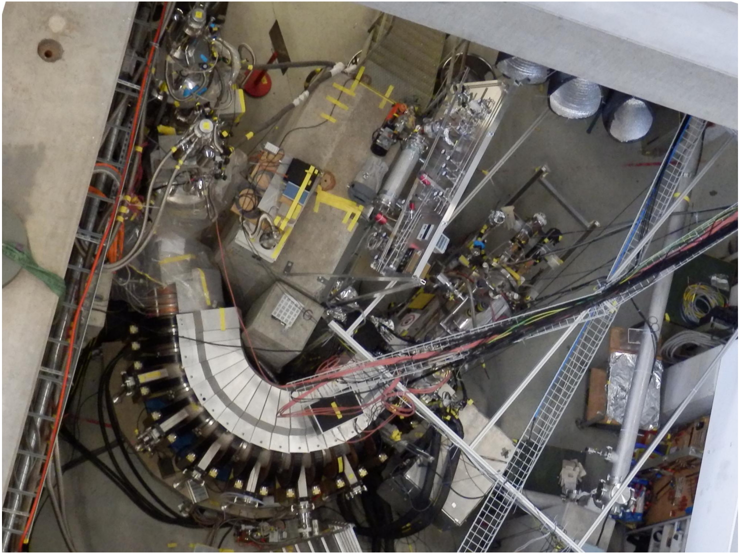
Experimental setup

muon beamline



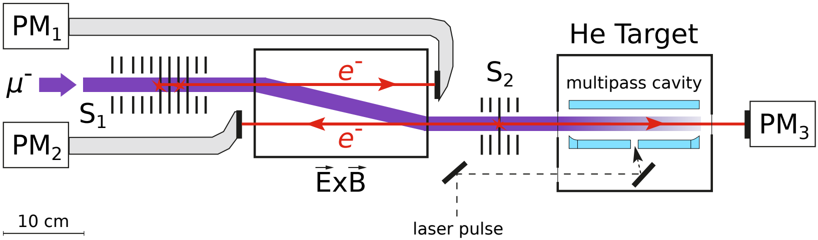
Experimental setup

zone PiE5



Experimental setup

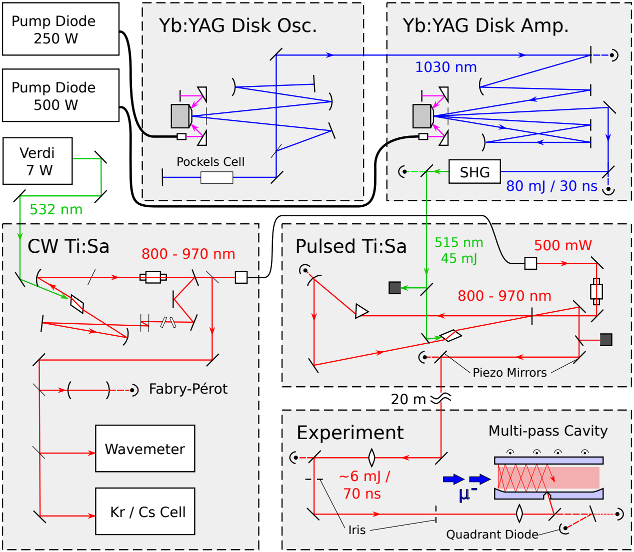
non-destructing muon detection



- 2 stacks of carbon foils
- pasing foils, muons lose energy and create secondary electrons
- velocity filter with: $\vec{F}_L = q(\vec{v} \times \vec{B})$ and $\vec{F}_E = q\vec{E}$

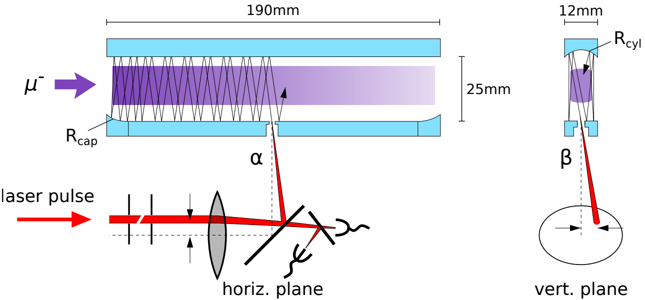
Experimental setup

Laser system for $\mu^3\mathrm{He}^+$ and $\mu^4\mathrm{He}^+$



Experimental setup

multipass cavity in $\mu^3\mathrm{He}^+$

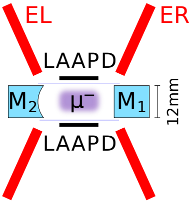


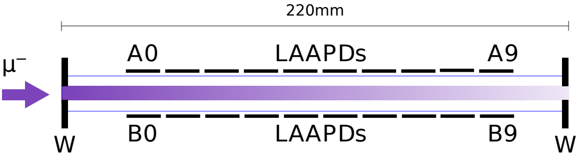
[J. Vogelsang *et al.*, Opt. Expr. 22, 13050 (2014)]

