



QCD with Zoltan

Giulia Zanderighi

Max Planck Institute for Physics & Technische Universität München



Symposium in the honour of Zoltan Kunszt's 80th birthday

Plan



- We are here to celebrate **Zoltan's 80th birthday**. Approximately, this means more than 50 years of physics!
- In fact, first papers of Zoltan indeed appeared in the late 60s. Very influential papers with Riccardo Barbieri very early on
- Aim of this talk: look back at the incredible progress done in the past 50 years in pQCD and at Zoltan's role in this endeavour

The talk will revolve around the dichotomy: what has changed dramatically in this time and what has remained essentially the same

Early stages of QCD

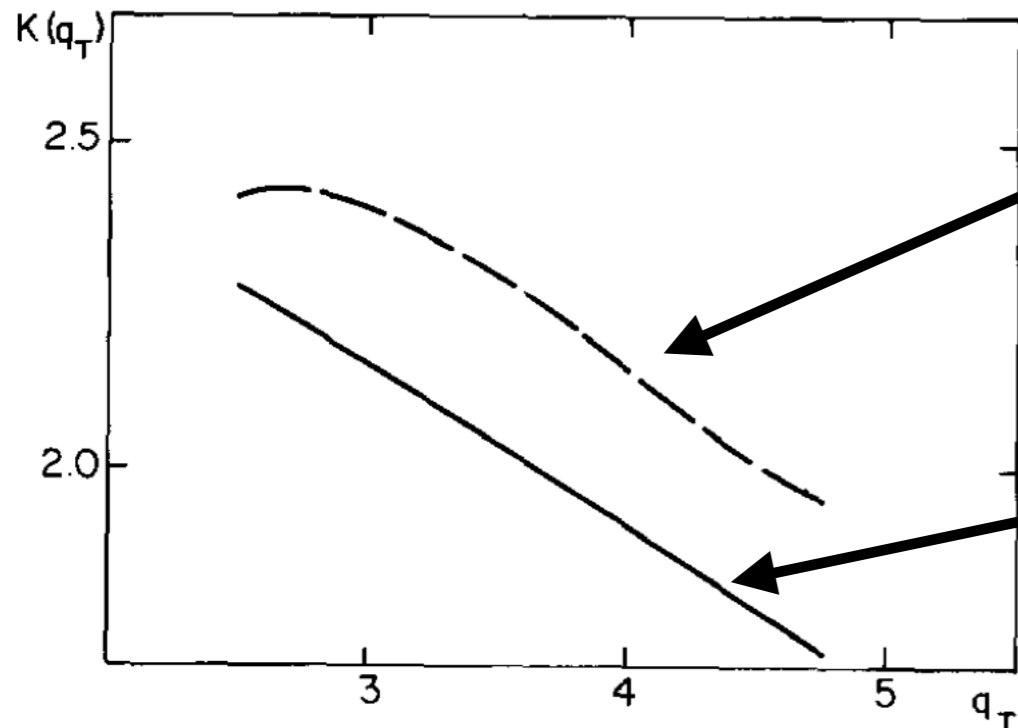
Perturbative calculations:

- Calculations done mostly analytically by hand (no big computer at the time)
- First few phenomenological milestones:
 - ▶ NLO calculation of three-jet production in e^+e^- collisions
Ellis, Ross, Terrano '80
 - ▶ Drell-Yan at NLO
Altarelli; Ellis, Martinelli, Petronzio '81

NB: Calculations already done in $\overline{\text{MS}}$ scheme using dimensional regularisation

Z+0 jets

K-factor for Z-boson transverse momentum



With
intrinsic k_t

Without
intrinsic k_t

Accurate modelling of
intrinsic k_t extremely relevant
today (see e.g. W-mass
measurement from transverse
momentum distributions)

$$K(q_T) = \left(\frac{1}{q_T} \frac{d\sigma}{dQ dq_T} \right) [O(\alpha_s) + O(\alpha_s^2)] /$$

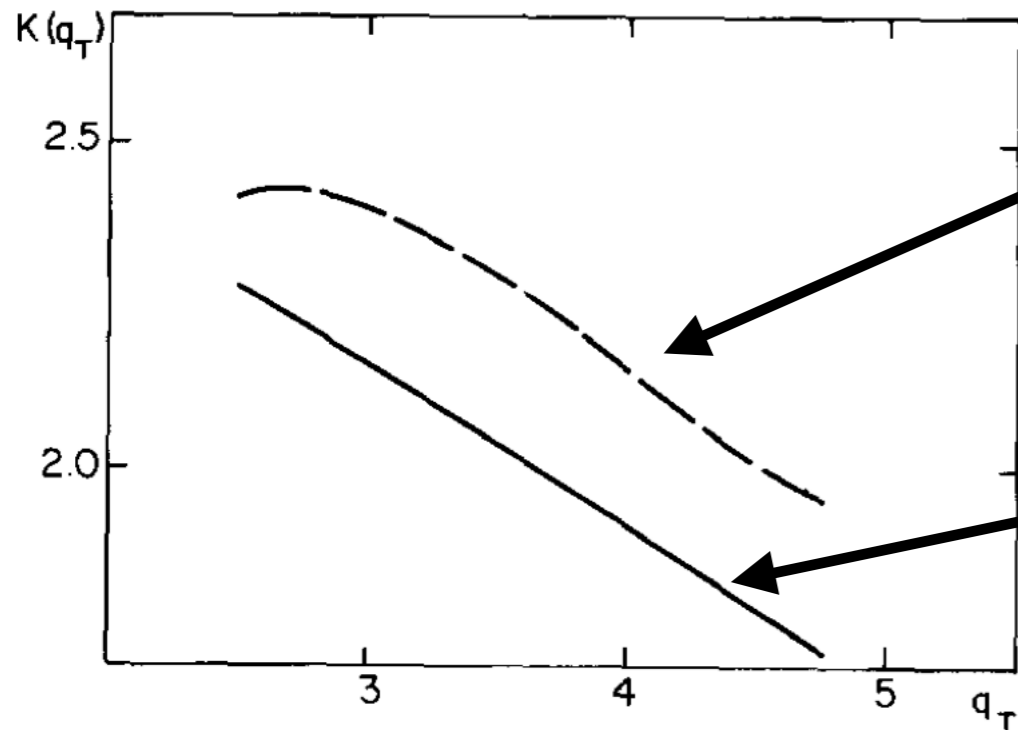
$$\left(\frac{1}{q_T} \frac{d\sigma}{dQ dq_T} \right) [O(\alpha_s)] ,$$

K-factors
ubiquitous
today

Ellis, Martinelli, Petronzio '81

Z+0 jets

K-factor for Z-boson transverse momentum



With
intrinsic k_t

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Accurate modelling of intrinsic k_t extremely relevant today (see e.g. W-mass measurement from transverse momentum distributions)

$$K(q_T) = \left(\frac{1}{q_T} \frac{d\sigma}{dQ dq_T} \right) [O(\alpha_s) + O(\alpha_s^2)] / \left(\frac{1}{q_T} \frac{d\sigma}{dQ dq_T} \right) [O(\alpha_s)]$$

K-factors ubiquitous today

SM@LHC2024

Intrinsic k_T model in generators

Intrinsic (primordial) k_T :
The transverse momenta of the partons in the incoming colliding hadrons
→ **Not calculable** in perturbative QCD
→ Described by **phenomenological models**

Free parameters to determine
In PYTHIA & HERWIG:
The intrinsic k_T is modelled by **Gaussian** distributions
→ **Width (σ)** of the distribution determined from **tuning to data**

PYTHIA parameter: $\sigma = \sqrt{2} * \text{BeamRemnants:primordialKTHard}$
HERWIG parameter: $\sigma = \text{ShowerHandler:IntrinsicPTGaussian}$

Intrinsic k_T + parton shower → $p_T(Z/\gamma)$
 $\sigma \uparrow$ → smears the intrinsic k_T → low $p_T(Z/\gamma)$ flattened

Intrinsic k_T tune to $p_T(Z/\gamma)$ has both **non-perturbative** & **perturbative** QCD effects

Fermi motion of partons, non-resolvable gluon emissions...

