α_s from the top quark pair production cross section

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Outline

1. Motivation
2. Extracting α_s from a σ_{tt} measurement
3. Combining α_s extractions
4. Conclusion

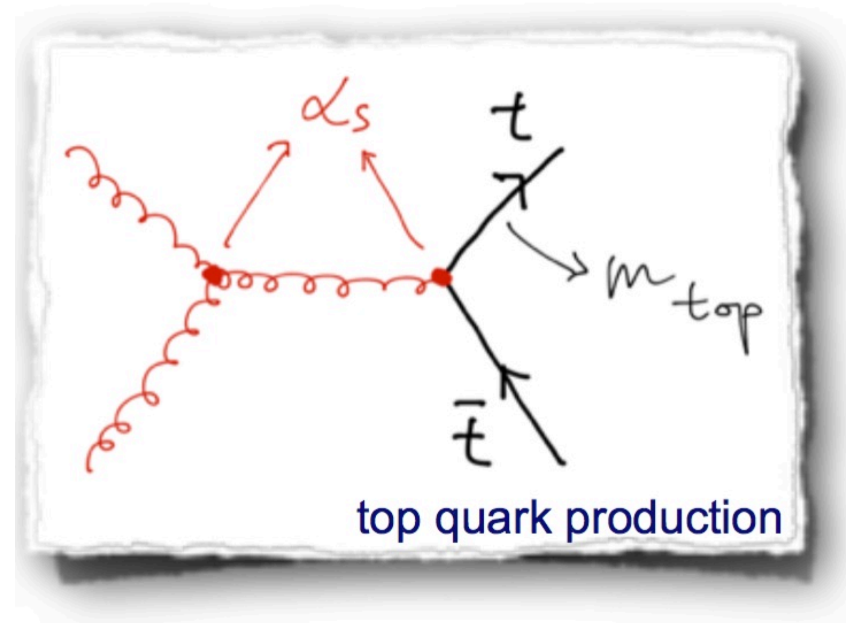
Why α_s?

- **Strong coupling constant α_s** enters in the calculation of every process that involves the strong interaction
    - Uncertainty on α_s leads to non-negligible uncertainties on many observables
        - Notable examples: Higgs production cross sections, branching ratios
- PDG world average (2015): **0.1181 ± 0.0013**; **~1.1%** relative uncertainty [http://pdg.lbl.gov/2015/reviews/rpp2015-rev-qcd.pdf]
    - Relative uncertainty of the fine structure constant: ~$2.3 \cdot 10^{-8}$% [http://physics.nist.gov/cgi-bin/cuu/Value?alph]

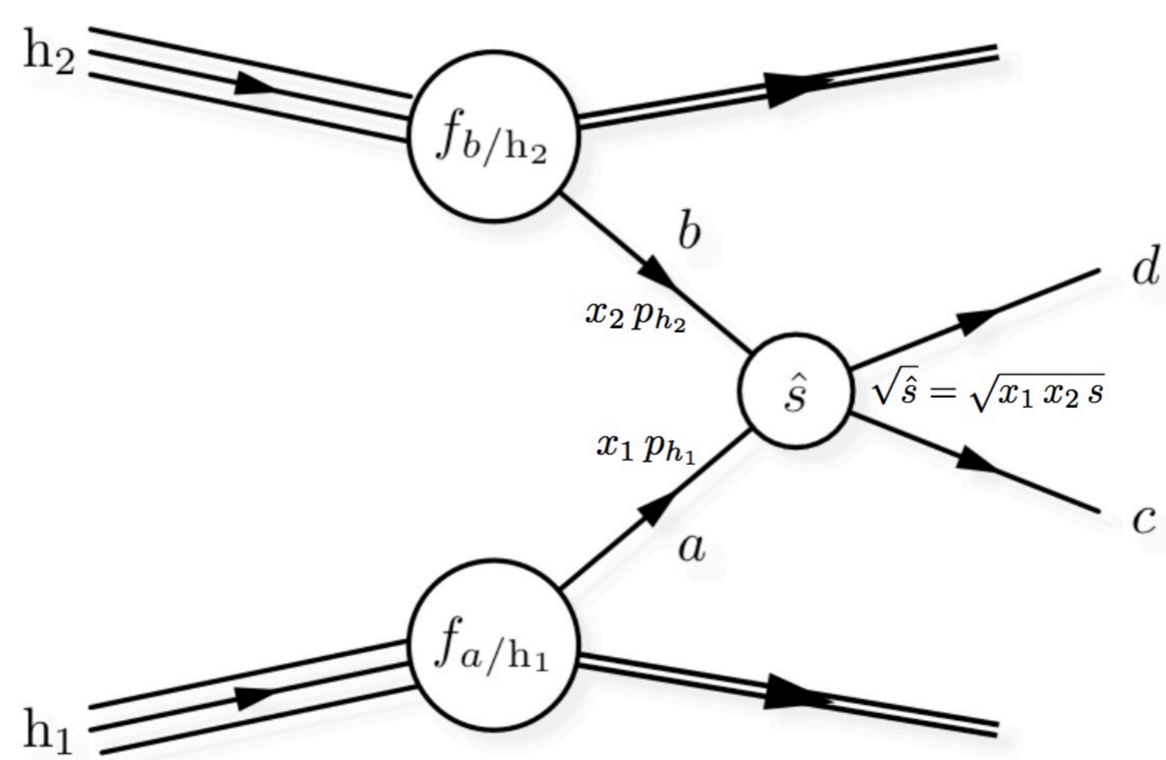
Why σ_{tt}?

σ_{tt} particularly sensitive to α_s

- α_s enters in the amplitude of the process (α_s^2 at LO)

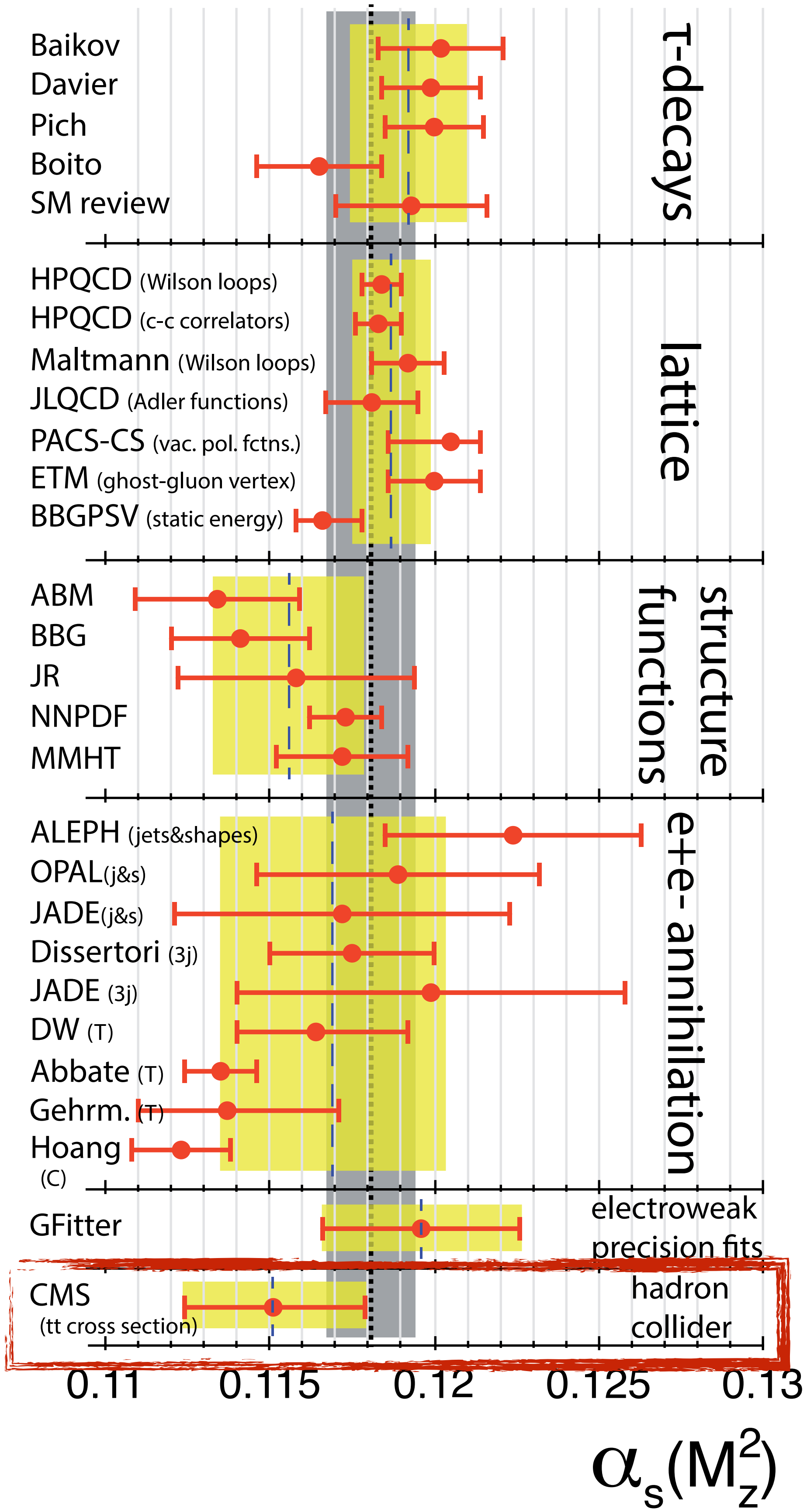


- α_s enters in the **parton distribution functions** (PDFs) through the DGLAP evolution equations



$$d\sigma(h_1 h_2 \to cd) = \int_0^1 dx_1 dx_2 \sum_{a,b} f_{a\,/\,h_1}(x_1, \mu_F^2, \alpha_s)\, f_{b\,/\,h_2}(x_2, \mu_F^2, \alpha_s)\; d\hat{\sigma}^{(ab\to cd)}(Q^2, \mu_F^2, \alpha_s)$$

Why σ_{tt}?



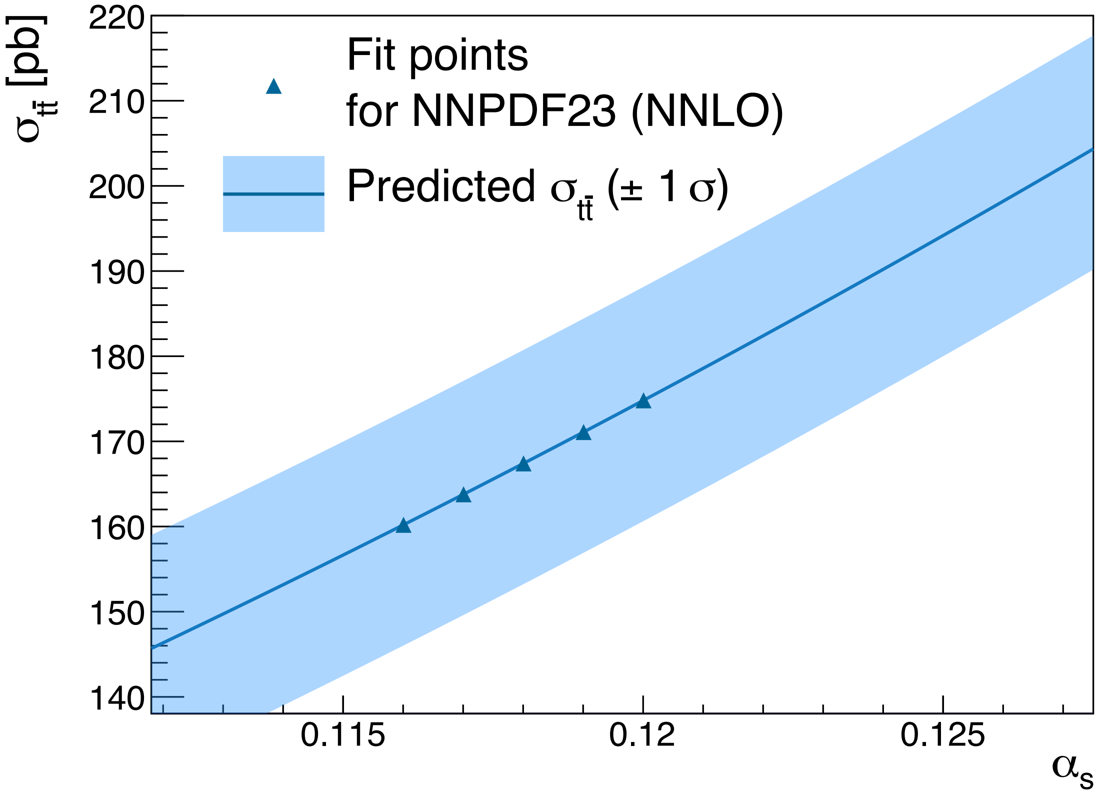
[http://pdg.lbl.gov/2015/reviews/rpp2015-rev-qcd.pdf]