- cavity lifetime $\sim 110\,\mathrm{ns} \rightarrow$ 1300 reflections
- lifetime constrained by injection hole and 50 $\mathrm{\mu m}$ gaps at 'ears'
- fluence reached: $\sim 0.5\,\mathrm{J\,cm^{-2}}$

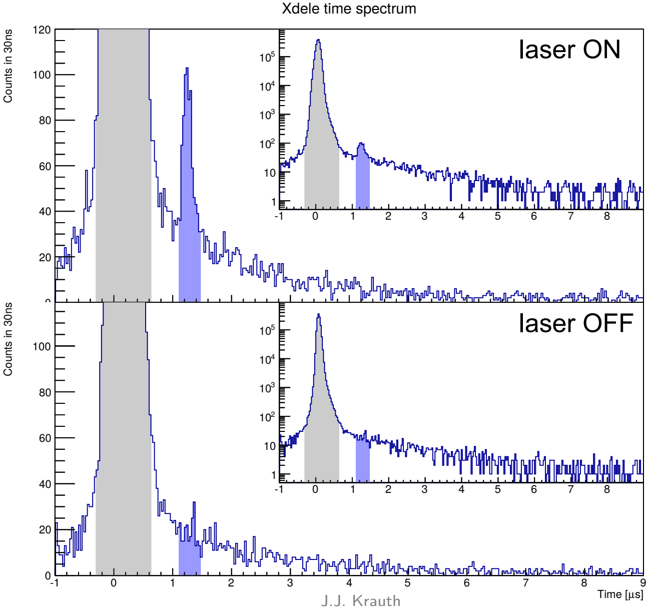
Experimental setup

Detection in target





Muonic helium-3 ($\mu^3\mathrm{He}^+$)



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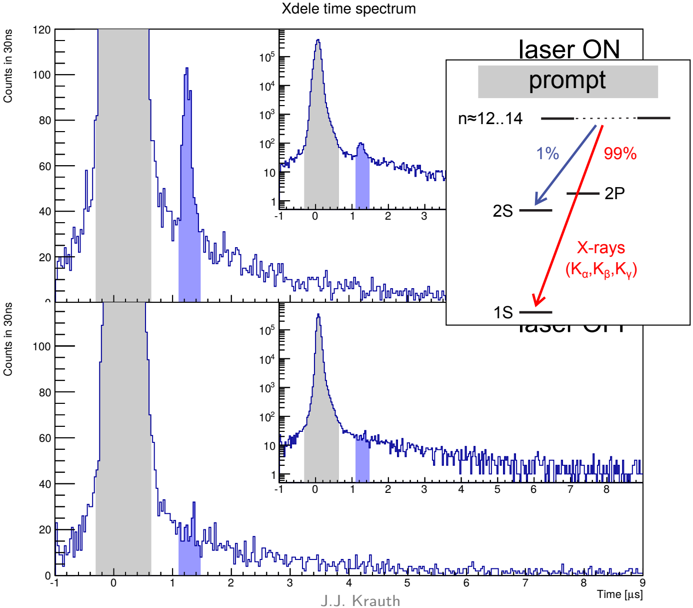

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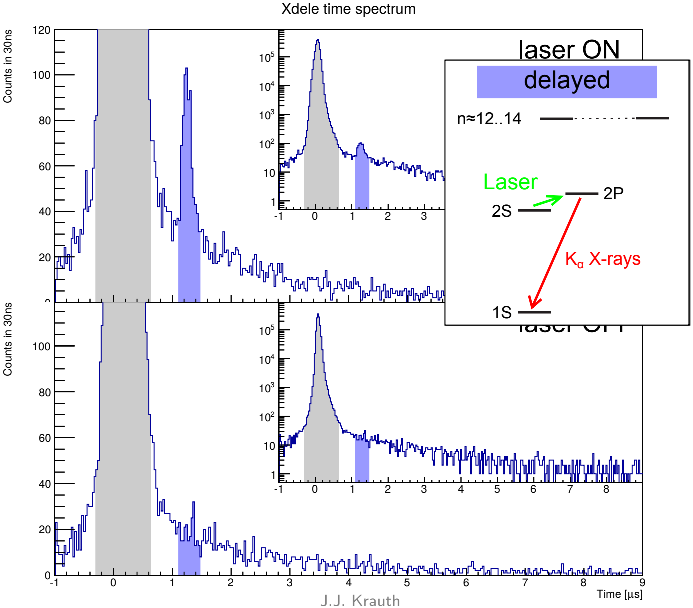

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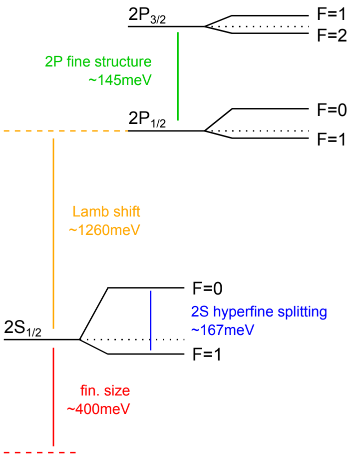