parton shower models

2

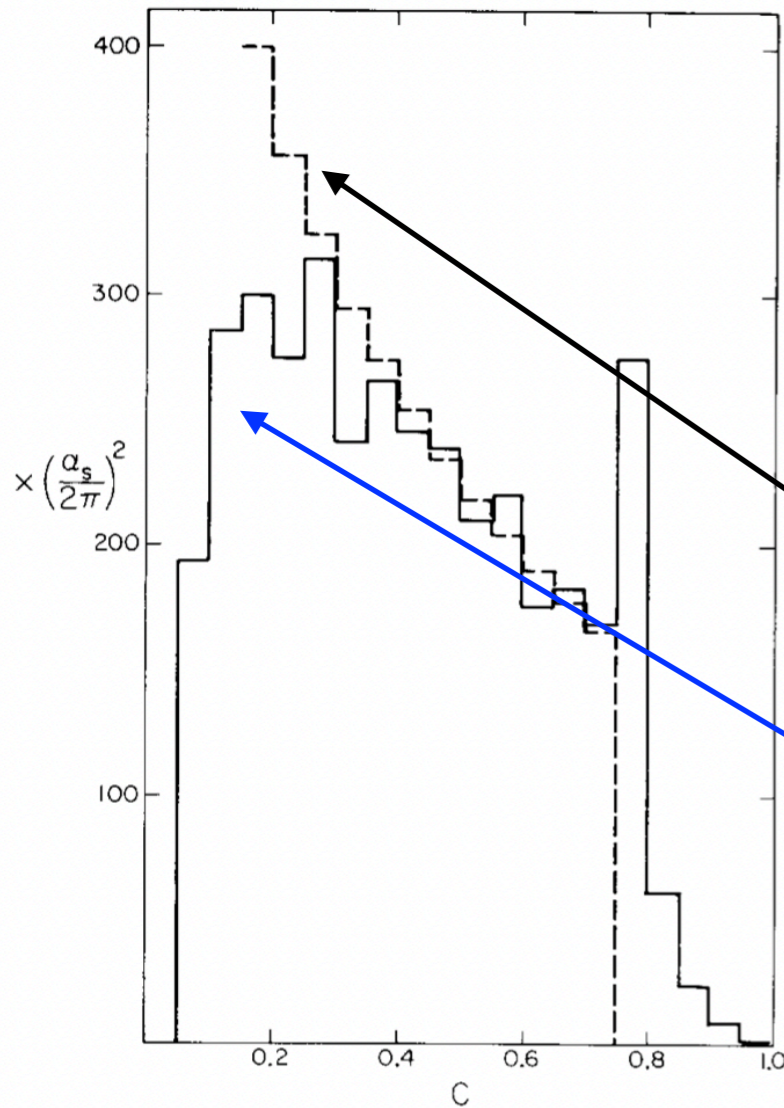
C-parameter in e^+e^-

R. K. Ellis et al. / Jet structure

Calculation of Event-Shape Parameters in e^+e^- Annihilation

R. K. Ellis, D. A. Ross, and A. E. Terrano
 Phys. Rev. Lett. **45**, 1226 – Published 13 October 1980

Received 2 July 1980



In eq. (2.24) we noted the appearance of π^2 terms related to the soft singularity. Since the soft logarithms exponentiate it is presumably true that the π^2 terms also exponentiate. Under this assumption eq. (4.5) becomes

$$\frac{1}{\sigma} \int_{1/2}^1 \frac{d\sigma}{dC} dC \sim \exp\left(\frac{\alpha_s(Q^2)}{2\pi} (C_F + \frac{1}{2}N_C)\pi^2\right) 2.8 \left(\frac{\alpha_s(Q^2)}{2\pi}\right) \left(1 + 9\frac{\alpha_s(Q^2)}{2\pi}\right).$$

NLO

Many debates on π^2 resummation even today

MACSYMA still used by Keith today!

Ellis, Ross, Terrano '80

We are pleased to acknowledge useful discussions with R. P. Feynman, R. D. Field, T. Goldman, Z. Kunszt, H. D. Politzer, and S. Wolfram. We thank the MATHLAB at MIT for the use of MACSYMA.

COMMENT ON THE $O(\alpha_s^2)$ CORRECTIONS TO JET-PRODUCTION IN e^+e^- ANNIHILATION

Zoltán KUNSZT

*Deutsches Elektronen-Synchrotron DESY, Hamburg, Germany
and L. Eötvös University, Budapest, Hungary*

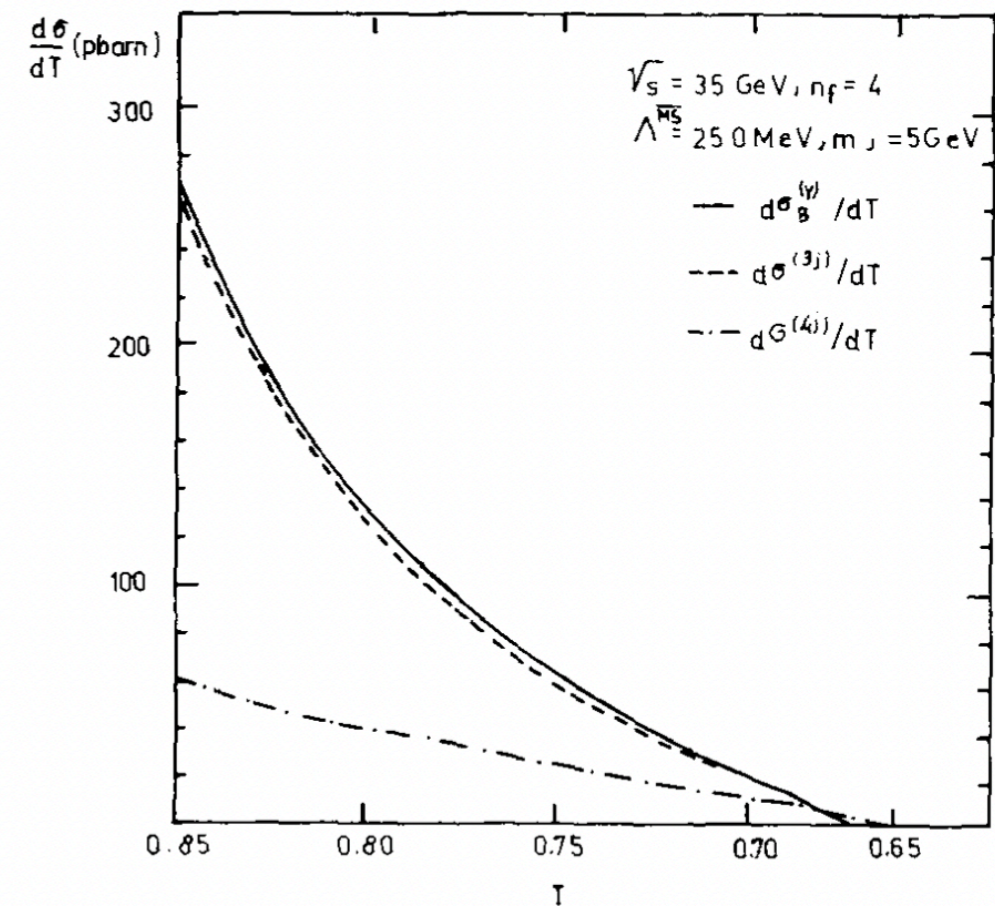
Received 28 August 1980

Using recent results of Ellis et al. I calculate the $O(\alpha_s^2)$ corrections to thrust distribution in e^+e^- annihilation. The numerical importance of the change of the four-momentum squared which determines the strength of the running coupling constant is studied in detail.

In ref. [1], however, all the important details are published so the above mentioned minor shortcomings can be easily eliminated. First of all with a slight modification of the pole terms (which have been subtracted from the four-jet matrix elements to regularize the infrared and mass singularities), any distribution can be calculated with high accuracy.

The same modification leads to formulae where jet fragmentation can be trivially introduced.

Many debates on choice of the scale in the running coupling even today



I am pleased to acknowledge illuminating discussions with R.K. Ellis and T. Walsh.

COMMENT ON THE $O(\alpha_s^2)$ CORRECTIONS TO JET-PRODUCTION IN e^+e^- ANNIHILATION

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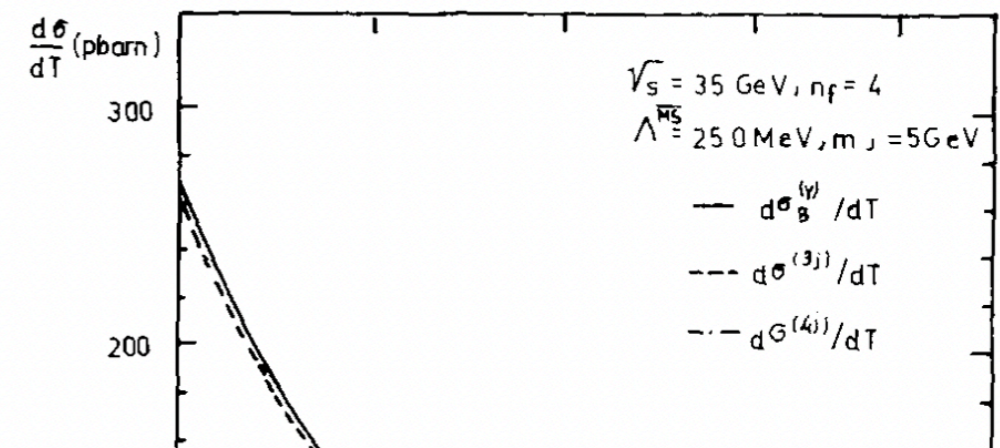
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Many debates on choice of the scale in the running coupling even today



Typical of Zoltan:

- ▶ Follow closely recent progress
- ▶ Find clever ways to improve on already very hard calculations and exploit them more widely, effectively contributing to improving theory predictions

I am pleased to acknowledge illuminating discussions with R.K. Ellis and T. Walsh.