- σ_{tt} is well-measured (known up to NNLO)
- Only few results from hadron colliders in the world average
- Currently one extraction like this available from CMS at 7 TeV [Phys. Let. B 728 (2014)]
  - Likely to be an underestimation (based on lower result for σ_{tt})
- New data available:
  - ATLAS at 7 TeV, 8 TeV and 13 TeV
    [Eur. Phys. J. C (2014) 74: 3109] [Phys. Lett. B761 (2016) 136]
  - CMS at 7 TeV, 8 TeV and 13 TeV
    [CMS-TOP-13-004] [CMS-TOP-16-005]
  - Tevatron (D0/CDF combination) at 1.96 TeV
    [Phys.Rev. D89, 072001 (2014)]

Extracting α_s from σ_{tt} measurements

Compare **theory** with **experiment**

- Theory dependence by fitting various evaluations of $\sigma_{tt}(\alpha_s)$ by `top++2.0` [Phys. Rev. Lett. 110, 252004]



Extracting α_s from σ_{tt} measurements

Compare theory with experiment

- Theory dependence by fitting various evaluations of $\sigma_{tt}(\alpha_s)$ by `top++2.0` [Phys. Rev. Lett. 110, 252004]

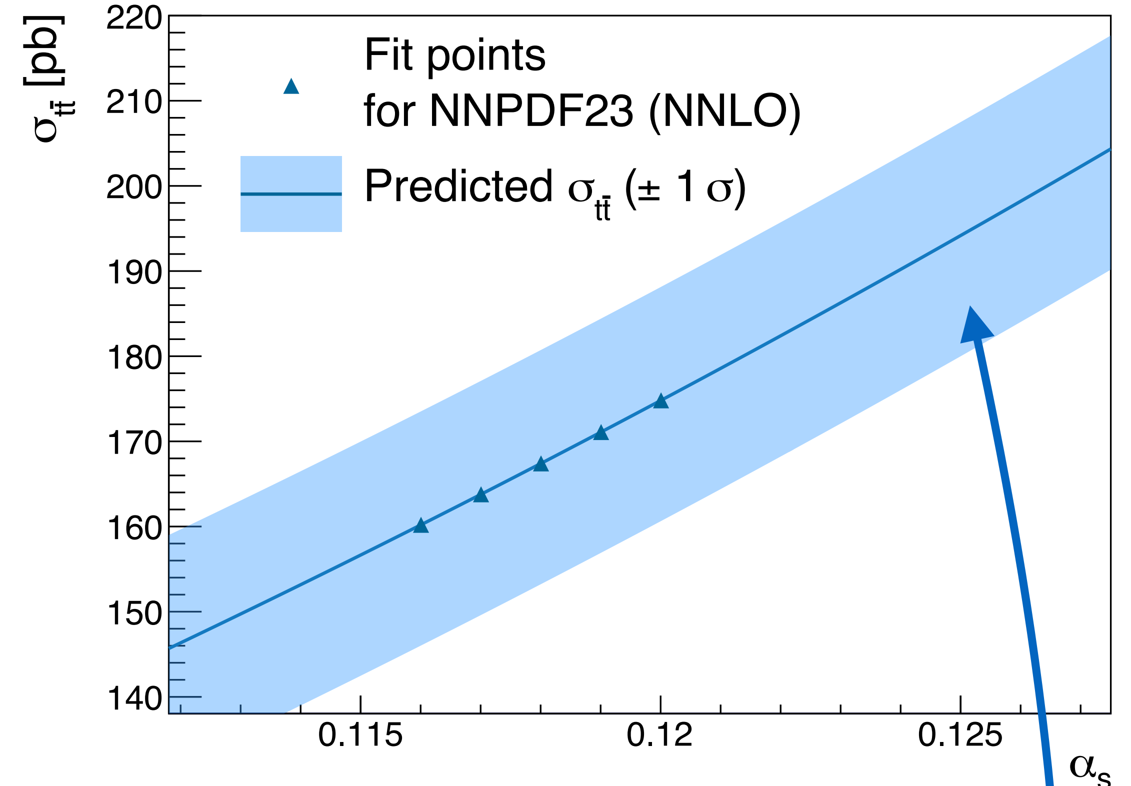
- Uncertainties from the **pdf**, **scale** and the **top mass** taken into account

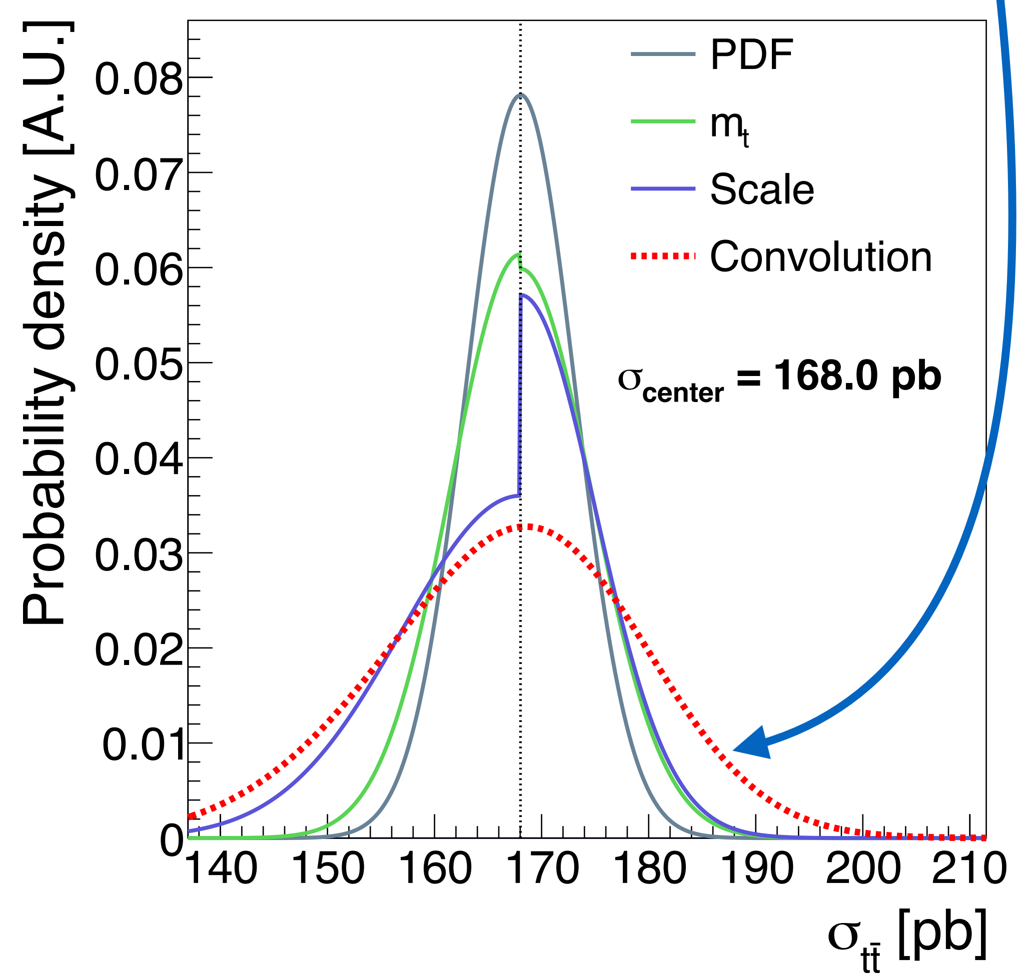
- Theory uncertainty composed of convoluted asymmetric Gaussians:

$$\text{Asym. Gauss.}(x) = \frac{1}{\sqrt{2\pi}\sigma_\pm} \exp\left(\frac{(x-\mu)^2}{\sigma_\pm^2}\right)$$

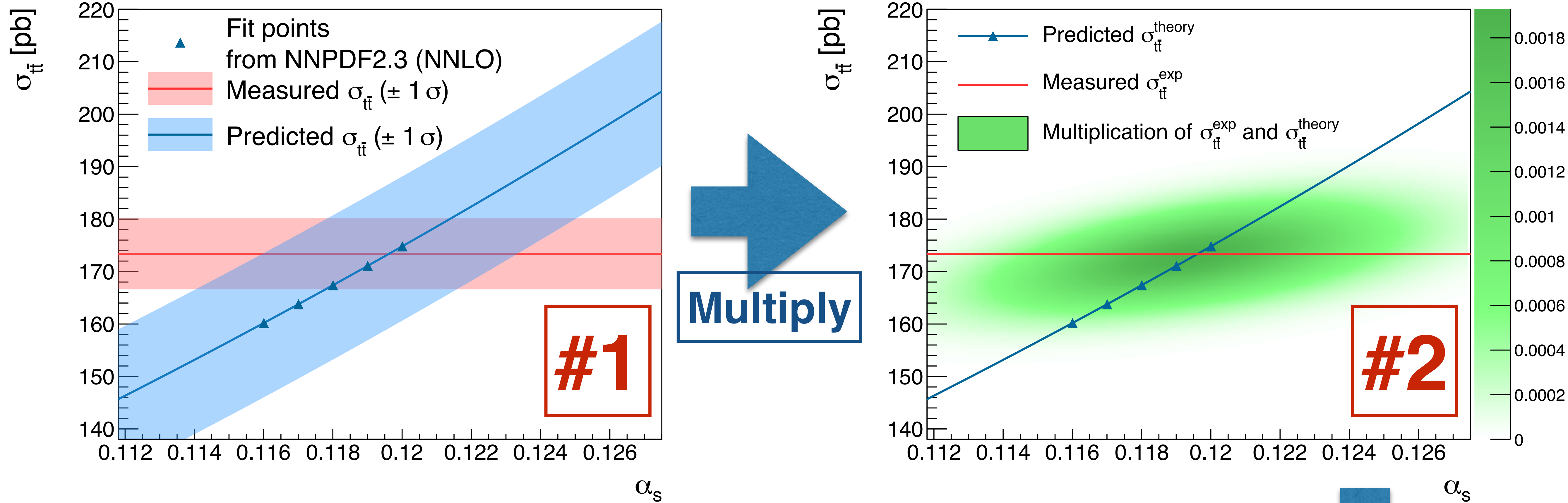
$$\sigma_\pm = \begin{cases} \sigma_+ & \text{if} \quad x > \mu \\ \sigma_- & \text{if} \quad x < \mu \end{cases}$$