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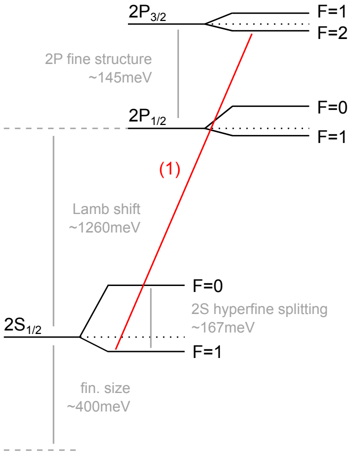

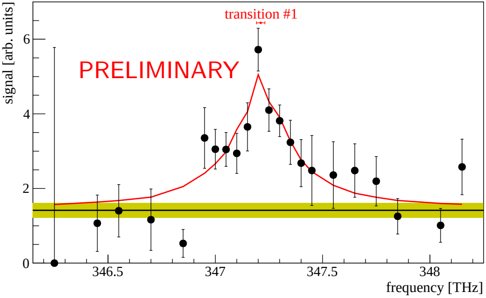

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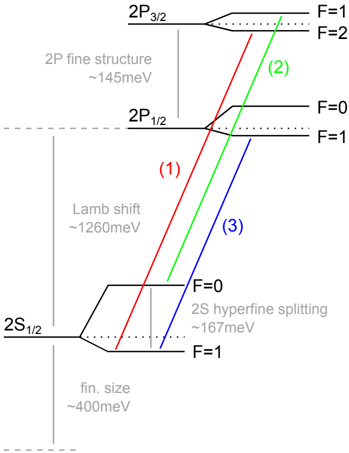

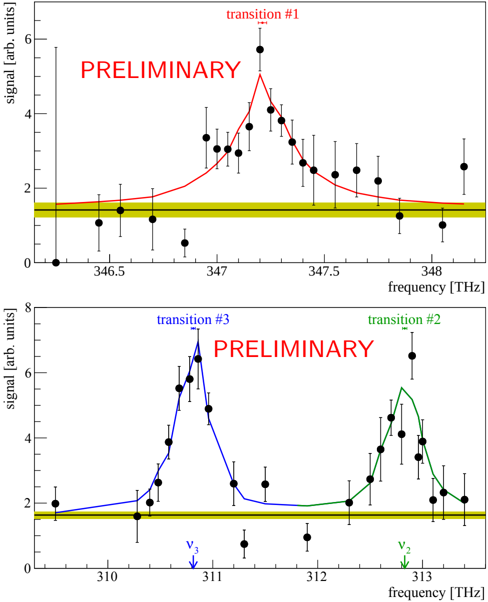

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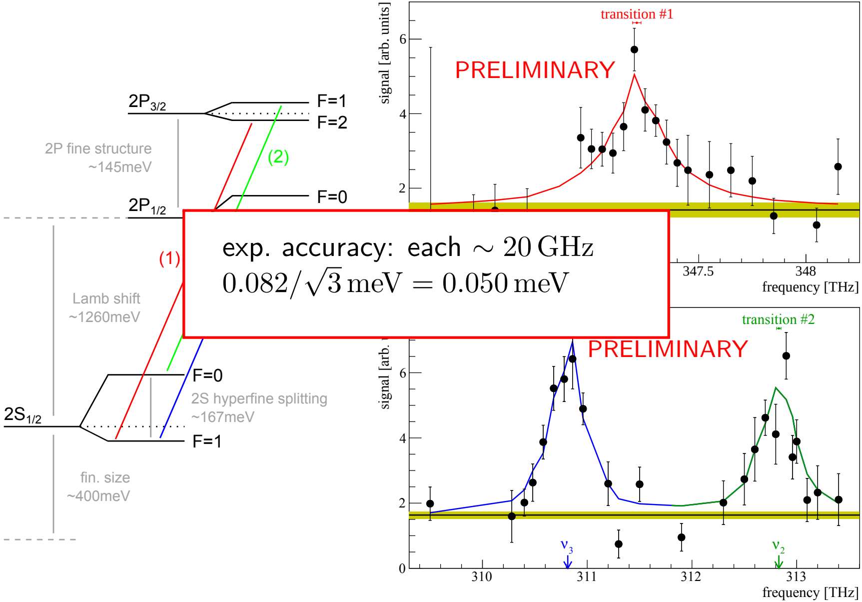

exp. accuracy: each $\sim 20\,\mathrm{GHz}$

$0.082/\sqrt{3}\,\mathrm{meV} = 0.050\,\mathrm{meV}$

Extracting the rms charge radius

Annals of Physics 331 (2013) 127–145

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Theory of the 2S–2P Lamb shift and 2S hyperfine splitting in muonic hydrogen

Aldo Antognini [a,*], Franz Kottmann [a], François Biraben [b], Paul Indelicato [b], François Nez [b], Randolf Pohl [c]