10 years before the discovery of the top-quark ...

ASSOCIATED PRODUCTION OF HEAVY HIGGS BOSON WITH TOP QUARKS

Z. KUNSZT* **

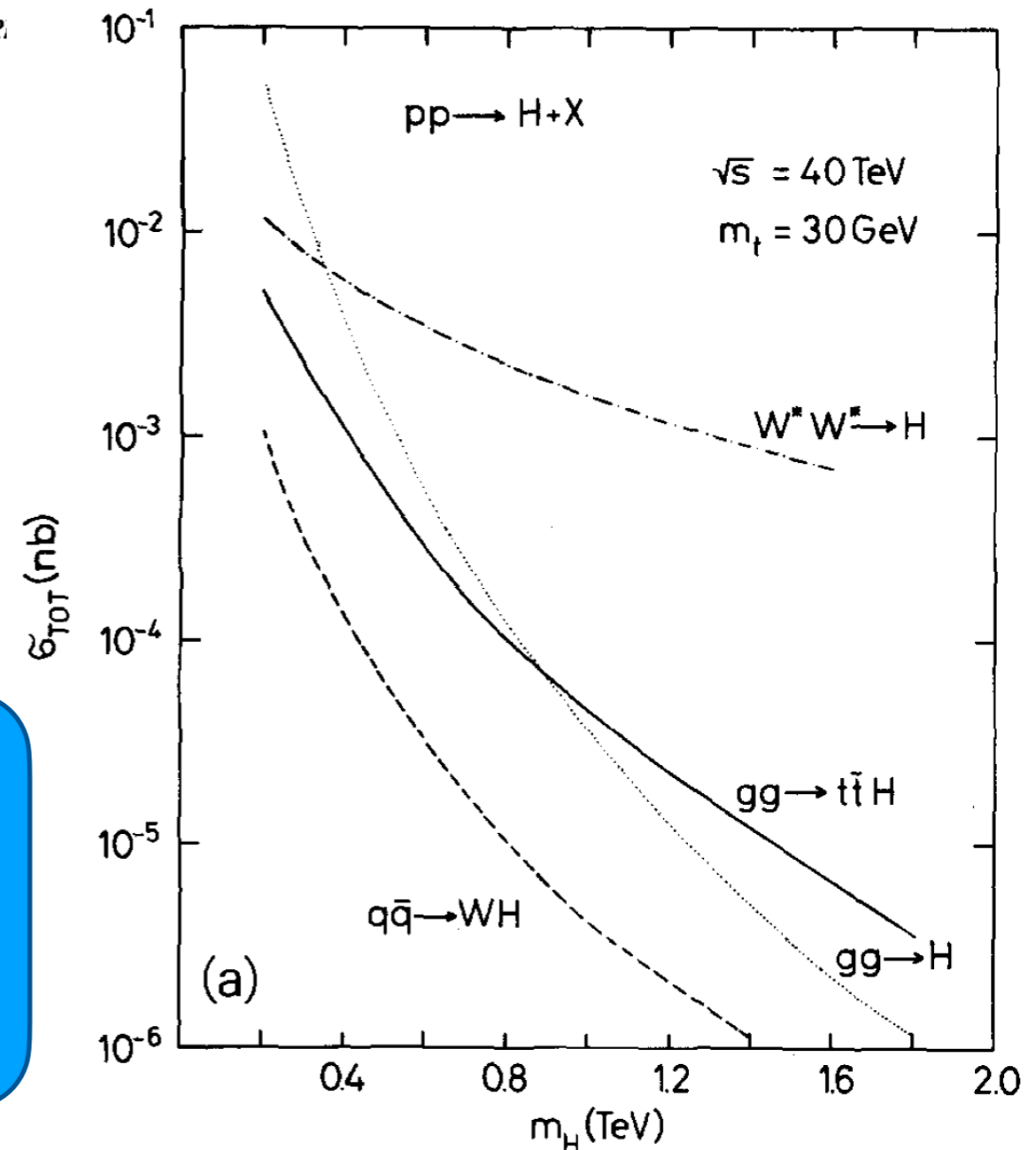
Institute for Theoretical Physics, University of Bern, Bern, Switze.

Received 13 June 1984

Typical of Zoltan:

- ▶ Close connection to phenomenology
- ▶ Focus of paper is on backgrounds as well

What I found however most astonishing: third solo paper I found of Zoltan during those years. I cannot imagine Zoltan working alone (without anybody to talk to!). Something must have changed over time!



IMPROVED ANALYTIC TECHNIQUES FOR TREE GRAPH CALCULATIONS AND THE $ggq\bar{q}\ell\bar{\ell}$ SUBPROCESS

J.F. GUNION

*Department of Physics, University of California at Davis, Davis, CA 95616, USA
and Institute for Theoretical Physics, University of Oregon, Eugene, OR 97403, USA*

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Received 14 June 1985

We demonstrate further improvements in the CALCUL approach to tree graph calculations. We employ polarization vectors referenced to a single momentum and obtain expressions in terms of spinor inner products. The method is crossing symmetric and can be easily implemented as an algebraic computer program. As an illustration we present an analytic calculation of the $ggq\bar{q}\ell\bar{\ell}$ subprocess in the standard model where $\ell\bar{\ell}$ can be in either the charged or the neutral channel.

- Paper pioneers the development of state-of-the art methods for the efficient calculation of LO high-multiplicity processes
- Use of “modern” computers to handle complex algebra

In conclusion we remark that numerical programs based on our expressions for the $ggq\bar{q}\ell\bar{\ell}$ process are both shorter and faster than those based on spin summed matrix elements squared computed by traditional trace techniques in terms of subprocess invariants. No sacrifice of numerical accuracy occurs; for example, the numerical gauge invariant check produces a result nineteen orders of magnitude smaller than the actual cross section (on an IBM 3081). Thus a wide variety of calculations involving massless external fermions may be usefully and economically performed using the techniques of this paper.

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- Use of “modern” computers to handle complex algebra

Typical of Zoltan:

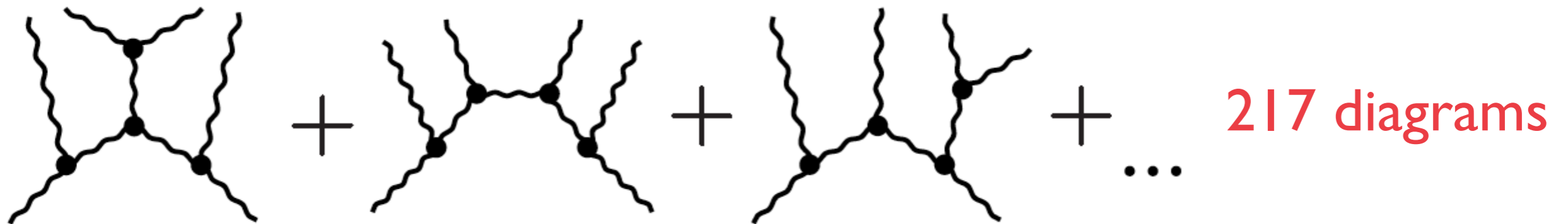
- ▶ Seek simplicity and analytic elegance
- ▶ Practical attitude: shorter and faster means more applications

In conclusion we remark that numerical programs based on our expressions for the $ggq\bar{q}\ell\bar{\ell}$ process are both shorter and faster than those based on spin summed matrix elements squared computed by traditional trace techniques in terms of subprocess invariants. No sacrifice of numerical accuracy occurs; for example, the numerical gauge invariant check produces a result nineteen orders of magnitude smaller than the actual cross section (on an IBM 3081). Thus a wide variety of calculations involving massless external fermions may be usefully and economically performed using the techniques of this paper.

$$gg \rightarrow gggg$$

Consider the amplitude for two gluons to collide and produce four gluons: $gg \rightarrow gggg$

Before modern computers, this would have been barely tractable even at leading order (LO)



gg → gggg

background to the detection of W^+W^- pairs in their nonleptonic decays. The cross sections for the elementary two→four processes have not been calculated, and their complexity is such that they may not be evaluated in the foreseeable future. It is worthwhile to seek estimates of the four-jet cross sections, even if these are only reliable in restricted regions of phase space.

Supercollider physics

(1984)

E. Eichten

Fermi National Accelerator Laboratory, P.O. Box 500, Batavia, Illinois 60510

I. Hinchliffe

Lawrence Berkeley Laboratory, Berkeley, California 94720

K. Lane

The Ohio State University, Columbus, Ohio 43210

C. Quigg

Fermi National Accelerator Laboratory, P.O. Box 500, Batavia, Illinois 60510

Eichten *et al.* summarize the motivation for exploring the 1-TeV ($=10^{12}$ eV) energy scale in elementary particle interactions and explore the capabilities of proton-(anti)proton colliders with beam energies between 1 and 50 TeV. The authors calculate the production rates and characteristics for a number of conventional processes, and discuss their intrinsic physics interest as well as their role as backgrounds to more exotic phenomena. The authors review the theoretical motivation and expected signatures for several new phenomena which may occur on the 1-TeV scale. Their results provide a reference point for the choice of machine parameters and for experiment design.