- Magnitude of uncertainty assumed not to depend on the cross section



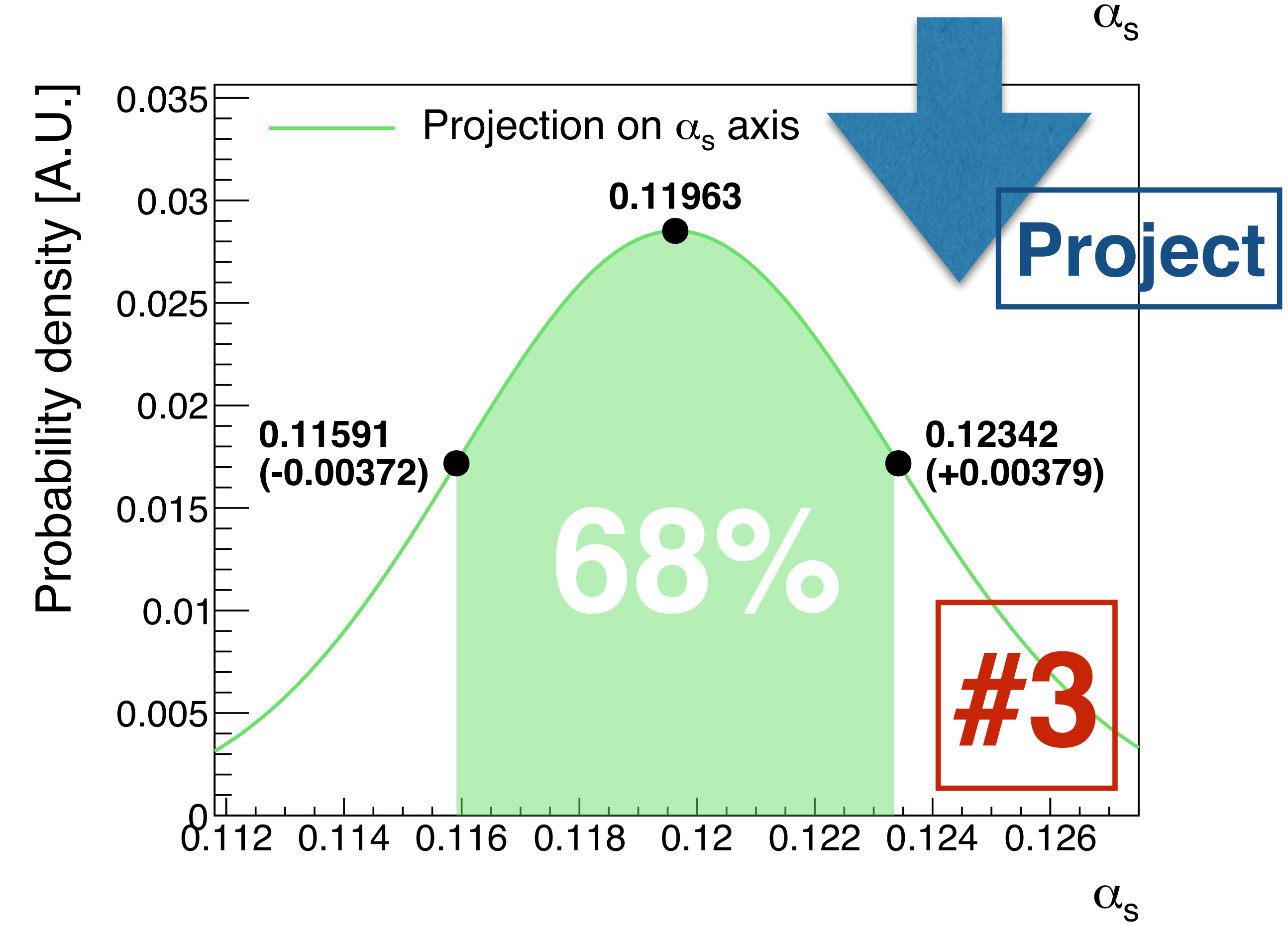


Extracting α_s from σ_{tt} measurements



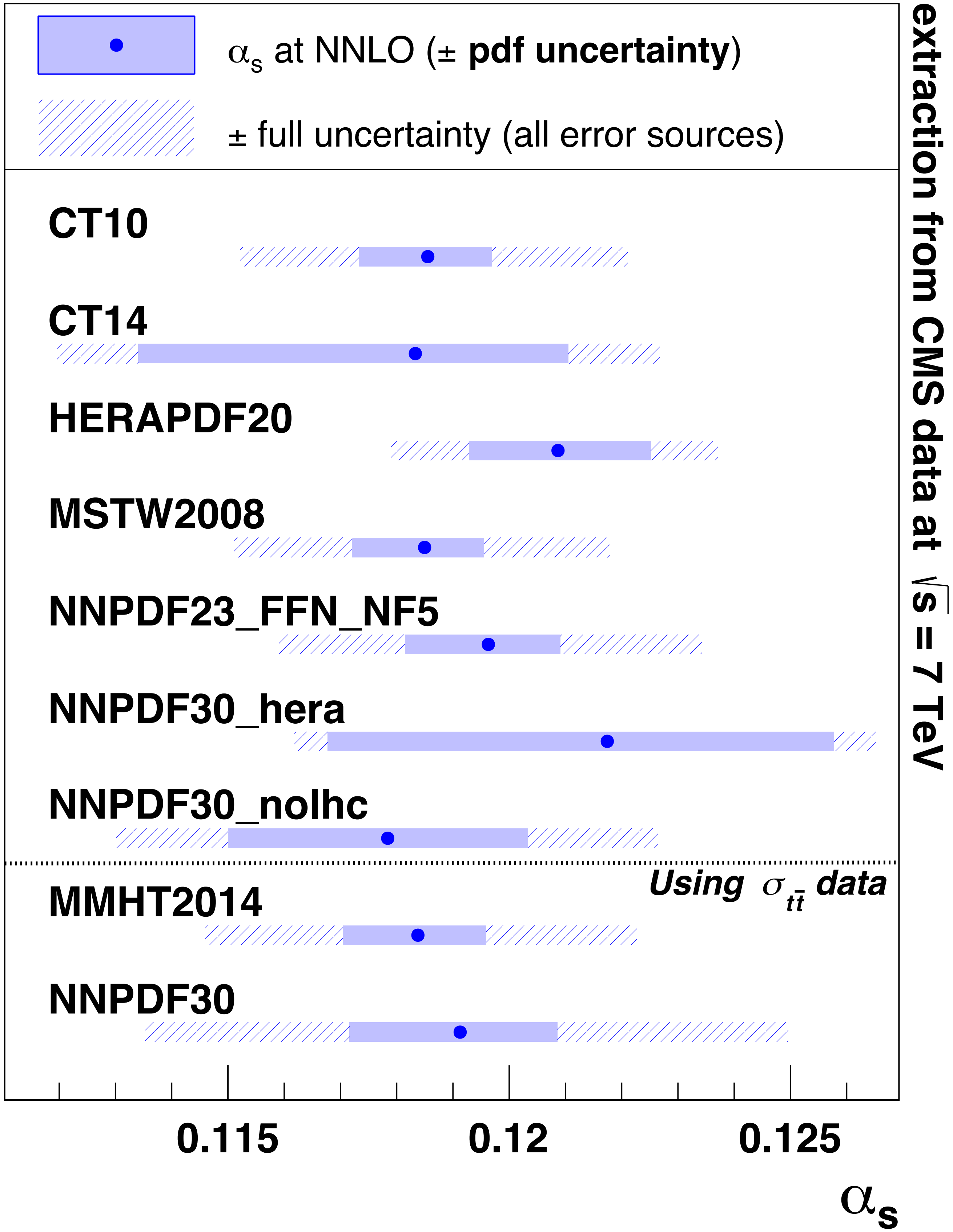
- Procedure from CMS [Phys. Let. B 728 (2014)]

- Dependence of acceptance corrections neglected (effect <1% in region of interest)



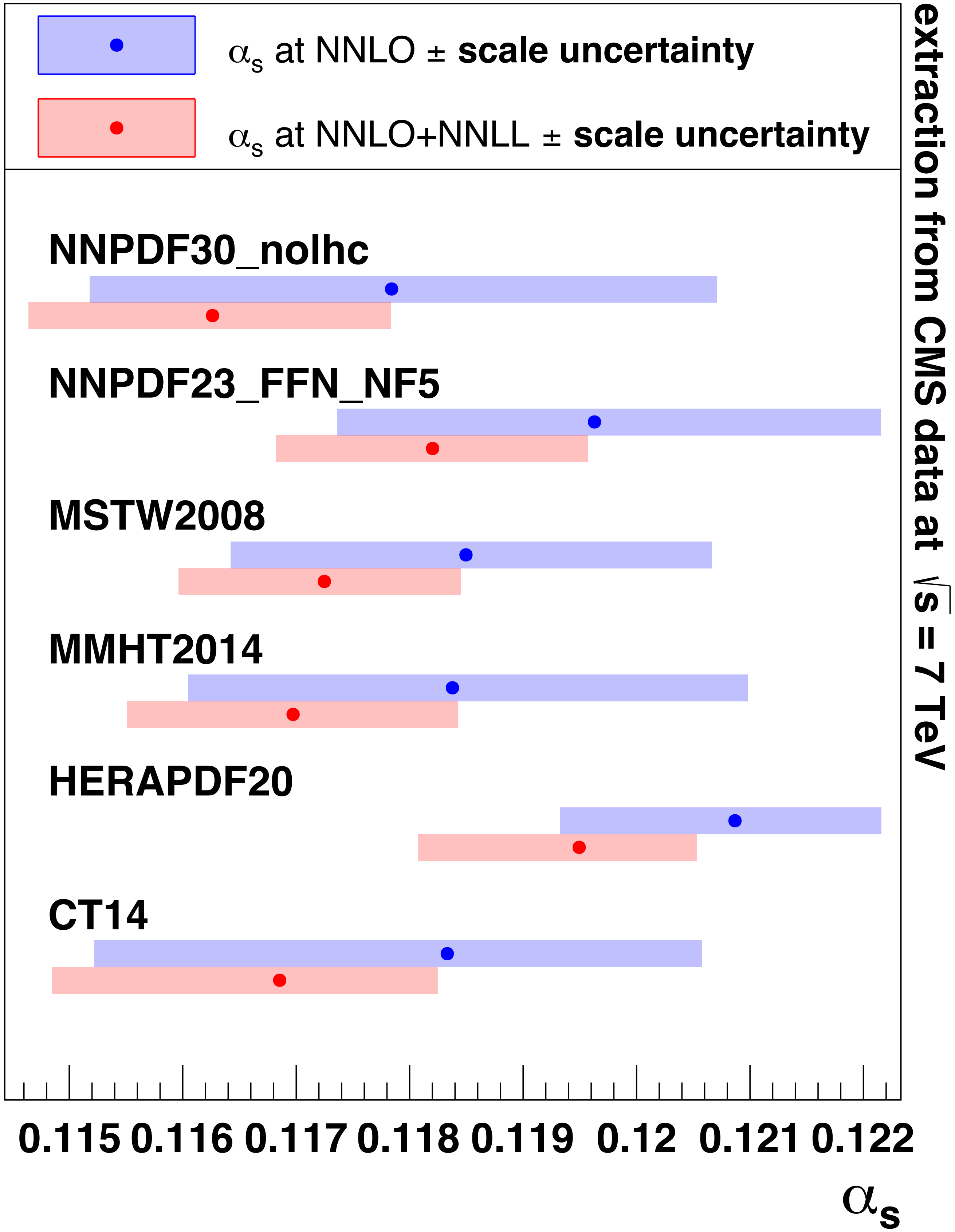
Challenges in the extraction procedure

- Some PDF sets use **σ_{tt} data in their global fits**
  - No bias immediately apparent, but using these PDF sets may introduce a bias
- PDF sets use a **different number** of evaluations of $\sigma_{tt}(\alpha_s)$, and **different range**
  - Extracted value of α_s for some sets *outside* the α_s range
- CT14 suits our criteria, but has a huge PDF uncertainty



Challenges in the extraction procedure

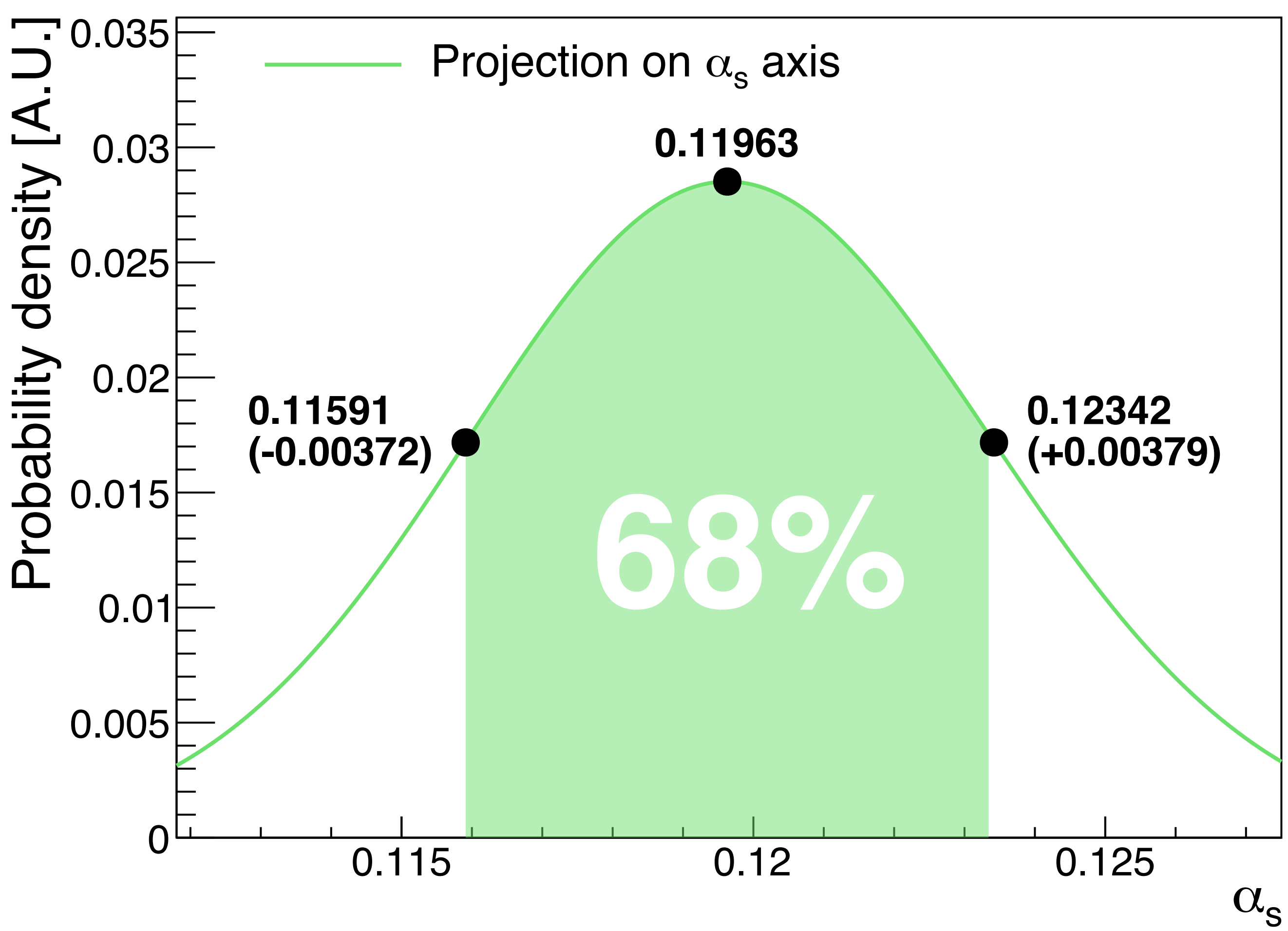
- Using resummation: **NNLO** or **NNLO+NNLL**
    - Scale uncertainty decreases by a factor of ~2 when using the resummation
    - Determined value of α_s goes down by ~0.001
- NNNLO shows small disagreements with NNLO +NNLL for gluon fusion
    - Now considering an average as the final determination