[a] *Institute for Particle Physics, ETH Zurich, 8093 Zurich, Switzerland*

[b] *Laboratoire Kastler Brossel, École Normale Supérieure, CNRS and Université P. et M. Curie, 75252 Paris, CEDEX 05, France*

[c] *Max-Planck-Institut für Quantenoptik, 85748 Garching, Germany*

Extracting the rms charge radius

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Annals of Physics 366 (2016) 168–196

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ANNALS of PHYSICS

Theory of the $n = 2$ levels in muonic deuterium

CrossMark

Julian J. Krauth [a,*], Marc Diepold [a], Beatrice Franke [a], Aldo Antognini [b,c], Franz Kottmann [b], Randolf Pohl [a]

[a] *Max-Planck-Institut für Quantenoptik, 85748 Garching, Germany*
[b] *Institute for Particle Physics, ETH Zurich, 8093 Zurich, Switzerland*
[c] *Paul-Scherrer-Institute, 5232 Villigen, Switzerland*

Extracting the rms charge radius

Annals of Physics 331 (2013) 127–145

Annals of Physics 366 (2016) 168–196

Theory of the Lamb shift and Fine Structure in $(\mu^4\text{He})^+$

Marc Diepold,[1,*] Julian J. Krauth,[1] Beatrice Franke,[1] Aldo Antognini,[2,3] Franz Kottmann,[2,3] and Randolf Pohl[4,1]

[1] *Max Planck Institute of Quantum Optics, 85748 Garching, Germany.*
[2] *Institute for Particle Physics, ETH Zurich, 8093 Zurich, Switzerland.*
[3] *Paul Scherrer Institute, 5232 Villigen-PSI, Switzerland.*
[4] *Johannes Gutenberg-Universität Mainz, Institut für Physik, QUANTUM, and PRISMA Cluster of Excellence, Mainz, Germany.*

(Dated: June 17, 2016)

An up to date review of the theoretical contributions to the 2S → 2P Lamb shift and the fine structure of the 2P-state in the $(\mu^4\text{He})^+$ ion is given. This summary will serve as the basis for the extraction of the alpha particle charge radius from the muonic helium Lamb shift measurements at the Paul Scherrer Institute Switzerland. Individual theoretical contributions needed for a charge radius extraction were compared and compiled into a consistent summary using the already established framework we used for muonic hydrogen and deuterium. The influence of the alpha particle charge distribution on the elastic two-photon exchange is studied to rule out possible model dependencies of the energy levels on the electric form factor of the nucleus.

I. INTRODUCTION

The CREMA Collaboration has recently performed

provide an improved value of the alpha particle charge radius. This charge radius will also be used for tests of fundamental bound state QED by planned measurements

Extracting the rms charge radius

Annals of Physics 331 (2013) 127–145

Annals of Physics 366 (2016) 168–196

Theory of the Lamb shift and Fine Structure in $(\mu^4\text{He})^+$

Theory of the $\boldsymbol{n = 2}$ levels in muonic helium-3 ions

Beatrice Franke[a,b,*], Julian J. Krauth[a,c,*], Aldo Antognini[d,e], Marc Diepold[a], Franz Kottmann[d], Randolf Pohl[c,a]

[a] *Max–Planck–Institut für Quantenoptik, 85748 Garching, Germany.*
[b] *TRIUMF, 4004 Wesbrook Mall, Vancouver, BC V6T 2A3, Canada*
[c] *Johannes Gutenberg-Universität Mainz, QUANTUM, Institut für Physik & Exzellenzcluster PRISMA, 55099 Mainz, Germany*
[d] *Institute for Particle Physics, ETH Zurich, 8093 Zurich, Switzerland.*
[e] *Paul Scherrer Institute, 5232 Villigen, Switzerland.*

30 Apr 2017

Abstract

The presen… es in muonic helium-3 io… 2P transition frequencies in the muonic helium-3 ion, $\mu^3\text{He}^+$. This ion is the bound state of a single negative muon μ^-

arXiv:1705.00352, submitted to EPJD

Extracting the rms charge radius

Annals of Physics 331 (2013) 127–145

Annals of Physics 366 (2016) 168–196

Theory of the Lamb shift and Fine Structure in $(\mu^4\mathrm{He})^+$

Theory of the $\boldsymbol{n} = \mathbf{2}$ levels in muonic helium-3 ions

Thanks for valuable discussions and remarks to:
S. Bacca, N. Barnea, M. Birse, E. Borie, C. E. Carlson, M. Eides, J.L. Friar, M. Gorchtein, O. J. Hernandez, C. Ji, S. Karshenboim, A.P. Martynenko, J. McGovern, N. Nevo Dinur, K. Pachucki, and M. Vanderhaeghen!