TeV. From Fig. 78 we find the corresponding two-jet cross section (at $p_{\perp}=0.5$ TeV/c) to be about 7×10^{-2} nb/GeV, which is larger by an order of magnitude. Let us next consider the cross section in the neighborhood of the peak in Fig. 102. The integrated cross section in the bin $0.3 \leq E_T \leq 0.4$ is approximately 0.1 nb/GeV, with transverse energy given roughly by $\langle E_T \rangle_{\text{bin}}(1 \text{ TeV}) \times \langle \cos\theta \rangle = 350$ GeV. The corresponding two-jet cross section, again from Fig. 78, is approximately 0.1 nb/GeV, which is larger by 2 orders of magnitude. In fact, we have certainly underestimated $\langle E_T \rangle$ and thus somewhat overestimated the two-jet/three-jet ratio in this second case.

We draw two conclusions from this very casual analysis:

At least at small-to-medium values of E_T , two-jet events should dominate for most of the cross section.

The three-jet cross section is large enough that a detailed study of this topology should be possible.

$$\sigma_4(E_T) = \int_{E_T}^{E_T+\epsilon} dE_{T1} \int_{E_T}^{E_T-\epsilon} dE_{T2} \frac{\sigma_2(E_{T1})\sigma_2(E_{T2})\delta(E_{T1}+E_{T2}-E_T)}{\sigma_{\text{total}}} \quad (3.47)$$

where $\sigma_2(E_{T1})$ is the two-jet cross section and ϵ denotes the minimum E_T required for a discernable two-jet event. For a recent study of double parton scattering at SppS and Tevatron energies, see Passar and Treleani (1983).

In view of the promise that multijet spectroscopy holds, improving our understanding of the QCD background is an urgent priority for further study.

D. Summary

We conclude this section with a brief summary of the ranges of jet energy which are accessible for various beam energies and luminosities. We find essentially no differences between pp and $p\bar{p}$ collisions, so only pp results will be given except at $\sqrt{s}=2$ TeV where $p\bar{p}$ rates are quoted. Figure 104 shows the E_T range which can be explored at the level of at least one event per GeV of E_T per unit rapidity at 90° in the c.m. (compare Figs. 77-79 and 83). The results are presented in terms of the transverse energy per event E_T , which corresponds to twice the transverse momentum p_{\perp} of a jet. In Fig. 105 we plot the values of E_T that distinguish the regimes in which the two-gluon, quark-gluon, and quark-quark final states are dominant. Comparing with Fig. 104, we find that while the accessible ranges of E_T are impressive, it seems extremely difficult to obtain a clean sample of quark jets. Useful for estimating trigger rates is the total cross section for two jets integrated over $E_T(-2p_{\perp}) > E_{T0}$ for both jets in a rapidity interval of -2.5 to $+2.5$. This is shown for pp collisions in Fig. 106.

It is apparent that these questions are amenable to detailed investigation with the aid of realistic Monte Carlo simulations. Given the elementary two→three cross sections and reasonable parametrizations of the fragmentation functions, this exercise can be carried out with some degree of confidence.

For multijet events containing more than three jets, the theoretical situation is considerably more primitive. A specific question of interest concerns the QCD four-jet background to the detection of W^+W^- pairs in their nonleptonic decays. The cross sections for the elementary two→four processes have not been calculated, and their complexity is such that they may not be evaluated in the foreseeable future. It is worthwhile to seek estimates of the four-jet cross sections, even if these are only reliable in restricted regions of phase space.

Another background source of four-jet events is double parton scattering, as shown in Fig. 103. If all the parton momentum fractions are small, the two interactions may be treated as uncorrelated. The resulting four-jet cross section with transverse energy E_T may then be approximated by

IV. ELECTROWEAK PHENOMENA

In this section we discuss the supercollider processes associated with the standard model of the weak and electromagnetic interactions (Glashow, 1961; Weinberg, 1967; Salam, 1968). By "standard model" we understand the $SU(2)_L \otimes U(1)_Y$ theory applied to three quark and lepton doublets, and with the gauge symmetry broken by a single complex Higgs doublet. The particles associated with the electroweak interactions are therefore the (left-handed) charged intermediate bosons W^\pm , the neutral intermedi-

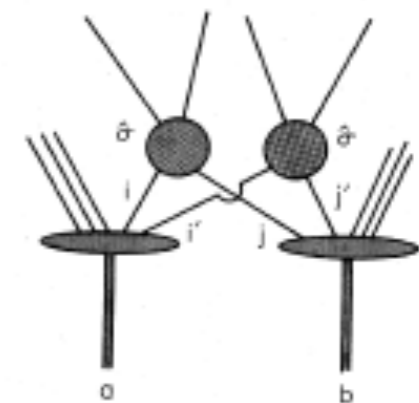


FIG. 103. Four-jet topology arising from two independent parton interactions.

gg → gggg

In 1985 Parke and Taylor took up the challenge, using

- ✓ the most advanced theoretical tools available
- ✓ the world best computers

they produced a final formula that would fit in **8 pages**

**THE CROSS SECTION FOR FOUR-GLUON PRODUCTION
BY GLUON-GLUON FUSION**

Stephen J. PARKE and T.R. TAYLOR

Fermi National Accelerator Laboratory, P.O. Box 500, Batavia, IL 60510 USA

Received 13 September 1985

The cross section for two-gluon to four-gluon scattering is given in a form suitable for fast numerical calculations.

Parke-Taylor

420

S.J. Parke, T.R. Taylor / Four gluon production

of our calculation, the most powerful test does not rely on the gauge symmetry, but on the appropriate permutation symmetries. The function $A_0(p_1, p_2, p_3, p_4, p_5, p_6)$ must be symmetric under arbitrary permutations of the momenta (p_1, p_2, p_3) and separately, (p_4, p_5, p_6) , whereas the function $A_1(p_1, p_2, p_3, p_4, p_5, p_6)$ must be symmetric under the permutations of (p_1, p_2, p_3, p_4) and separately, (p_5, p_6) . This test is extremely powerful, because the required permutation symmetries are hidden in our supersymmetry relations, eqs. (1) and (3), and in the structure of amplitudes involving different species of particles. Another, very important test relies on the absence of the double poles of the form $(s_{ij})^{-2}$ in the cross section, required by general arguments based on the helicity conservation. Further, in the leading $(s_{ij})^{-1}$ pole approximation, the answer should reduce to the two to three cross section [3, 4], convoluted with the appropriate Altarelli-Parisi probabilities [5]. Our result has successfully passed both these numerical checks.

Details of the calculation, together with a full exposition of our techniques, will be given in a forthcoming article. Furthermore, we hope to obtain a simple analytic form for the answer, making our result not only an experimentalist's, but also a theorist's delight.

We thank Keith Ellis, Chris Quigg and especially, Estia Eichten for many useful discussions and encouragement during the course of this work. We acknowledge the hospitality of Aspen Center for Physics, where this work was being completed in a pleasant, strung-out atmosphere.

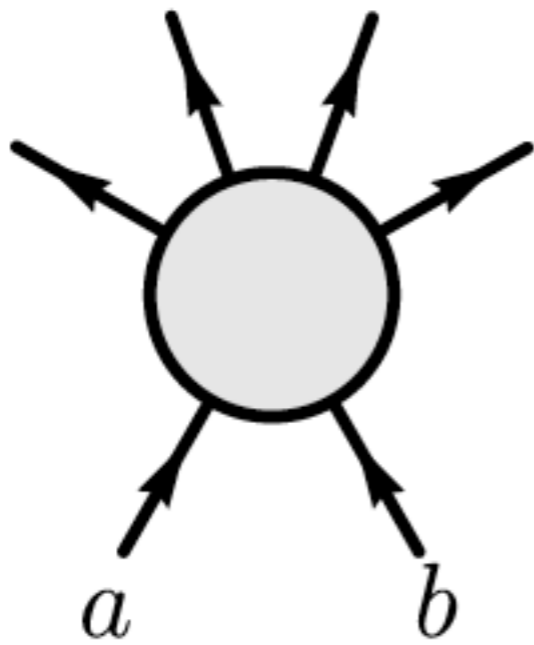
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- [1] E. Eichten, I. Hinchliffe, K. Lane and C. Quigg, *Rev. Mod. Phys.* 56 (1984) 579
- [2] Z. Kunszt, *Nucl. Phys.* B247 (1986) 339
- [3] S.J. Parke and T.R. Taylor, *Phys. Lett.* 157B (1985) 81
- [4] T. Gottsche and D. Sivers, *Phys. Rev.* D21 (1980) 302;
F.A. Berends, R. Kleiss, P. de Causmaecker, R. Gastmans and T.T. Wu, *Phys. Lett.* 103B (1981) 124
- [5] G. Altarelli and G. Parisi, *Nucl. Phys.* B126 (1977) 298