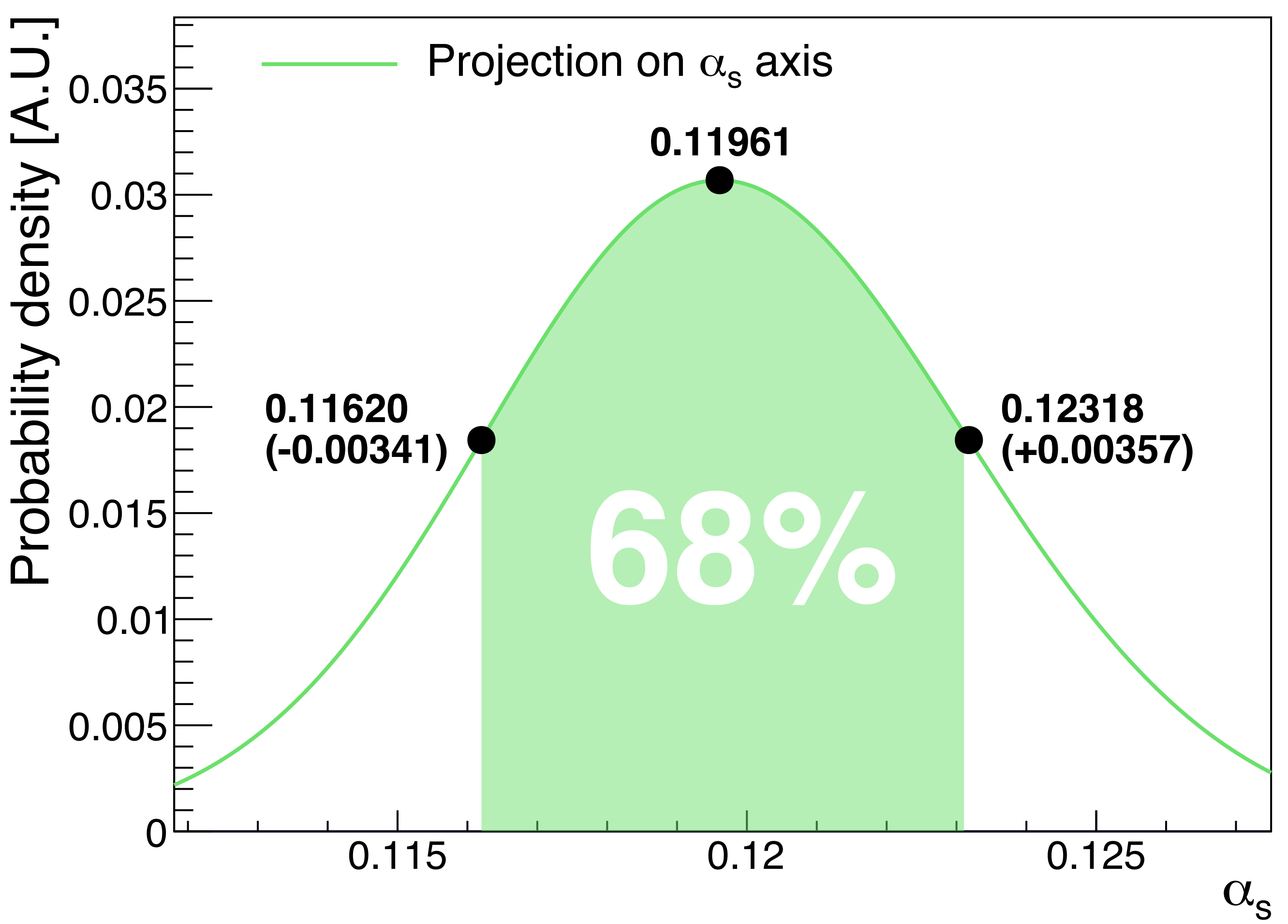
Estimating impact per error source

- Impact per source of uncertainty is evaluated by repeating the extraction with an error source omitted

With PDF error



Without PDF error



$$\text{PDF error}^{+} = \sqrt{0.00379^2 - 0.00357^2} = 0.00128$$

$$\text{PDF error}^{-} = -\sqrt{-0.00372^2 - -0.00341^2} = -0.00148$$

Combining correlated measurements

- One extraction yields **1 central value** and **7 uncertainties** (*statistical, systematic, luminosity, beam energy, pdf, scale, top mass*)

Using NNPDF2.3 (NNLO)

	Central value	Stat.	Syst.	Lumi.	E_{beam}	PDF	m_{top}	Scale
ATLAS (7 TeV)	0.12204	0.00081	0.00110	0.00094	0.00086	0.00134	0.00170	0.00232
ATLAS (8 TeV)	0.11819	0.00034	0.00110	0.00150	0.00084	0.00132	0.00183	0.00248
CMS (7 TeV)	0.11963	0.00057	0.00115	0.00102	0.00081	0.00138	0.00178	0.00240
CMS (8 TeV)	0.11861	0.00028	0.00117	0.00127	0.00084	0.00131	0.00169	0.00247
Tevatron (~2 TeV)	0.12150	0.00161	0.00234	0.00169		0.00097	0.00135	0.00256

- These uncertainties are (often strongly) correlated
- Aim is to combine these measurements into a single determination

Maximum Likelihood Estimate Method

- Idea: Fit α_S to the individual probability distribution functions per experiment simultaneously
- Correlations are split up:

“Fully correlated uncertainties” (100%) —> **Nuisance parameters**
“Uncorrelated uncertainties” (0%) —> **Statistical uncertainties**

  - The nuisance parameters affect individual experiments simultaneously, and are fitted together with α_S
- Correlation coefficients between 0 and 1 are split up in a nuisance parameter and a statistical uncertainty
  - E.g. Luminosity has a correlated part at LHC (the uncertainty from the Van der Meer scans) and an uncorrelated part (from long-term luminosity monitoring per experiment)

Maximum Likelihood Estimate Method

$$L(\alpha_s, \boldsymbol{\theta}) = \prod_i \mathrm{Gauss.}(\alpha_s,\, \mu_i + \sum_j \theta_j \delta_j,\, \sigma_i) \times \prod_j \mathrm{Gauss.}(\theta_j,\, 0,\, 1)$$

- μ_i : The determination for experiment i
- σ_i : Statistical uncertainty for experiment i
- θ_j : The nuisance parameter j
- δ_j : Impact of nuisance parameter j

- Same likelihood estimate as in the *combine* tool
- Gaussians replaced by convolutions of asymmetric Gaussians when working with asymmetry

Maximum Likelihood Estimate Method

$$L(\alpha_s, \boldsymbol{\theta}) = \prod_i \text{Gauss.}(\alpha_s,\ \mu_i + \sum_j \theta_j \delta_j,\ \sigma_i) \times \boxed{\prod_j \text{Gauss.}(\theta_j,\ 0,\ 1)}$$

- μ_i : The determination for experiment i
- σ_i : Statistical uncertainty for experiment i
- θ_j : The nuisance parameter j
- δ_j : Impact of nuisance parameter j

- Same likelihood estimate as in the *combine* tool
- Gaussians replaced by convolutions of asymmetric Gaussians when working with asymmetry
- Second part can strongly influence the final determination if a nuisance parameter has a large δ

Maximum Likelihood Estimate Method

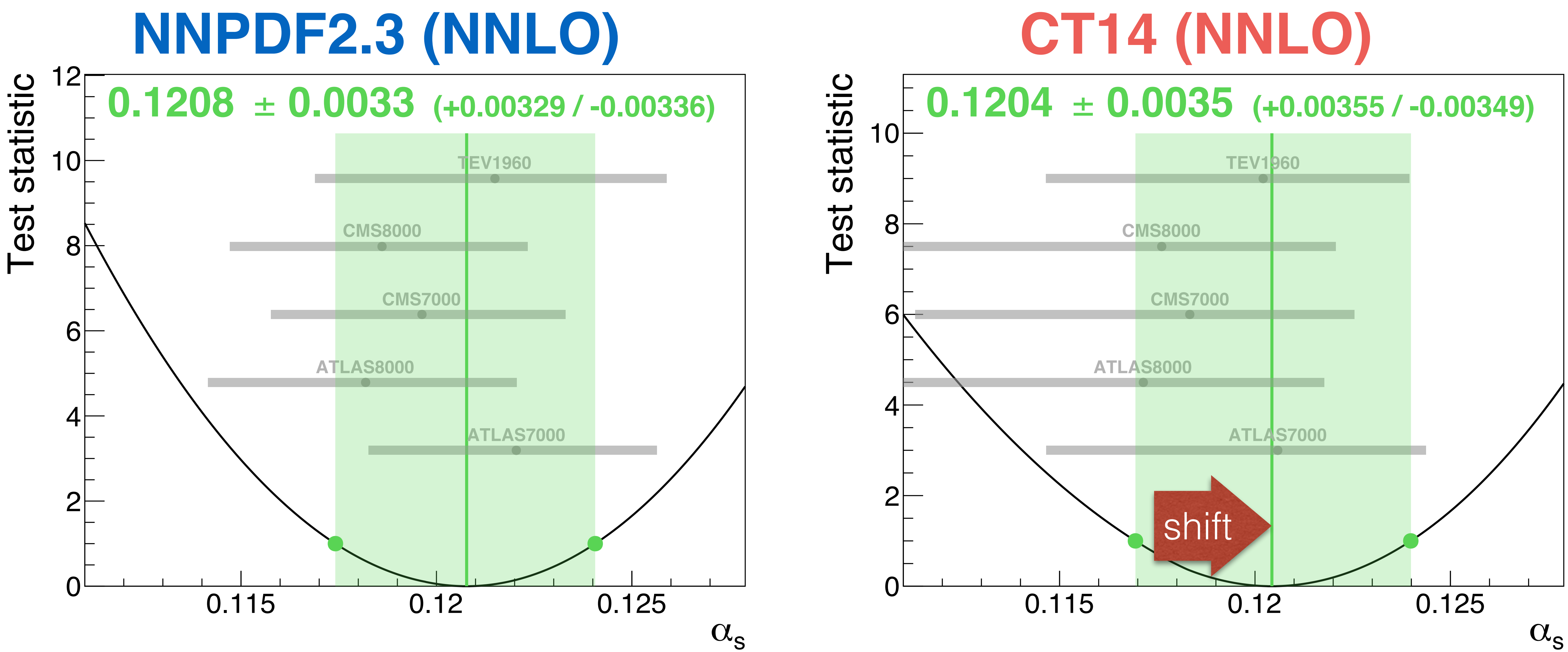
$$\boxed{L(\alpha_s, \boldsymbol{\theta}) = \prod_i \text{Gauss.}(\alpha_s,\, \mu_i + \sum_j \theta_j \delta_j,\, \sigma_i) \times \prod_j \text{Gauss.}(\theta_j,\, 0,\, 1)}$$