Abstract

arXiv:1705.00352, submitted to EPJD

The presen... es in muonic helium-3 ions ... 2P transition frequencies in the muonic helium-3 ion, $\mu^3\mathrm{He}^+$. This ion is the bound state of a single negative muon μ^-

Theory of n=2 levels

The measured transition energy is theoretically predicted by

$$E_{1,2,3} = \Delta E_{\mathrm{LS}} + \Delta E_{\mathrm{2S}}(i) + \Delta E_{\mathrm{2P}}(f), \qquad (13)$$

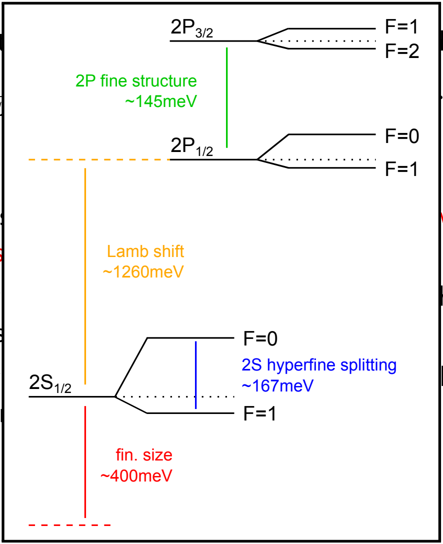
- the Lamb shift energy E_{LS} $(\mathrm{2S}_{1/2} \rightarrow \mathrm{2P}_{1/2})$, which contains the finite size effect!
- the energy difference $\Delta E_{\mathrm{2S}}(i)$ from $2S_{1/2}$ to the initial 2S hyperfine state
- the energy difference $\Delta E_{\mathrm{2P}}(f)$ from $2P_{1/2}$to the final 2P state, given by fine- and hyperfine splitting

Theory of n=2 levels

The measured t … licted by

$$E \ldots), \qquad (13)$$

- the Lamb … which contains the finite s…
- the energy … he initial 2S hyperfine s…
- the energy … he final 2P state, give…



Lamb shift $(2S_{1/2} - 2P_{1/2})$

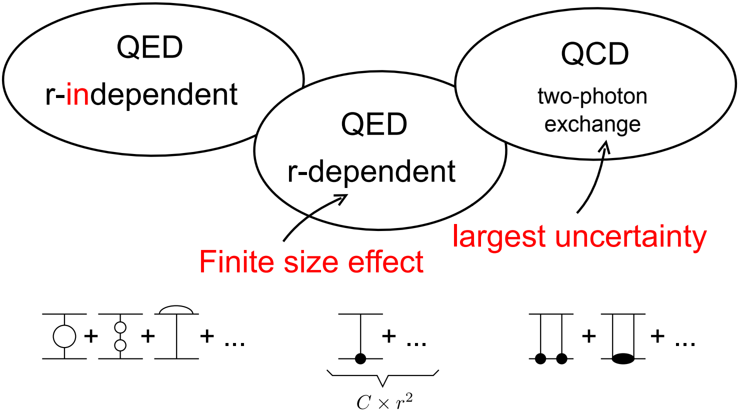


Table 1: All known **nuclear structure-independent** contributions to the Lamb shift in $\mu^3\mathrm{He}^+$. Values are in meV. Item numbers "#" in the 1st column follow the nomenclature of Refs. [1, 2], which in turn follow the supplement of Ref. [4]. Items "#" with a dagger † were labeled "New" in Ref. [1], but we introduced numbers in Ref. [2] for definiteness. For Borie [25] we refer to the most recent arXiv version-7 which contains several corrections to the published paper [24] (available online 6 Dec. 2011). For Martynenko *et al.*, numbers #1 to #29 refer to rows in Tab. I of Ref. [26]. Numbers in parentheses refer to equations in the respective paper.