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Game-changer

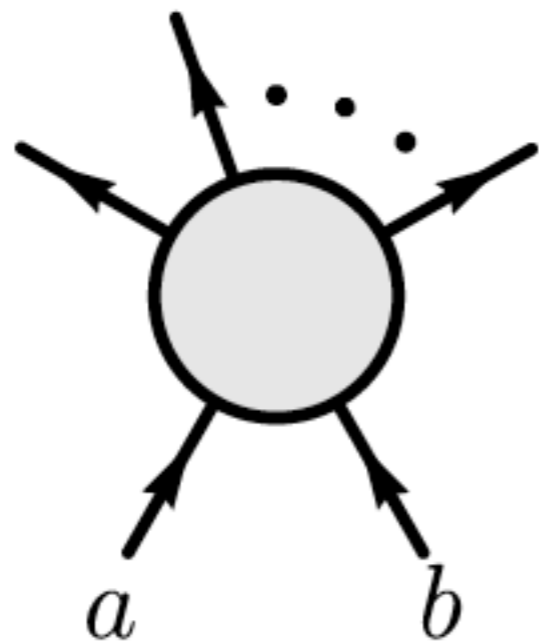
Soon afterwards they could guess an incredible, unanticipated simple form (for a fixed helicity configuration) ...


$$= \frac{\langle a b \rangle^4}{\langle 1 2 \rangle \langle 2 3 \rangle \langle 3 4 \rangle \langle 4 5 \rangle \langle 5 6 \rangle \langle 6 1 \rangle}$$

Game-changer

... which naturally suggested the result for an arbitrary number of gluons

Parke and Taylor, Phys. Rev. Lett 56 (1986) 2459



$$= \frac{\langle ab \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 45 \rangle \cdots \langle n1 \rangle}$$

The surprise about this result is that all denominators are simple dot products of two momenta. The Feynman diagrams for $n (> 5)$ gluon scattering contain propagators $(p_i + p_j + p_k)^2$, $(p_i + p_j + p_k + p_m)^2$, These propagators must cancel for eqn(3) to be correct; this occurs for $n=6$. Of course, Altarelli and Parisi have taught us that many cancellations are expected.

COMBINED USE OF THE CALKUL METHOD AND $N = 1$ SUPERSYMMETRY TO CALCULATE QCD SIX-PARTON PROCESSES

Z. KUNSZT¹

CERN, Geneva, Switzerland

Received 23 December 1985

Concise expressions are presented for the two independent helicity amplitudes of the subprocess $4g2q$. The result has been derived using the improved CALKUL method and is given in terms of spinor inner products in manifestly covariant and crossing symmetric forms. Changing the color factors of the quarks from the fundamental to the adjoint representation we obtain the helicity amplitudes of the four-gluon-two-gluino subprocess. Simple $N = 1$ supersymmetric relations have been found which express the helicity amplitudes of the six gluon parton process in terms of the helicity amplitudes of the $4g2\tilde{g}$ process. In this way we have avoided the direct calculation of 220 Feynman diagrams. Gauge invariance and the validity of the supersymmetric relations have been tested with an independent numerical calculation.

In order to check the results I also have calculated the amplitudes and cross sections for the $6g$, $4g2q$ and $4g2\tilde{g}$ subprocesses with a completely numerical program. This way I have checked gauge invariance and the validity of the supersymmetry relations. The numerical program is about 10 times slower than the Fortran program based on the formulae presented in this paper.

Phenomenological implications of the results of this paper will be discussed in a forthcoming publication [14]. Here I would like only to emphasize that we are finally in a position to carry out the phenomenological cross section calculations based on all subprocesses without resorting to any approximations.

Note added

After completing this work I have been informed by S. Parke and T. Taylor that they have also computed the amplitudes of the $4g2q$ subprocesses [15]. We have compared our results numerically for both the subprocesses involving six gluons and four gluons plus two quarks and we have found complete agreement. I thank S. Parke for correspondence. The six-gluon amplitude has been also calculated recently by the helicity method [16].

The One-Jet Inclusive Cross Section at Order α_s^3 : Quarks and Gluons

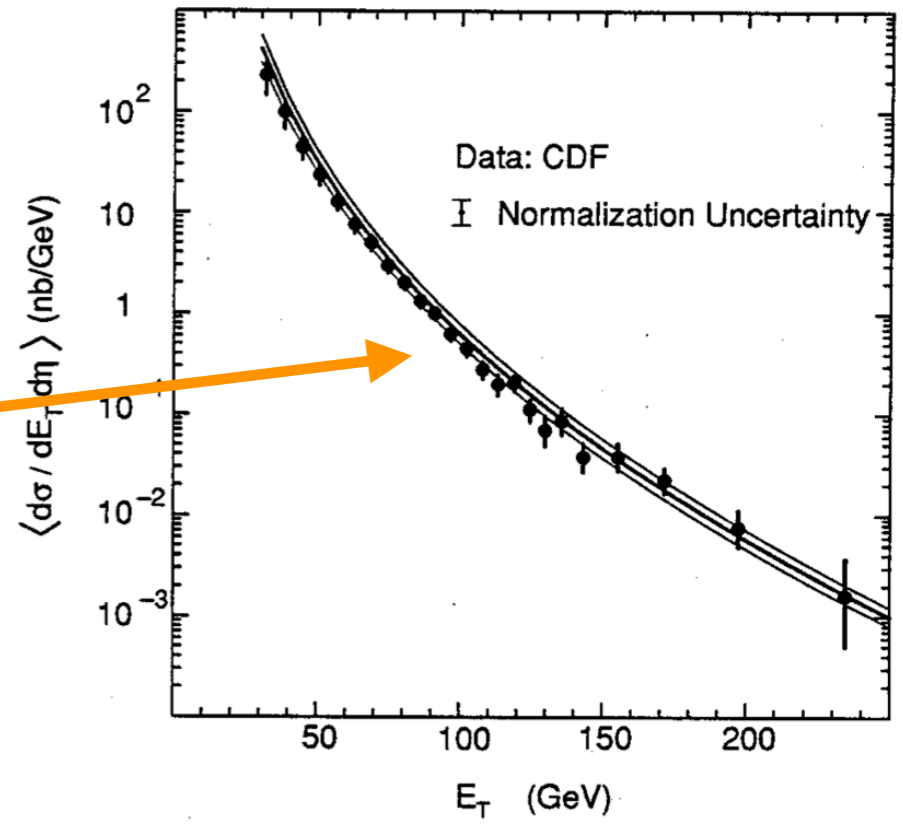
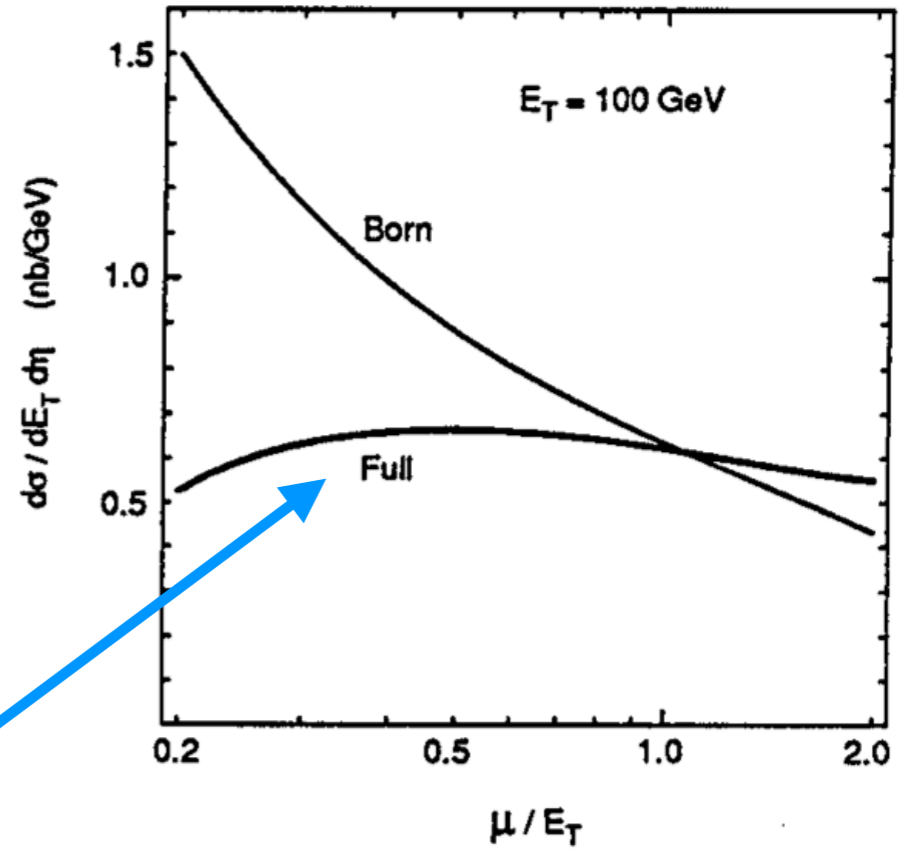
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Pioneering full NLO calculation
Characteristic shapes of LO and
NLO scales dependencies