- To extract the uncertainties, a **scan** is performed over α_s, while the nuisance parameters are **profiled**
- For each scan point a **test statistic** q is calculated:

$$q(\alpha_s) = -2 \ln \left(\frac{L(\alpha_s,\, \boldsymbol{\theta}_{\alpha_s})}{L(\hat{\alpha}_s,\, \hat{\boldsymbol{\theta}}_{\hat{\alpha}_s})} \right)$$

- $L(\hat{\alpha}_s,\, \hat{\boldsymbol{\theta}}_{\hat{\alpha}_s})$: Likelihood maximised for α_s and **θ**
- $L(\alpha_s,\, \boldsymbol{\theta}_{\alpha_s})$: Likelihood maximised for **θ** (α_s is input)
- -2 and the natural logarithm make q χ^2-distributed

Preliminary combination results

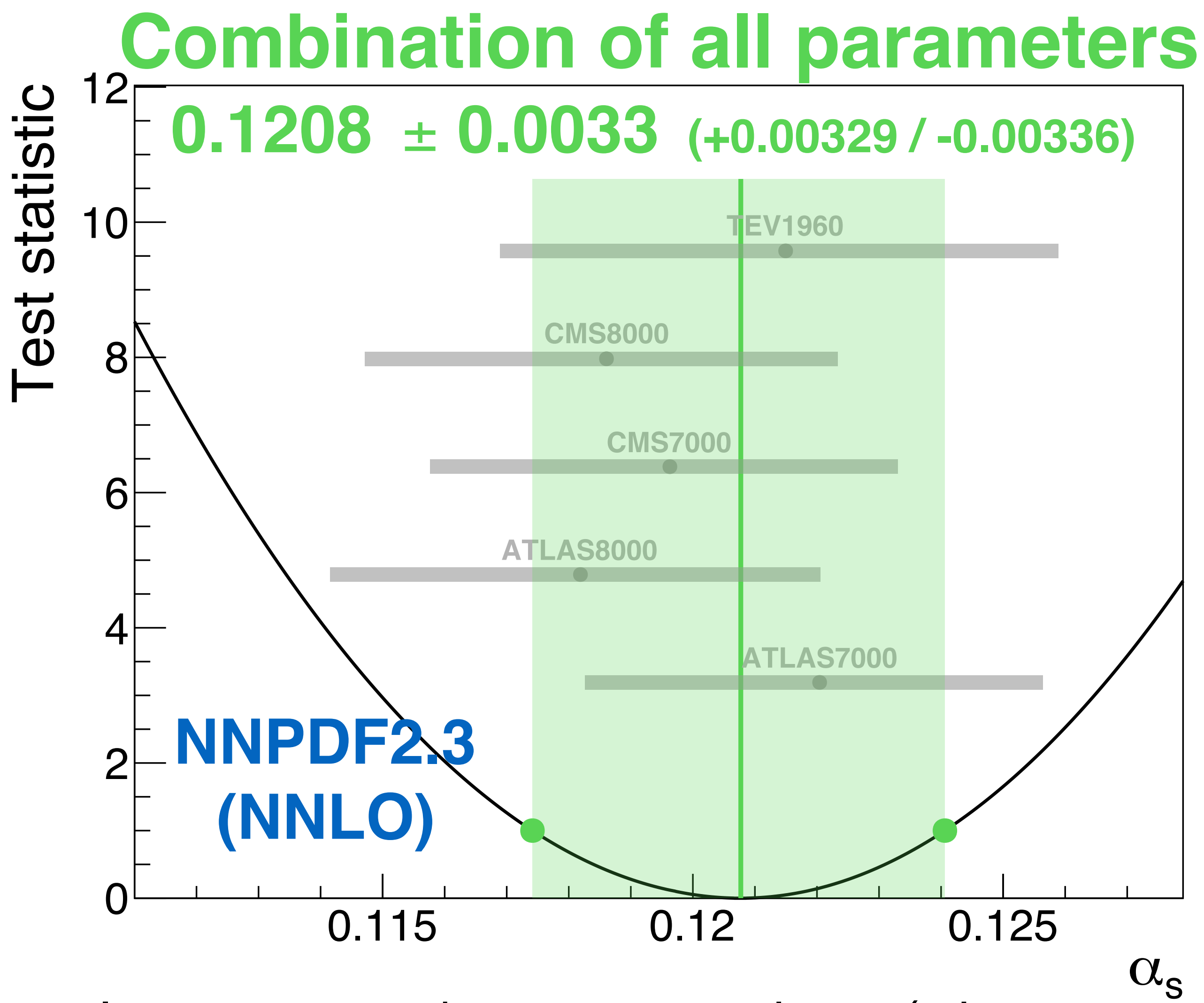


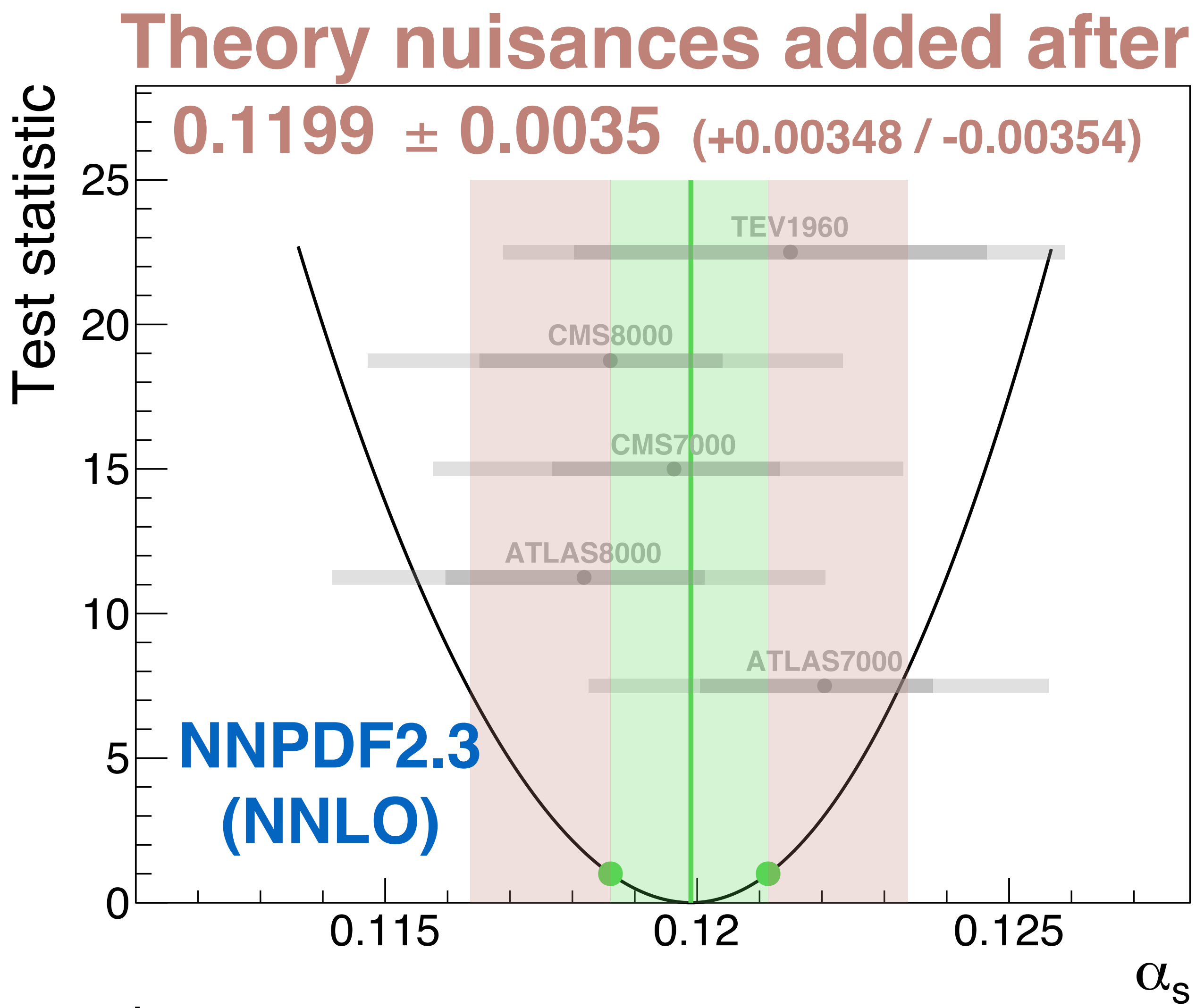
- Slightly asymmetric probability distribution functions return a reasonable combination
- Asymmetric functions are strongly influenced by the nuisance parameters
  - Different combination techniques (BLUE[1]) show the same pattern

1: **B**est **L**inear **U**nbiased **E**stimate [Nucl. Instrum. Methods A 270 (1988)]

Preliminary combination results

- **Theory nuisance parameters** are very large, and tend to drive the final determination
    - Solution is to add theory uncertainties after combining