#	Contribution	Borie (B)		Martynenko group (M)		Jentschura (J)		Karshenboim group (K)		Our choice			
		[25]		Krutov *et al.* [26]		Jentschura, Wundt [31]		Karshenboim *et al.* [34]		value		source	Fig.
						Jentschura [32]		Korzinin *et al.* [33]					
1	NR one-loop electron VP (eVP)			1641.8862	#1	1641.885	[32]						
2	Rel. corr. (Breit-Pauli)	(0.50934) [a]	Tab. 1	0.5093	#7+#10	0.509344	[31](17), [32]	(0.509340)	[34] Tab. IV				
3	Rel. one-loop eVP	1642.412	Tab. p. 4										
19	Rel. RC to eVP, $\alpha(Z\alpha)^4$	−0.0140	Tab. 1+6										
	Sum of the above	1642.3980	3+19	1642.3955	1+2	1642.3943	1+2	1642.3954	[33] Tab. I	1642.3962	± 0.0018	avg	2
4	Two-loop eVP (Källén-Sabry)	11.4107	Tab. p. 4	11.4070	#2					11.4089	± 0.0019	avg.	3
5	One-loop eVP in 2-Coulomb lines $\alpha^2(Z\alpha)^2$	1.674	Tab. 6	1.6773	#9	1.677290	[31](13)			1.6757	± 0.0017	avg.	4
	Sum of 4 and 5	13.0847	4+5	13.0843	4+5			13.0843	[33] Tab. I	(13.0846) [b]			
6+7	Third order VP	0.073(3)	p. 4	0.0689	#4+#12+#11			0.073(3)	[33] Tab. I	0.0710	± 0.0036	avg.	
29	Second-order eVP contribution $\alpha^2(Z\alpha)^4 m$			0.0018	#8+#13			0.00558	[33] Tab. VIII "eVP2"	0.0037	± 0.0019	avg	
9	Light-by-light "1:3": Wichmann-Kroll	−0.01969	p. 4	−0.0197	#5								5a
10	Virtual Delbrück, "2:2" LbL			} 0.0064	#6								5b
9a†	"3:1" LbL												5c
	Sum: Total light-by-light scatt.	−0.0134(6)	p.5+Tab.6	−0.0133	9+10+9a			−0.0134(6)	[33] Tab. I	−0.0134	± 0.0006	K	
20	μSE and μVP	−10.827368	Tab. 2+6	−10.8286	#24					−10.8280	± 0.0006	avg.	6
11	Muon SE corr. to eVP $\alpha^2(Z\alpha)^4$	(−0.1277) [c]	Tab. 16	−0.0627	#28	−0.06269	[31](29)	−0.06269	[33] Tab. VIII (a)	−0.06269		J, K	7
12	eVP loop in self-energy $\alpha^2(Z\alpha)^4$	incl. in 21		−0.0299	#27			−0.02992	[33] Tab. VIII (d)	incl. in 21		B	8
30	Hadronic VP loop in self-energy $\alpha^2(Z\alpha)^4 m$							−0.00040(4)	[33] Tab. VIII (e)	−0.00040	± 0.00004	K	9
13	Mixed eVP + μVP	0.00200	p. 4	0.0022	#3			0.00383	[33] Tab. VIII (b)	0.0029	± 0.0009	avg	10
31	Mixed eVP + hadronic VP							0.0024(2)	[33] Tab. VIII (c)	0.0024	± 0.0002	K	11
21	Higher-order corr. to μSE and μVP	−0.033749	Tab. 2+6							−0.033749		B	
	Sum of 12, 30, 13, 31, and 21	−0.031749	13+21	−0.0277	12+13			−0.0241(2)	12+30+13+31	−0.0288		sum	
14	Hadronic VP	0.221(11)	Tab. 6	0.2170	#29					0.219	± 0.011	avg.	
17	Recoil corr. $(Z\alpha)^4 m_r^3/M^2$ (Barker-Glover)	0.12654	Tab. 6	0.1265	#21	0.12654	[31](A.3) [32](15)			0.12654		B, J	
18	Recoil, finite size	(0.4040(10)) [d]											
22	Rel. RC $(Z\alpha)^5$	−0.55811	p.9+Tab.6	−0.5581	#22	−0.558107	[31](32)			−0.558107		J	
23	Rel. RC $(Z\alpha)^6$			0.0051	#23					0.0051		M	
24	Higher order radiative recoil corr.	−0.08102	p.9+Tab.6	−0.0656	#25					−0.0733	± 0.0077	avg.	
28†	Rad. (only eVP) RC $\alpha(Z\alpha)^5$					0.004941				0.004941		J	
	Sum	1644.3916 [e]		1644.3431						**1644.3466**	**± 0.0146**		

[a] Does not contribute to the sum in Borie's approach.

[b] Sum of *our choice* of item #4 and #5, written down for comparison with the Karshenboim group.

[c] In App. C of [25], incomplete. Does not contribute to the sum in Borie's approach, see text.

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[e] Including item #18 and #r3' yields 1644.9169 meV, which is Borie's value from Ref. [25] page 15. On that page she attributes an uncertainty of 0.6 meV to that value. This number is far too large to be correct, so we ignore it.

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$$\Delta E^{\mathrm{LS}}_{\mathrm{r-indep.}} = 1644.3466 \pm 0.0146\,\mathrm{meV}$$

[a] Does not contribute to the sum in Borie's approach.
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Lamb shift

radius-dependent (finite size)

There are calculations from Borie, Martynenko, and Karshenboim.

The main finite size contributions are given to order $(Z\alpha)^6$ by

$$\Delta E_{\text{fin. size}} = \frac{2\pi Z\alpha}{3}|\Psi_{n=2}(0)|^2 \left[\langle r^2\rangle - \frac{Z\alpha m_r}{2}\langle r^3\rangle_{(2)} + (Z\alpha)^2(F_{\text{REL}} + m_r^2 F_{\text{NREL}})\right] \tag{14}$$

[J. L. Friar, Annals of Physics 122, 151196 (1979)]

- The second term is the Friar moment contribution.
- The last term is partly evaluated with an exp. model.