Validation of NLO through comparison to data



NB: no k_t -algorithm, cone algorithm still used

New clustering algorithm for multijet cross sections in e^+e^- annihilation[☆]

S. Catani^{a,b,1}, Yu.L. Dokshitzer^{c,d}, M. Olsson^d, G. Turnock^a and B.R. Webber^a

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^c *Leningrad Nuclear Physics Institute, Gatchina, SU-188 350 Leningrad, USSR*

^d *Department of Theoretical Physics, University of Lund, Sölvegatan 14A, S-22362 Lund, Sweden*

$$y_{kl} = 2(1 - \cos \theta_{kl}) \min(E_k^2, E_l^2) / s,$$

Received 2 August 1991

Longitudinally-invariant k_{\perp} -clustering algorithms for hadron-hadron collisions^{*}

S. Catani^{**}

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Nuclear Physics Institute, Gatchina, 188 350 St. Petersburg, Russian Federation

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M.H. Seymour

Department of Theoretical Physics, University of Lund, Sölvegatan 14A, S-22362 Lund, Sweden

$$d_{kB} \approx E_k^2 \theta_{kB}^2 \approx k_{\perp kB}^2, \quad \text{for } \theta_{kB} \rightarrow 0,$$

$$d_{kl} \approx \min(E_k^2, E_l^2) \theta_{kl}^2 \approx k_{\perp kl}^2, \quad \text{for } \theta_{kl} \rightarrow 0.$$

B.R. Webber^{***}

Theory Division, CERN, CH-1211 Geneva 23, Switzerland

Received 9 March 1993

Accepted for publication 5 April 1993

New jet cluster algorithms: next-to-leading order QCD and hadronization corrections

S. Bethke

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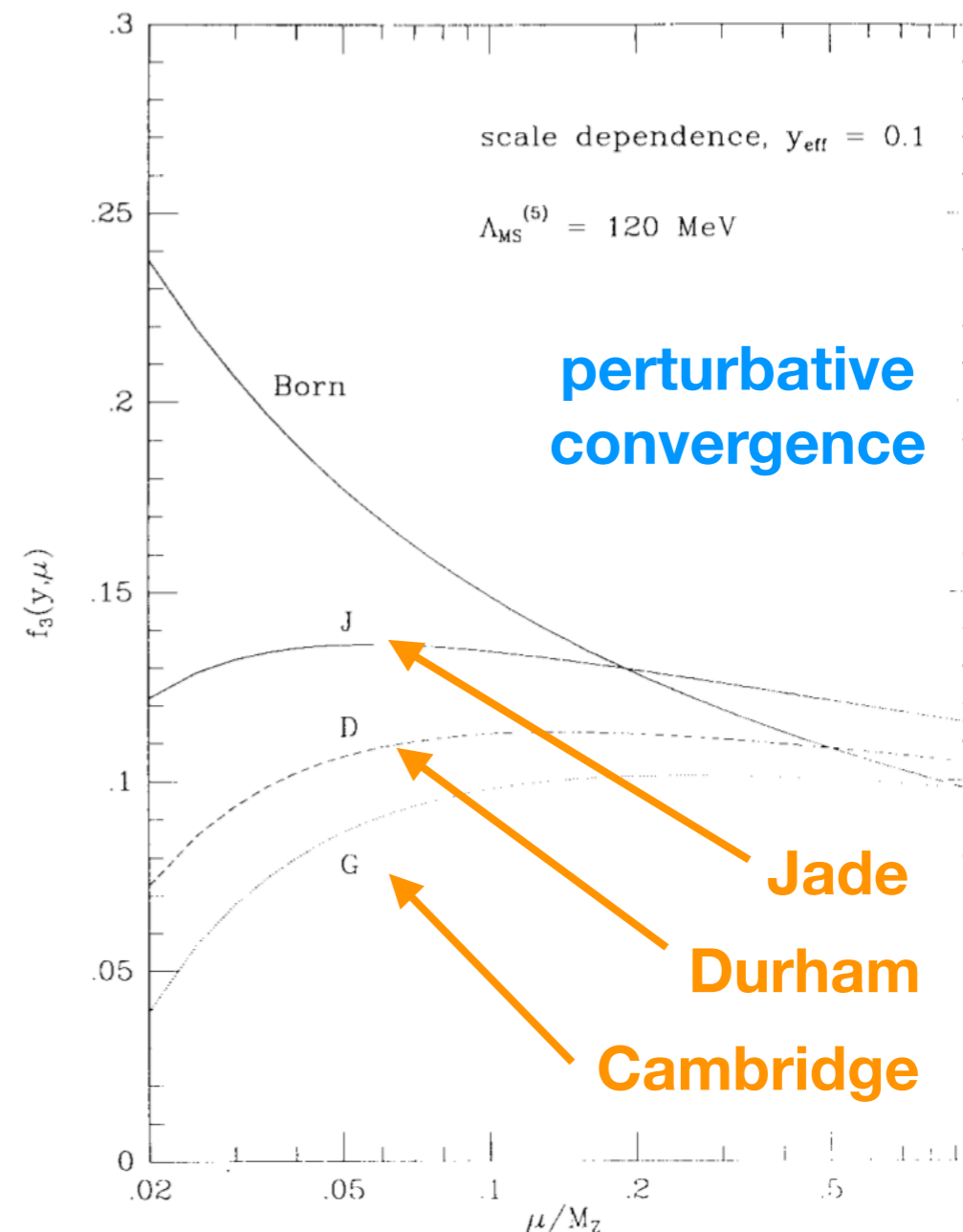
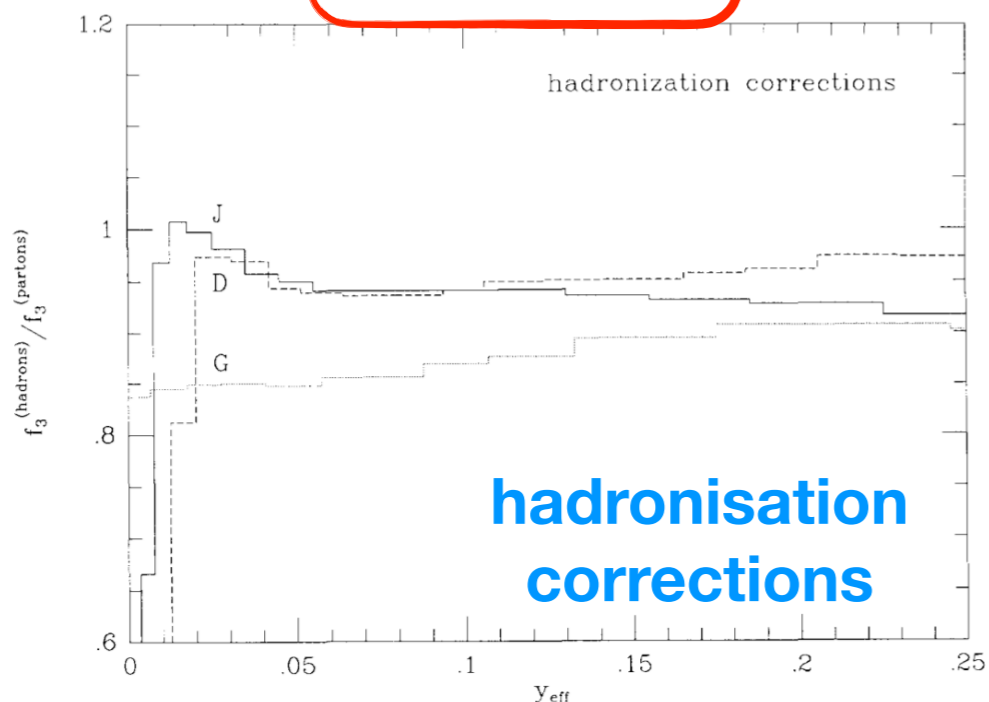
D.E. Soper

Institute of Theoretical Science, University of Oregon, Eugene, OR 97403, USA

W.J. Stirling *

TH Division, CERN, Geneva, Switzerland

Received 2 September 1991



Our study also raises some wider questions about the QCD perturbation theory.

...


With these issues of the behaviour of the perturbation series at large y , resummability at small y , and non-perturbative hadronization effects, there is clearly much interesting and important work still to be done in this area of jet physics.

Other “pheno” highlights of the 90s

Calculation of jet cross-sections in hadron collisions at order α_s^3

Zoltan Kunszt (Zurich, ETH), Davison E. Soper (Oregon U.) (Jan, 1992)

Published in: *Phys.Rev.D* 46 (1992) 192-221

 pdf  DOI  cite  claim  reference search  339 citations

Two jet production in hadron collisions at order α_s^3 in QCD

Stephen D. Ellis (Washington U., Seattle), Zoltan Kunszt (Zurich, ETH), Davison E. Soper (Oregon U.)