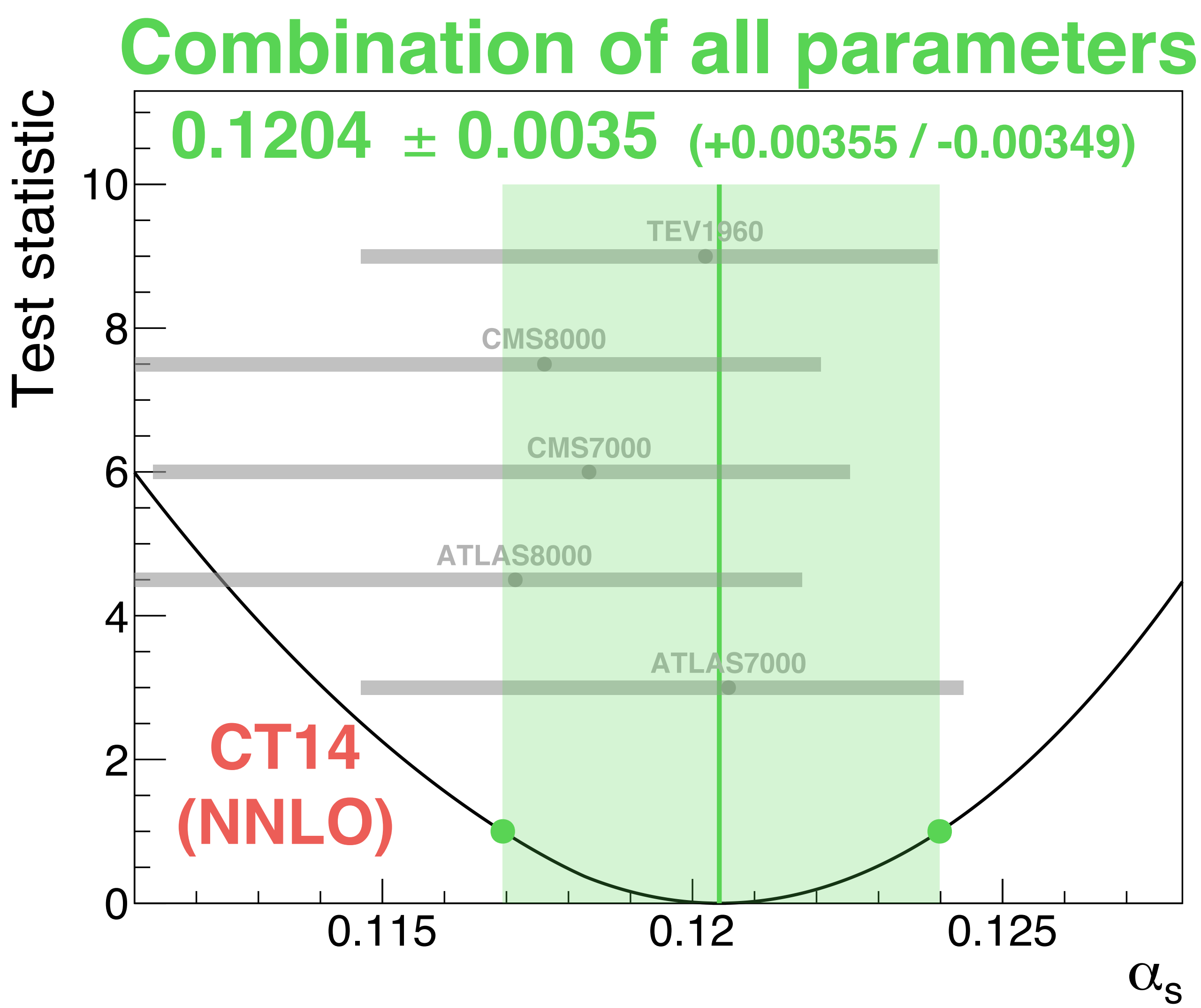


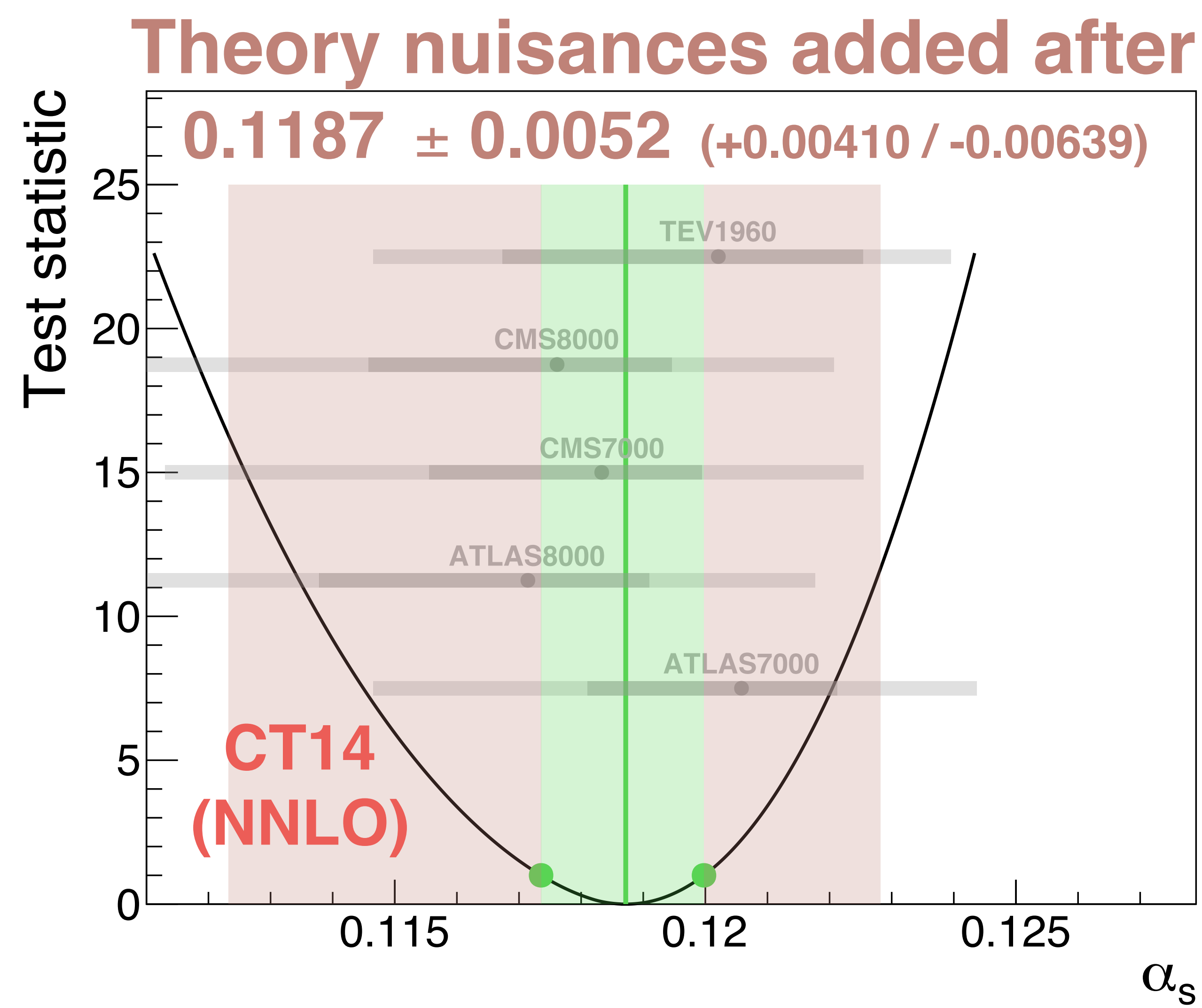


- Increases the uncertainty (since some nuisance parameters are not optimally fitted)
- For **NNPDF2.3 (NNLO)**, final determinations are not too different

Preliminary combination results

- For **CT14 (NNLO)**: Larger theory uncertainties and more asymmetry, differences are more pronounced
  - Uncertainty goes up, central value shifts (but well within the 1σ band)





- Same method under different combination schemes yields similar results

Conclusion

- Machinery to extract α_s from σ_{tt} measurements and to combine these is in place
  - Precision of ~2.5% to ~4%, dependent on which PDF set is used
- Several important decisions need to be made:
  - NNLO, NNLO+NNLL or a weighted average thereof?
  - What are the final PDF sets to be used?
    - CT14 fits our criteria, but has very large theory uncertainties compared to other PDF sets
- In due time more experiments will be added to the combination (13 TeV and 5 TeV measurements from LHC)

Backup

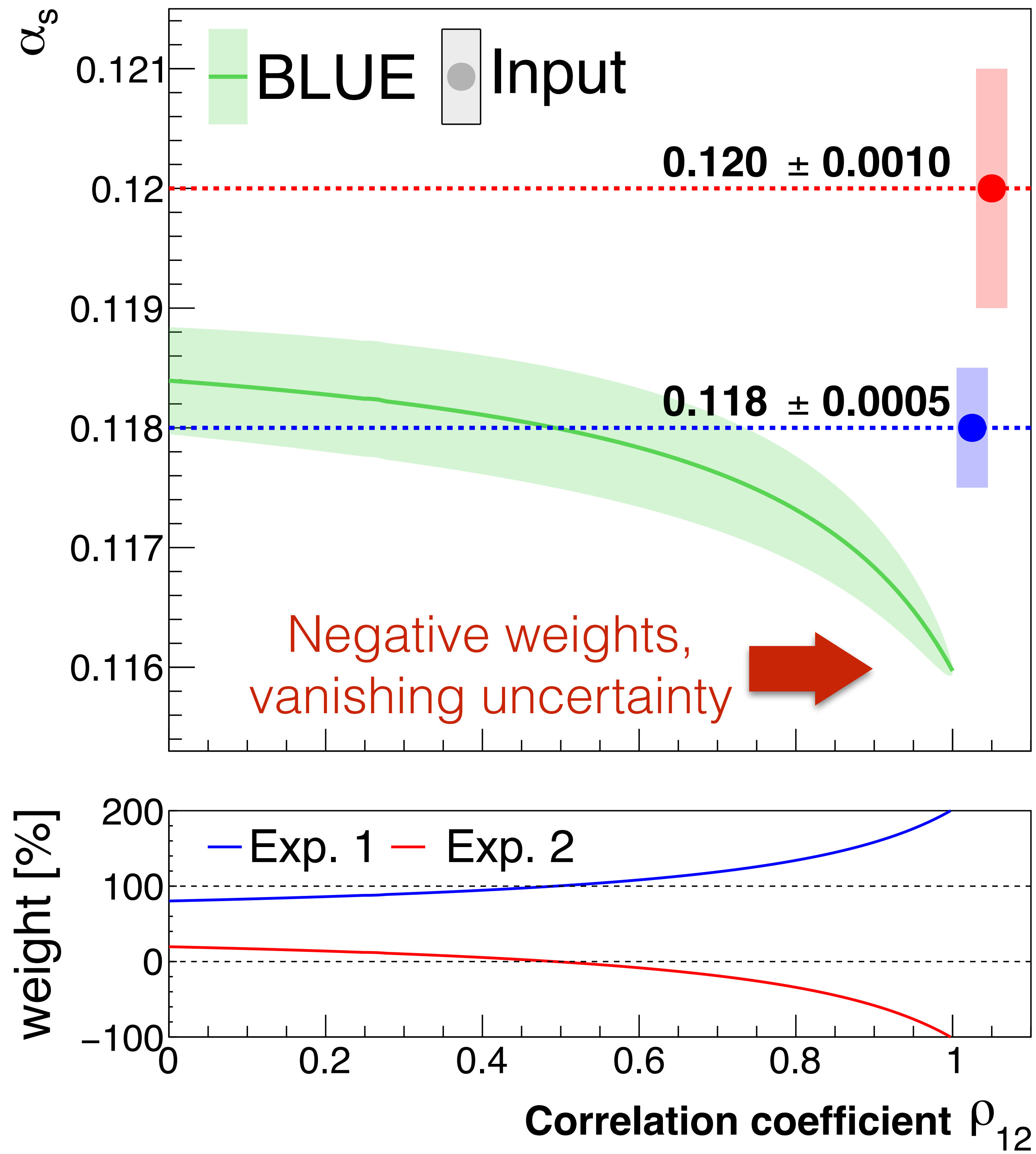
Combining correlated measurements

- One extraction yields **1 central value** and **7 uncertainties** (*statistical, systematic, luminosity, beam energy, pdf, scale, top mass*)
    - Many uncertainties are correlated between experiments
- Combinations have been performed using the **BLUE**[1] method:

$$y_{BLUE} = \sum_i w_i \, y_i \qquad \sigma^2_{BLUE} = w^T \, \mathbf{E} \, w$$

- Weights found by minimising σ_{BLUE}
- Correlation coefficients ρ have to be set carefully
    - ρ = 1.0 is <u>not</u> conservative

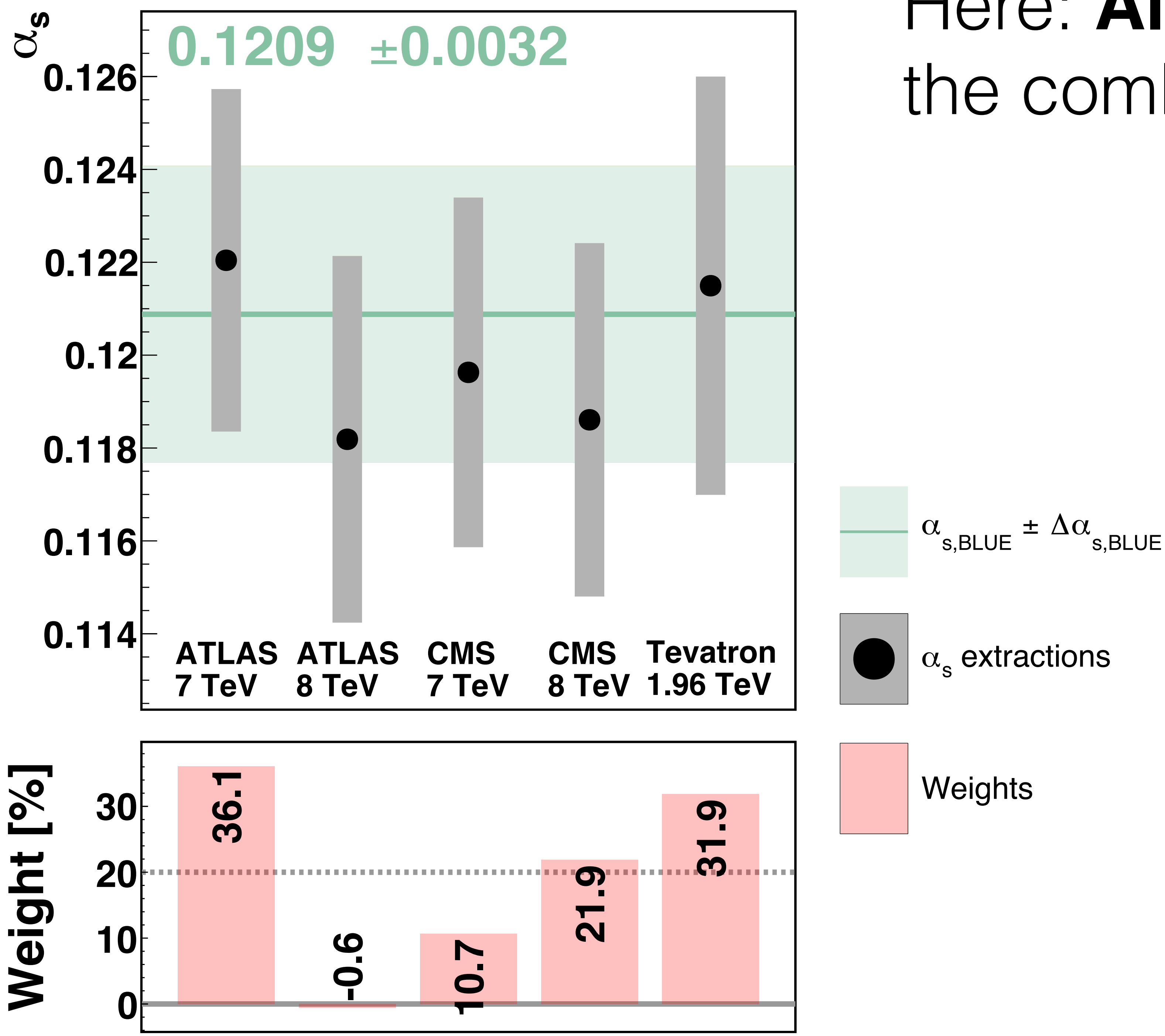
Example combination



1: **B**est **L**inear **U**nbiased **E**stimate
[Nucl. Instrum. Methods A 270 (1988)]