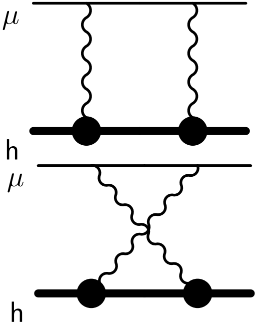
$$\Delta E^{\text{LS}}_{\text{rad.-dep.}} = -103.5184(98)\, r_{\text{h}}^2\, \text{meV/fm}^2 + 0.1177(33)\, \text{meV} \tag{15}$$

Lamb shift

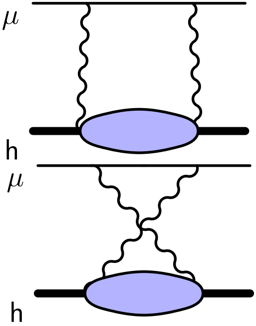
two-photon exchange (TPE)

$$\Delta E^{\mathrm{LS}}_{\mathrm{TPE}} = \Delta E^{\mathrm{LS}}_{\mathrm{Friar}} + \Delta E^{\mathrm{LS}}_{\mathrm{inelastic}} = 15.30(52)\,\mathrm{meV} \qquad (16)$$

elastic (Friar moment)



inelastic (polarizability)



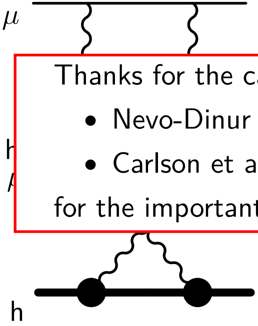
→ TPE: main limitation for determination of r_{h}!

Lamb shift

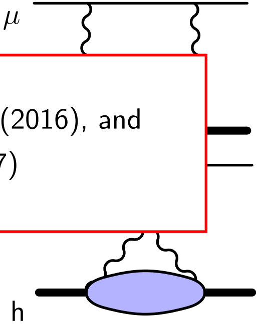
two-photon exchange (TPE)

$$\Delta E^{\rm LS}_{\rm TPE} = \Delta E^{\rm LS}_{\rm Friar} + \Delta E^{\rm LS}_{\rm inelastic} = 15.30(52)\,{\rm meV} \qquad (16)$$

elastic (Friar moment)



inelastic (polarizability)



Thanks for the calculations to

- Nevo-Dinur et al., PLB 755 (2016), and
- Carlson et al., PRA 95 (2017)

for the important work.

$\rightarrow$ TPE: main limitation for determination of $r_{\rm h}$!

Lamb shift

helion rms charge radius

extract charge radius from muonic data and theory:

- 3 measured transitions, 2 fit parameters (LS, 2S HFS)
- $\Delta E_{\mathrm{LS}} = \Delta E_{\mathrm{QED}} + \Delta E_{\mathrm{fin.size}}(C \times r_{\mathrm{h}}^2) + \Delta E_{\mathrm{TPE}}^{\mathrm{LS}}$

This yields:

$\rightarrow$ $r_{\mathrm{h}}(\mu^3\mathrm{He}^+) = 1.97xxx(12)^{\mathrm{exp}}(128)^{\mathrm{theo}}\,\mathrm{fm}$ Preliminary!

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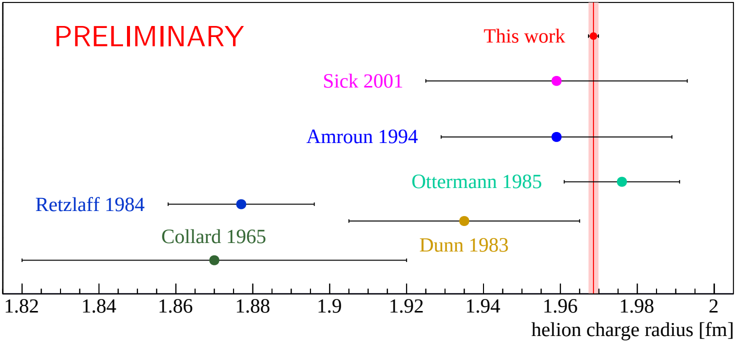
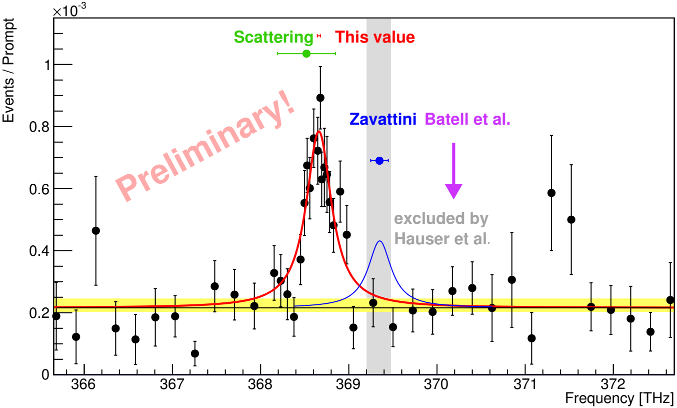