(Jul, 1992)

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
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Jets at hadron colliders at order α_s^3 : A Look inside

Stephen D. Ellis (Washington U., Seattle), Zoltan Kunszt (Zurich, ETH), Davison E. Soper (Oregon U.)

(Aug, 1992)



Published in: *Phys.Rev.Lett.* 69 (1992) 3615-3618 • e-Print: [hep-ph/9208249](https://arxiv.org/abs/hep-ph/9208249) [hep-ph]

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Higgs production with large transverse momentum in hadronic collisions at next-to-leading order

D. de Florian (Zurich, ETH), M. Grazzini (Zurich, ETH), Z. Kunszt (Zurich, ETH) (Feb, 1999)

Published in: *Phys.Rev.Lett.* 82 (1999) 5209-5212 • e-Print: [hep-ph/9902483](https://arxiv.org/abs/hep-ph/9902483) [hep-ph]

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“Amplitudes” highlights of the 90s

One loop radiative corrections to the helicity amplitudes of QCD processes involving four quarks and one gluon

Zoltan Kunszt (Zurich, ETH), Adrian Signer (Zurich, ETH), Zoltan Trocsanyi (Zurich, ETH and Kossuth Lajos U., Debrecen) (May, 1994)

Published in: *Phys.Lett.B* 336 (1994) 529-536 • e-Print: [hep-ph/9405386](#) [hep-ph]

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Singular terms of helicity amplitudes at one loop in QCD and the soft limit of the cross-sections of multiparton processes

Zoltan Kunszt (Zurich, ETH), Adrian Signer (Zurich, ETH), Zoltan Trocsanyi (Zurich, ETH) (Jan, 1994)

Published in: *Nucl.Phys.B* 420 (1994) 550-564 • e-Print: [hep-ph/9401294](#) [hep-ph]

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One loop helicity amplitudes for all $2 \rightarrow 2$ processes in QCD and N=1 supersymmetric Yang-Mills theory

Zoltan Kunszt (Zurich, ETH), Adrian Signer (Zurich, ETH), Zoltan Trocsanyi (Zurich, ETH) (May, 1993)

Published in: *Nucl.Phys.B* 411 (1994) 397-442 • e-Print: [hep-ph/9305239](#) [hep-ph]

Helicity amplitudes for $O(\alpha_s)$ production of W^+W^- , $W^\pm Z$, ZZ , $W^\pm\gamma$, or $Z\gamma$ pairs at hadron colliders

Lance J. Dixon (SLAC), Z. Kunszt (Zurich, ETH), A. Signer (CERN) (Mar, 1998)

Published in: *Nucl.Phys.B* 531 (1998) 3-23 • e-Print: [hep-ph/9803250](#) [hep-ph]

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The game-changer: FKS

Three jet cross-sections to next-to-leading order

S. Frixione (Zurich, ETH), Z. Kunszt (Zurich, ETH), A. Signer (SLAC) (Dec, 1995)

Published in: *Nucl.Phys.B* 467 (1996) 399-442 • e-Print: [hep-ph/9512328](https://arxiv.org/abs/hep-ph/9512328) [hep-ph]

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The subtraction method presented in the literature is based on a systematic use of boost-invariant kinematical variables, and therefore its application to three-jet production is quite cumbersome. In this paper we re-analyze the subtraction method and point out the advantage of using angle and energy variables. This leads to simpler results and it has complete generality, extending its validity to n -jet production. The formalism is also applicable to n -jet production in e^+e^- annihilation and in photon-hadron collisions. All the analytical results necessary to construct an efficient numerical program for next-to-leading order three-jet inclusive quantities in hadroproduction are given explicitly. As new analytical result, we also report the collinear limits of all the two-to-four processes.

The game-changer: FKS

Three jet cross-sections to next-to-leading order

S. Frixione (Zurich, ETH), Z. Kunszt (Zurich, ETH), A. Signer (SLAC) (Dec, 1995)

Published in: *Nucl.Phys.B* 467 (1996) 399-442 • e-Print: [hep-ph/9512328](https://arxiv.org/abs/hep-ph/9512328) [hep-ph]

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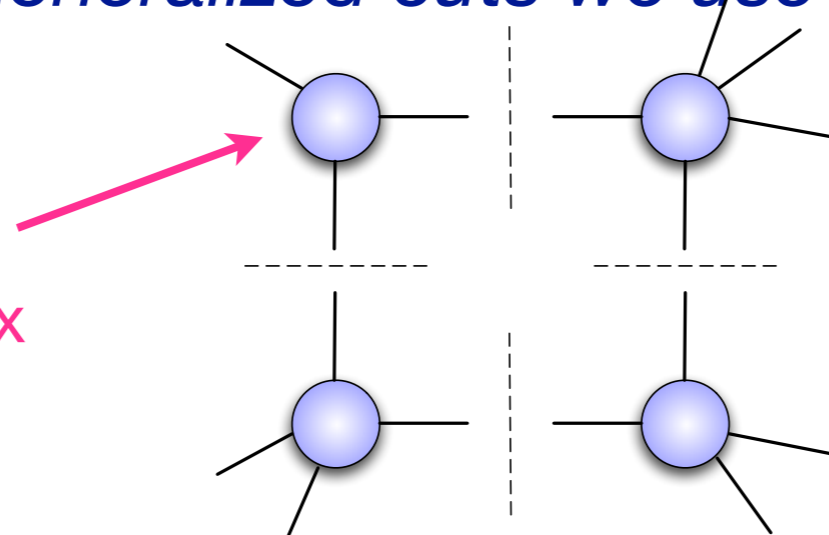
- 😊 Together with Catani-Seymour, one of the two subtraction methods used in virtually all NLO calculations
- 😊 Implemented e.g. in POWHEG, MC@NLO, ...
- 😊 Precursor of NNLO subtraction methods
- 😞 But ...

NLO revolution

Two breakthrough ideas:

1) “... we show how to use generalized unitarity to read off the (box) coefficients. The generalized cuts we use are quadrupole cuts ...”

NB: non-zero
because cut
gives complex
momenta



Britto, Cachazo, Feng '04

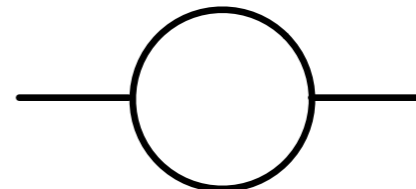
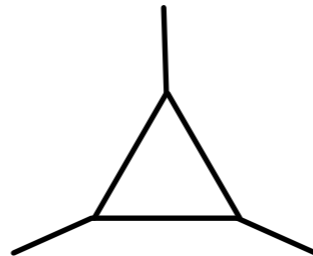
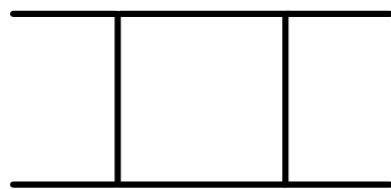
Quadrupole cuts: four on-shell conditions on four dimensional loop momentum freezes the integration. **Rational part** of the amplitude, computed separately

NLO revolution

Two breakthrough ideas:

2) *The OPP method: “We show how to extract the coefficients of 4-, 3-, 2- and 1-point one-loop scalar integrals....”*

$$\mathcal{A}_N = \sum_{[i_1|i_4]} \left(d_{i_1 i_2 i_3 i_4} I_{i_1 i_2 i_3 i_4}^{(D)} \right) + \sum_{[i_1|i_3]} \left(c_{i_1 i_2 i_3} I_{i_1 i_2 i_3}^{(D)} \right) + \sum_{[i_1|i_2]} \left(b_{i_1 i_2} I_{i_1 i_2}^{(D)} \right) + \mathcal{R}$$



Ossola, Pittau, Papadopolous '06

Coefficients can be determined by solving a system of equations

Highlights after 2000

With a series of seminal papers on D-dimensional unitarity, Zoltan played a key role in the development of the so-called NLO revolution

D-dimensional unitarity cut method

[Charalampos Anastasiou](#) (Zurich, ETH), [Ruth Britto](#) (Amsterdam U.), [Bo Feng](#) (Imperial Coll., London and Zhejiang U.), [Zoltan Kunszt](#) (Zurich, ETH), [Pierpaolo Mastrolia](#) (Zurich U.) (Sep, 2006)

Published in: *Phys.Lett.B* 645 (2007) 213-216 • e-Print: [hep-ph/0609191](#) [hep-ph]

 pdf  DOI  cite  claim  reference search  267 citations

Unitarity cuts and Reduction to master integrals in d dimensions for one-loop amplitudes

[Charalampos Anastasiou](#) (CERN), [Ruth Britto](#) (Amsterdam U.), [Bo Feng](#) (Imperial Coll., London and Zhejiang U.), [Zoltan Kunszt](#) (Zurich, ETH), [Pierpaolo Mastrolia](#) (Zurich U.) (Dec, 2006)