Preliminary combination results

Full combination for NNPDF2.3 at NNLO



Here: **All** error sources used in the combination

Alternative:

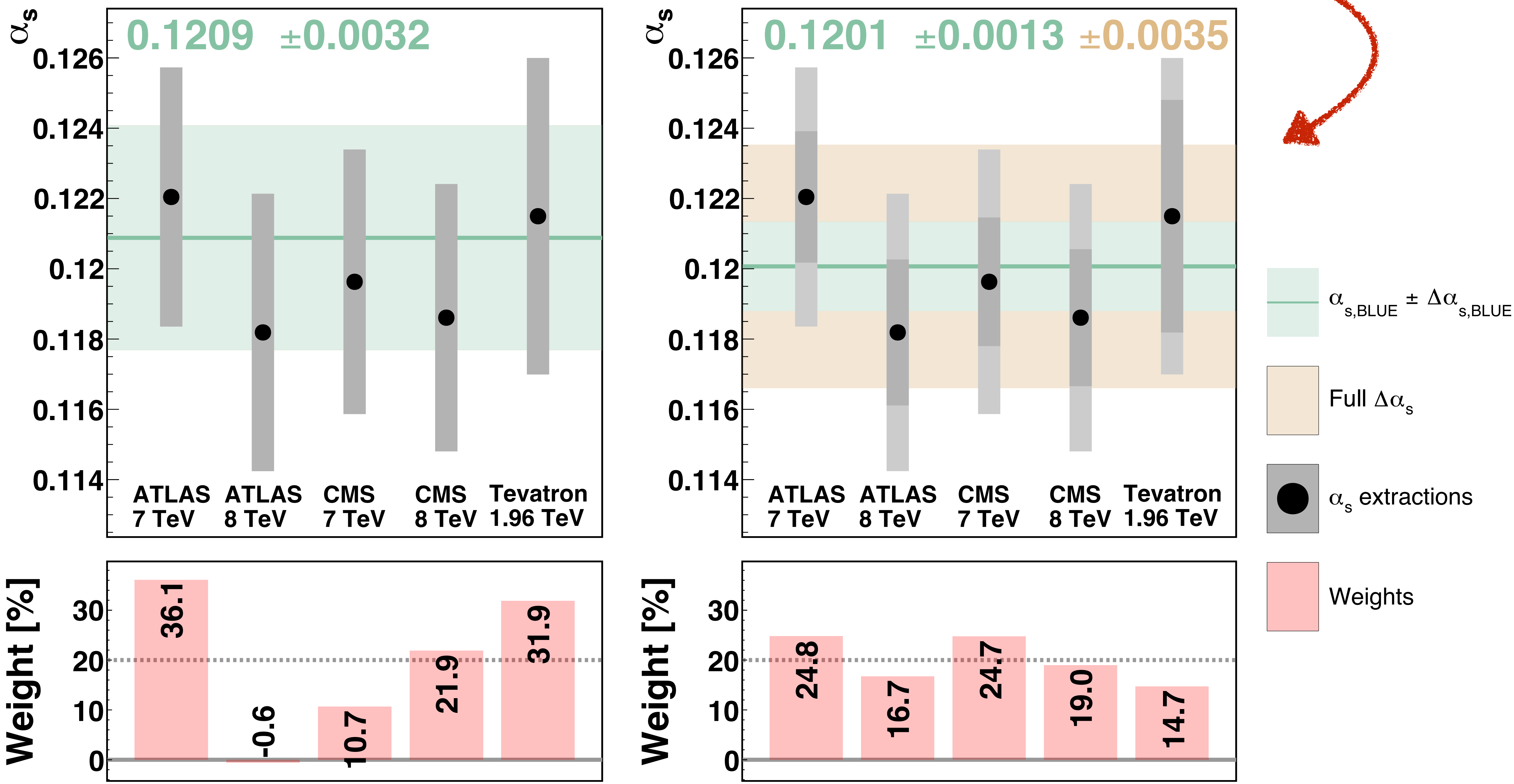
- Run the combination without some error sources
- Add these error sources after the combination

Can be useful to study the effects of the (strongly correlated) theory uncertainties

Preliminary combination results

Full combination for NNPDF2.3 at NNLO

Combination of only experimental uncertainties
(theoretical uncertainties added after combination)

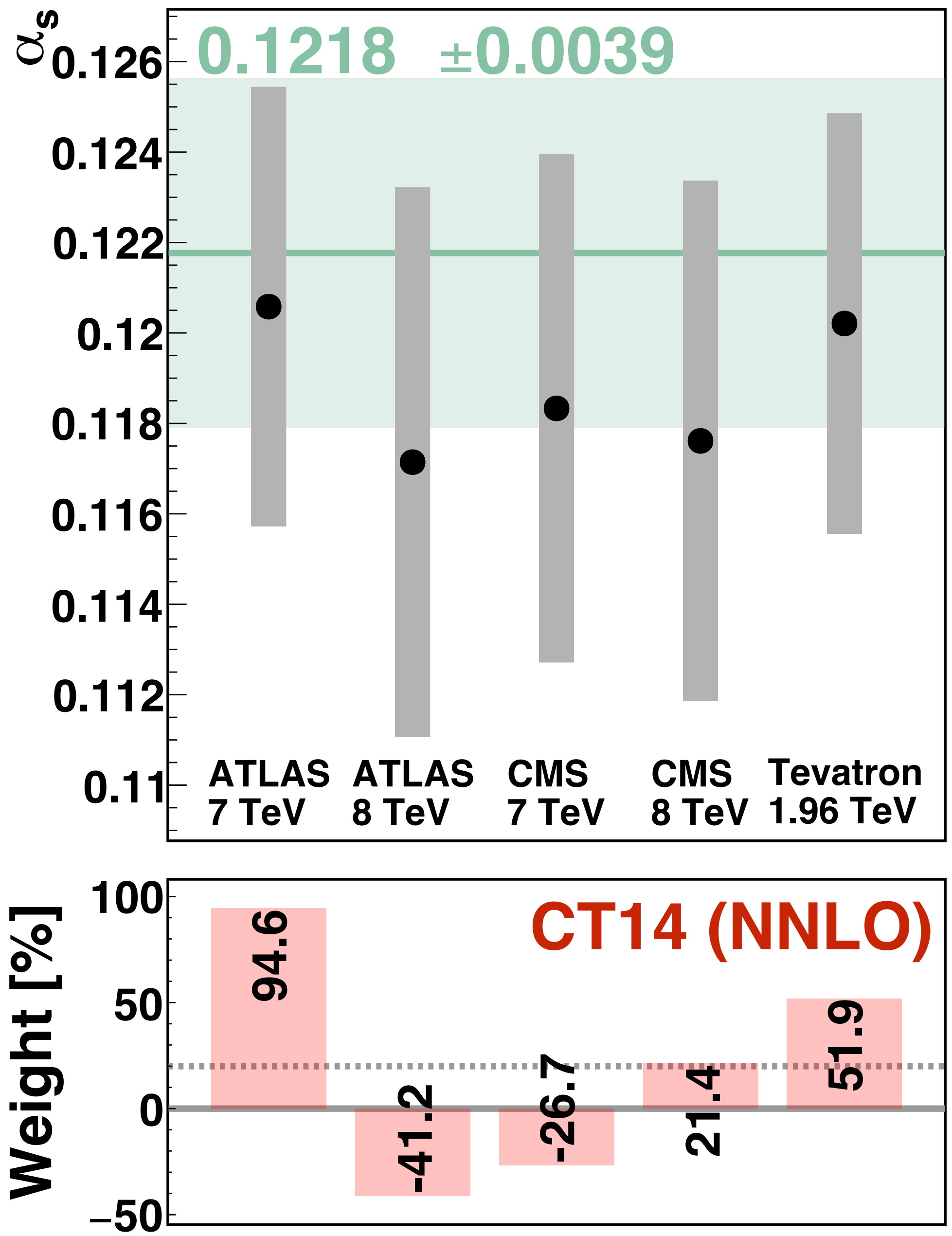


Preliminary combination results

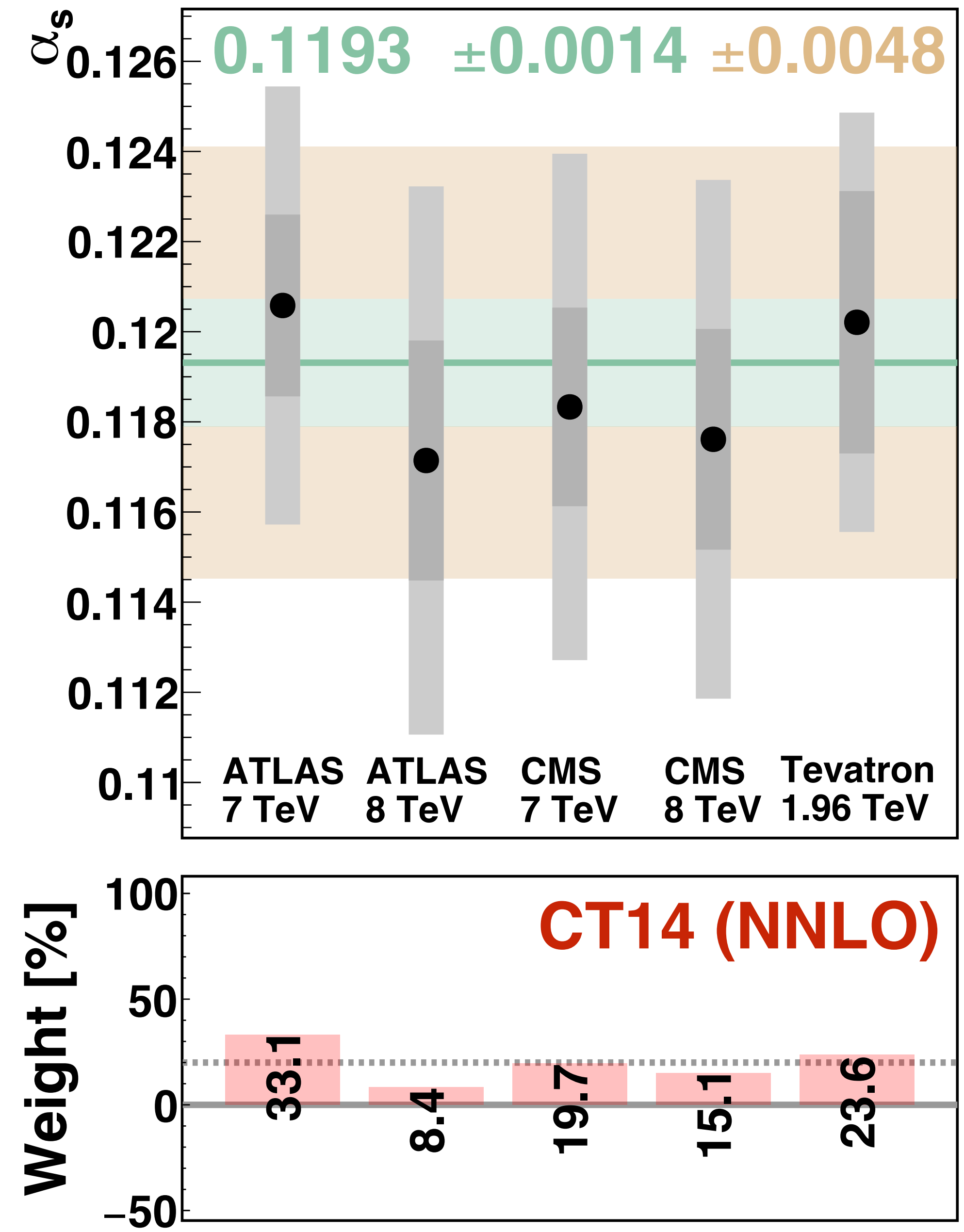
For larger theoretical uncertainties, combination yields very negative weights, caused by strong correlations

- Is the large influence caused by strong correlations trustworthy?