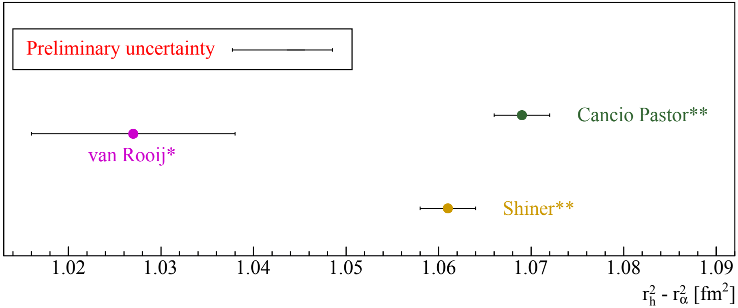
muonic helium-4 (μ^4He$^+$)

Muonic helium-4 ($\mu^4\mathrm{He}^+$)



experimental accuracy of $17\,\mathrm{GHz}$
with theory we get: $r_\alpha(\mu^4\mathrm{He}^+) = 1.68xxx(19)_{\mathrm{exp}}(58)_{\mathrm{theo}}\,\mathrm{fm}$
compared to $1.68100(400)\,\mathrm{fm}$ from e-scatt..

Charge radius difference in muonic helium



value from re-evaluated theory in

* Patkos et al., PRA 95, 012508 (2017)

** Patkos et al., PRA 94, 052508 (2016)

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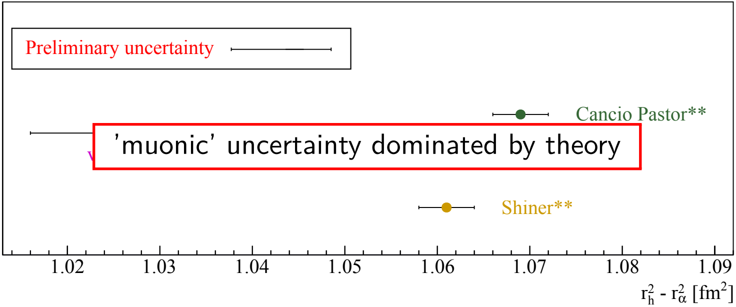

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Muonic Summary

$r_{\mathrm{p}}(\mu\mathrm{d})$	6σ smaller than CODATA
$r_{\mathrm{d}}(\mu\mathrm{d})$	6σ smaller than CODATA
	consistent with $r_{\mathrm{p}}(\mu\mathrm{d})$

$r_{\mathrm{h}}(\mu^3\mathrm{He}^+)$, $r_{\alpha}(\mu^4\mathrm{He}^+)$: no big discrepancy! prelim.

Muonic Summary

$r_p(\mu d)$ 6σ smaller than CODATA
$r_d(\mu d)$ 6σ smaller than CODATA
consistent with $r_p(\mu d)$
$r_h(\mu^3\mathrm{He}^+)$, $r_\alpha(\mu^4\mathrm{He}^+)$: no big discrepancy! prelim.

Scenario 1:

$r_p(H)$ and $r_d(D)$ wrong?
$\rightarrow R_\infty$ wrong by 7σ
$\rightarrow r_p(e-p)$ wrong

$\Rightarrow$

$r_p = 0.84087(39)\,\mathrm{fm}$
$r_d = 2.12771(22)\,\mathrm{fm}\ (\mu p + \mathrm{iso})$
$\Delta E^{\mathrm{LS}}_{\mathrm{TPE}} = 1.7638(68)\,\mathrm{meV}\ (\mu d)$
$\rightarrow$ shift R_∞ by 7σ!

Muonic Summary

$r_\mathrm{p}(\mu\mathrm{d})$ 6σ smaller than CODATA
$r_\mathrm{d}(\mu\mathrm{d})$ 6σ smaller than CODATA
consistent with $r_\mathrm{p}(\mu\mathrm{d})$
$r_\mathrm{h}(\mu^3\mathrm{He}^+)$, $r_\alpha(\mu^4\mathrm{He}^+)$: no big discrepancy! prelim.

Scenario 2:

Beyond SM
$\mu\mathrm{p}$: $0.33\,\mathrm{meV}$
$\mu\mathrm{d}$: $0.45\,\mathrm{meV}$
$(g-2)_\mu$: 3.5σ

$\Rightarrow$

New force carrier w/ $\sim$ MeV mass
– Batell *et al.* (2011)
$\rightarrow$ excluded by $\mu^4\mathrm{He}^+$! (prelim.!)
– Tucker-Smith, Yavin (2011)

Outlook

- proton smaller (*Pohl et al. Nature 2010, Antognini et al. Science 2013*)
- deuteron smaller (*Pohl et al. Science 2016*)

- r_α and r_{he} agree with e^--scattering (preliminary)
- New insights into charge radius difference.
- helion $\leftrightarrow$ triton: first of the two mirror nuclei measured.

- more experiments to come: H(2S-4P), H(2S-6P), H(2S-2P), MUSE, He^+, ISR, PRAD, μp(HFS), μ^3He^+(HFS) and many more

Thank you for your attention!