Published in: *JHEP* 03 (2007) 111 • e-Print: [hep-ph/0612277](#) [hep-ph]

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A Numerical Unitarity Formalism for Evaluating One-Loop Amplitudes

[R.Keith Ellis](#) (Fermilab), [W.T. Giele](#) (Fermilab), [Z. Kunszt](#) (Zurich, ETH) (Aug, 2007)

Published in: *JHEP* 03 (2008) 003 • e-Print: [0708.2398](#) [hep-ph]

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Highlights after 2000

With a series of seminal papers on D -dimensional unitarity, Zoltan played a key role in the development of the so-called NLO revolution

Full one-loop amplitudes from tree amplitudes

[Walter T. Giele \(Fermilab\)](#), [Zoltan Kunszt \(Zurich, ETH\)](#), [Kirill Melnikov \(Hawaii U.\)](#) (Jan, 2008)

Published in: *JHEP* 04 (2008) 049 • e-Print: [0801.2237](#) [hep-ph]

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Masses, fermions and generalized D -dimensional unitarity

[R.Keith Ellis \(Fermilab\)](#), [Walter T. Giele \(Fermilab\)](#), [Zoltan Kunszt \(Zurich, ETH\)](#), [Kirill Melnikov \(Hawaii U.\)](#) (Jun, 2008)

Published in: *Nucl.Phys.B* 822 (2009) 270-282 • e-Print: [0806.3467](#) [hep-ph]

 pdf  links  DOI  cite  claim  reference search  244 citations

One-loop amplitudes for W^+ 3 jet production in hadron collisions

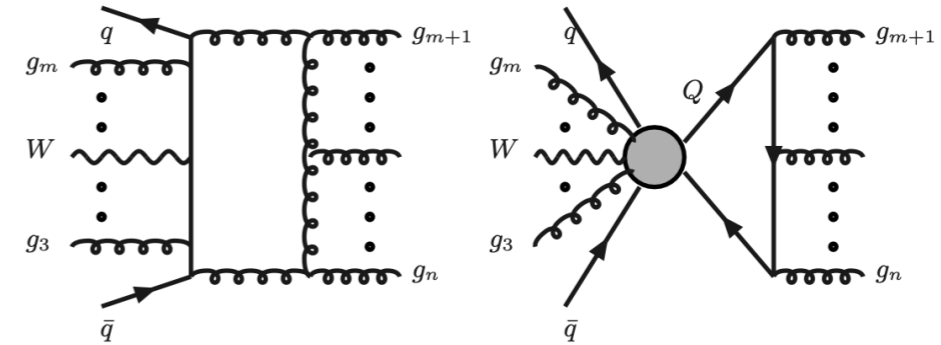
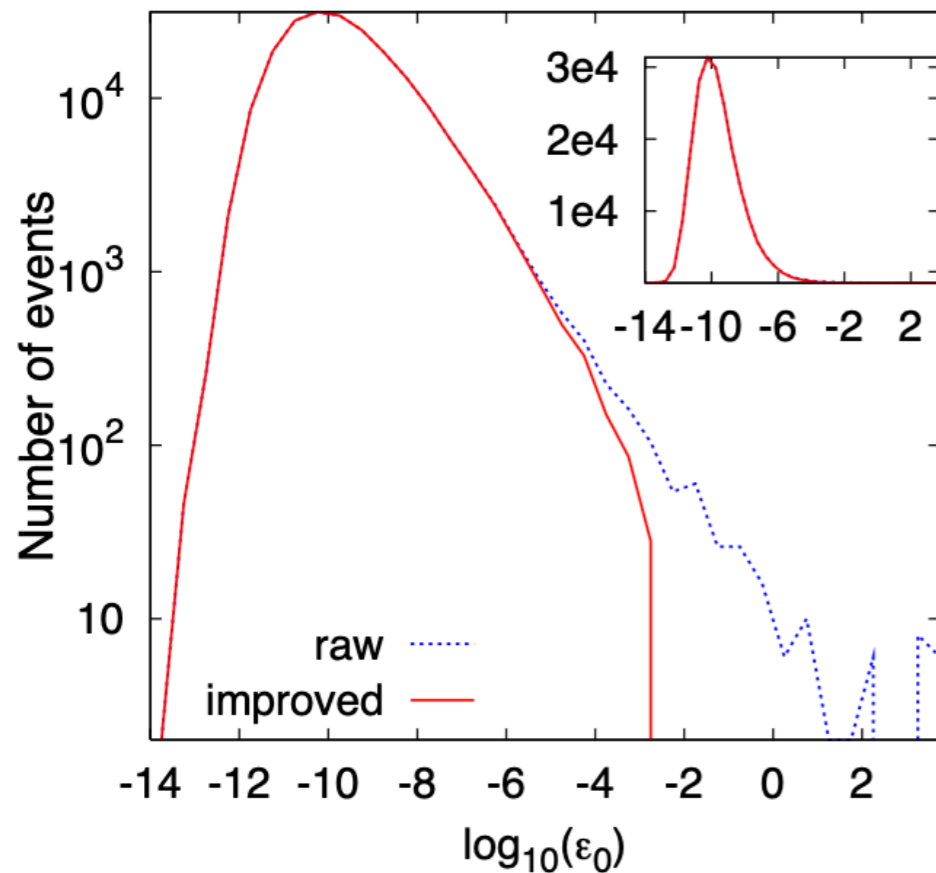
[R.Keith Ellis \(Fermilab\)](#), [W.T. Giele \(Fermilab\)](#), [Zoltan Kunszt \(Zurich, ETH\)](#), [Kirill Melnikov \(Johns Hopkins U.\)](#), [Giulia Zanderighi \(Oxford U., Theor. Phys.\)](#) (Oct, 2008)

Published in: *JHEP* 01 (2009) 012 • e-Print: [0810.2762](#) [hep-ph]

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W+3 jets

accuracy for one primitive amplitude



Helicity	$1/\epsilon^2$	$1/\epsilon$	ϵ^0
$A^{\text{tree}}(1_{\bar{q}}^+ 2_q^- 3_g^+ 4_g^+ 5_g^+ 6_l^+ 7_l^-)$			$-0.006873 + i 0.011728$
$r_L^{[1]}(1_{\bar{q}}^+ 2_q^- 3_g^+ 4_g^+ 5_g^+ 6_l^+ 7_l^-)$	-4.00000	$-10.439578 - i 9.424778$	$5.993700 - i 19.646278$
$A^{\text{tree}}(1_{\bar{q}}^+ 2_q^- 3_g^+ 4_g^+ 5_g^- 6_l^+ 7_l^-)$			$0.010248 - i 0.007726$
$r_L^{[1]}(1_{\bar{q}}^+ 2_q^- 3_g^+ 4_g^+ 5_g^- 6_l^+ 7_l^-)$	-4.00000	$-10.439578 - i 9.424778$	$-14.377555 - i 37.219716$
$A^{\text{tree}}(1_{\bar{q}}^+ 2_q^- 3_g^- 4_g^+ 5_g^+ 6_l^+ 7_l^-)$			$0.495774 - i 1.274796$
$r_L^{[1]}(1_{\bar{q}}^+ 2_q^- 3_g^- 4_g^+ 5_g^+ 6_l^+ 7_l^-)$	-4.00000	$-10.439578 - i 9.424778$	$-1.039489 - i 30.210418$
$A^{\text{tree}}(1_{\bar{q}}^+ 2_q^- 3_g^- 4_g^+ 5_g^- 6_l^+ 7_l^-)$			$-0.294256 - i 0.223277$
$r_L^{[1]}(1_{\bar{q}}^+ 2_q^- 3_g^- 4_g^+ 5_g^- 6_l^+ 7_l^-)$	-4.00000	$-10.439578 - i 9.424778$	$-1.444709 - i 26.101951$

Paper written in the phase when unitarity methods were still being tested and validated; all ingredients provided for the computation of the full NLO cross section

Review on unitarity

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⇒ **Very pedagogical, many worked out examples. Input and contributions of Zoltan absolutely crucial**

Conclusions

The field of perturbative computations for multi-particle processes went through a remarkable transformation in the past few years. During these years, the ability to perform specific computations that are of importance for the Tevatron and the LHC physics program has increased beyond the most optimistic expectations. The improvement in our understanding of perturbative quantum field theory – that is a byproduct of these exciting developments – gives us hope that the momentum of the past several years can be carried forward, so that even more complicated physics – both in terms of the number of external particles and in terms of the number of loops – can be addressed.

A few personal
recollections

Looking for pictures of Zoltan ...

Looking for pictures of Zoltan ...



Santa Barbara 2004!?

Santa Barbara 2004!?



Santa Barbara 2004!? Crazy times, but no pictures of Zoltan ...



Loopfest in Buffalo in 2006!!!



Zoltan's free time at Loopfest ...

EXCLUSIVE

Zoltan's free time at Loopfest ...











Happy
Birthday
Zoltan!