Full combination



Only experimental



Extracting α_s (1) — Getting $\sigma_{tt}(\alpha_s)$

- For $\sigma_{tt,\ theory}(\alpha_s)$, uncertainties include:

  1. Uncertainty due to pdf (**pdf**)
     Calculated by `top++2.0` by computing σ_{tt} for all members of the pdf set

     - For the *replicas* type pdfs, uncertainty due to pdf is simply the standard deviation of σ_{tt} for different members. Calculation can be a bit more involved depending on the pdf.

  2. Uncertainty due to scale (**scale**)

     - By recomputing σ_{tt} in `top++2.0` at different renormalisation scale variations ($1/2 \leq \mu_R/\mu_F \leq 2$), and taking minimum and maximum variations

  3. Uncertainty due to uncertainty on the top mass (**mtop**)

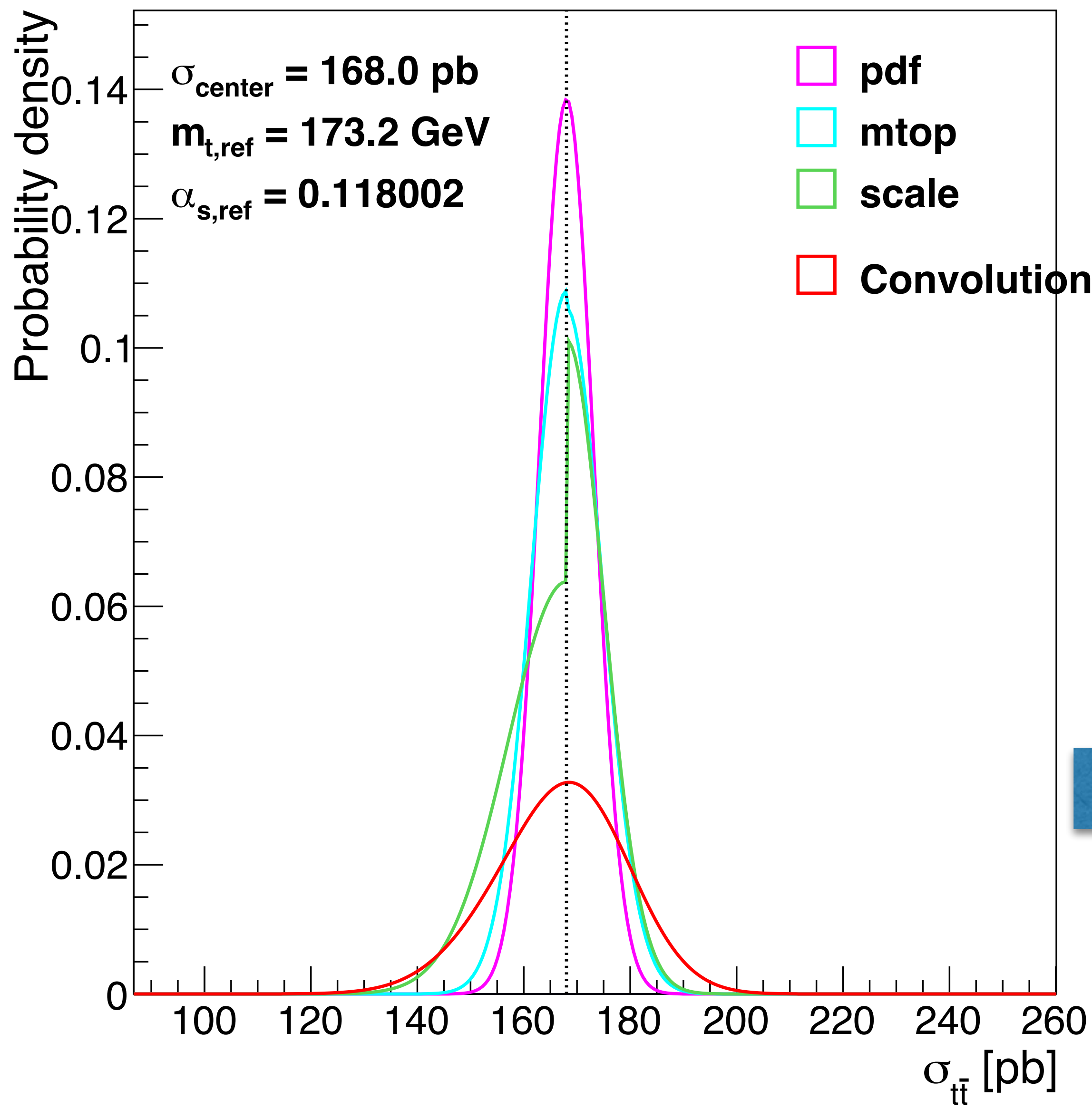
     - Recompute σ_{tt} in `top++2.0` at $(m_{top,\ pole} + \Delta m_{top,\ pole})$ and $(m_{top,\ pole} - \Delta m_{top,\ pole})$

     - Experimental σ_{tt} also depends on $m_{top,\ pole}$, so bounds should be scaled:

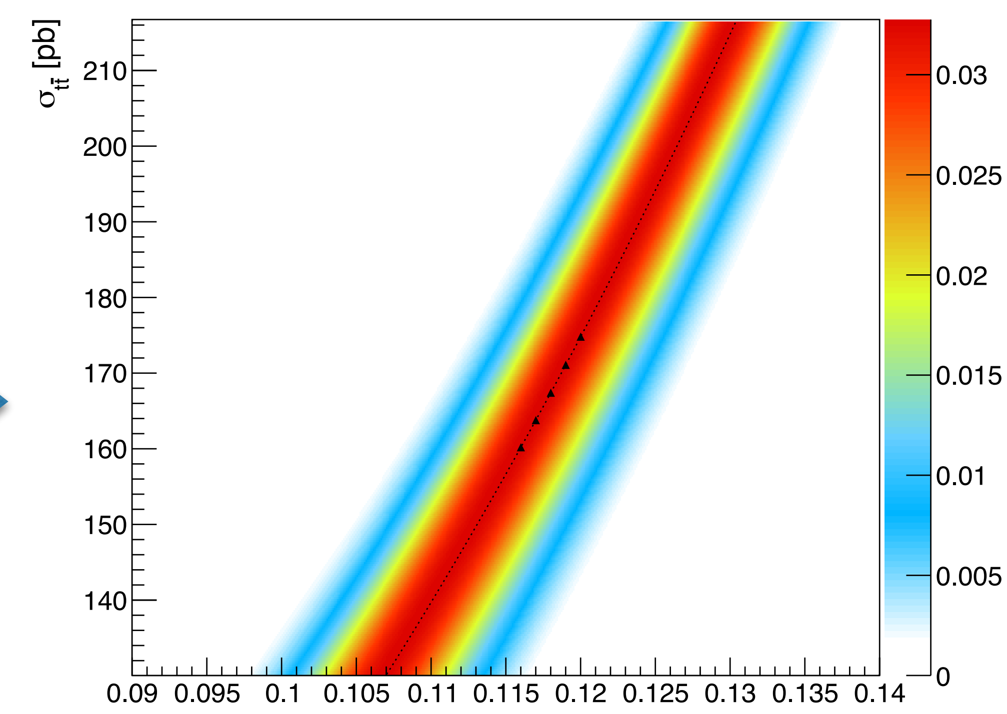
$$\sigma_{t\bar{t}}^{+} = \sigma_{t\bar{t}}(m_{\text{top}} + \Delta m_{\text{top}}^{\text{pole}})_{\text{theory}} \cdot \frac{\sigma_{t\bar{t}}(m_{\text{top}})_{\text{experimental}}}{\sigma_{t\bar{t}}(m_{\text{top}} + \Delta m_{\text{top}}^{\text{pole}})_{\text{experimental}}}$$

$$\sigma_{t\bar{t}}^{-} = \sigma_{t\bar{t}}(m_{\text{top}} - \Delta m_{\text{top}}^{\text{pole}})_{\text{theory}} \cdot \frac{\sigma_{t\bar{t}}(m_{\text{top}})_{\text{experimental}}}{\sigma_{t\bar{t}}(m_{\text{top}} - \Delta m_{\text{top}}^{\text{pole}})_{\text{experimental}}}$$

- Combining asymmetric error sources done by convoluting asymmetric gaussians
  (Cleanest approach of combining asymmetric errors)



- Magnitude of theoretical uncertainties are assumed to be independent of α_s



Examples shown here concern NNPDF2.3 at NNLO and the CMS experiment at 7 TeV

Extracting α_s (3) — Getting $\sigma_{tt}(\alpha_s)$

- For $\sigma_{tt,\ experimental}(\alpha_s)$, uncertainties include:

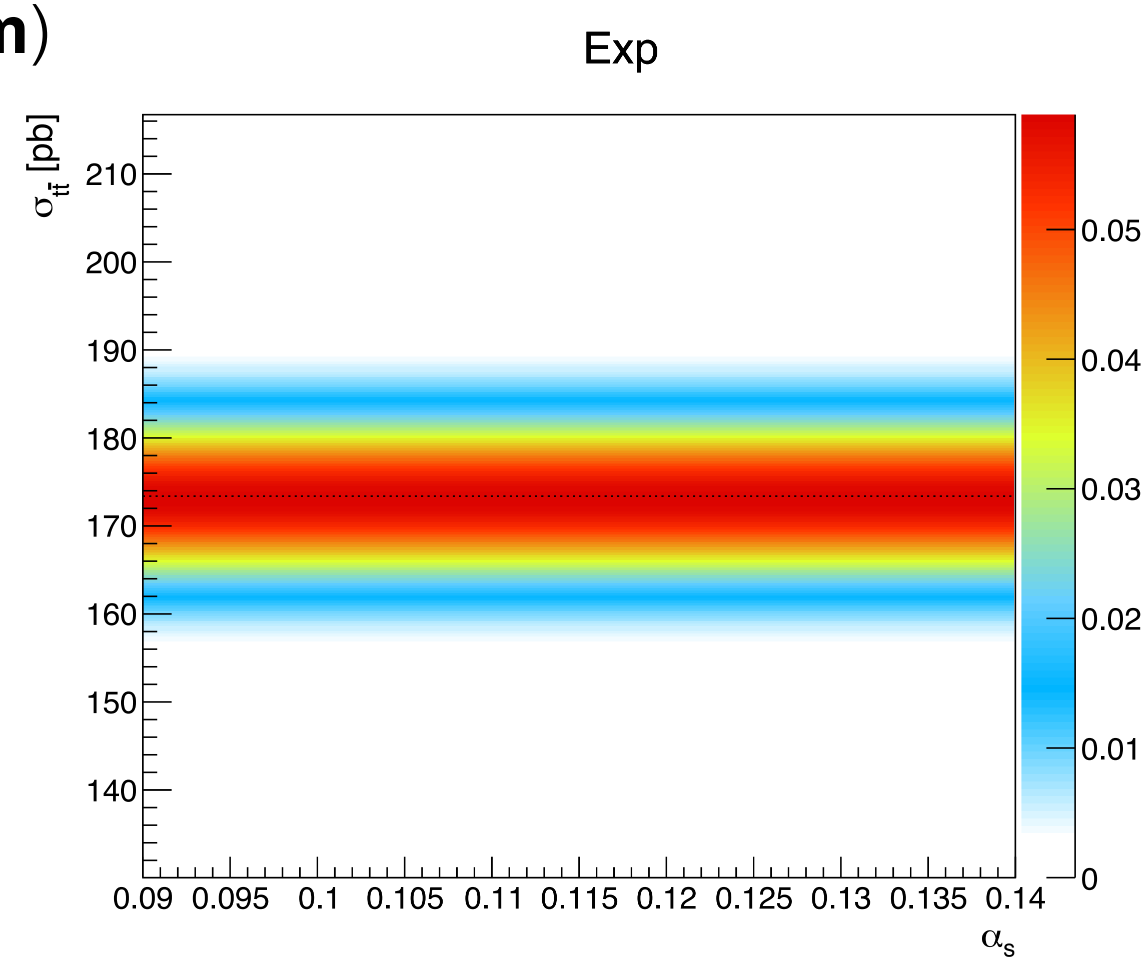
    1. Statistical uncertainty (**stat**)
    2. Systematic uncertainty (**syst**)
    3. Uncertainty due to luminosity measurement (**lumi**)
    4. Uncertainty due to beam energy (**Ebeam**)

- These numbers are quoted as symmetric errors, and can thus be added in quadrature

- Distribution of σ_{tt} is assumed to be Gaussian

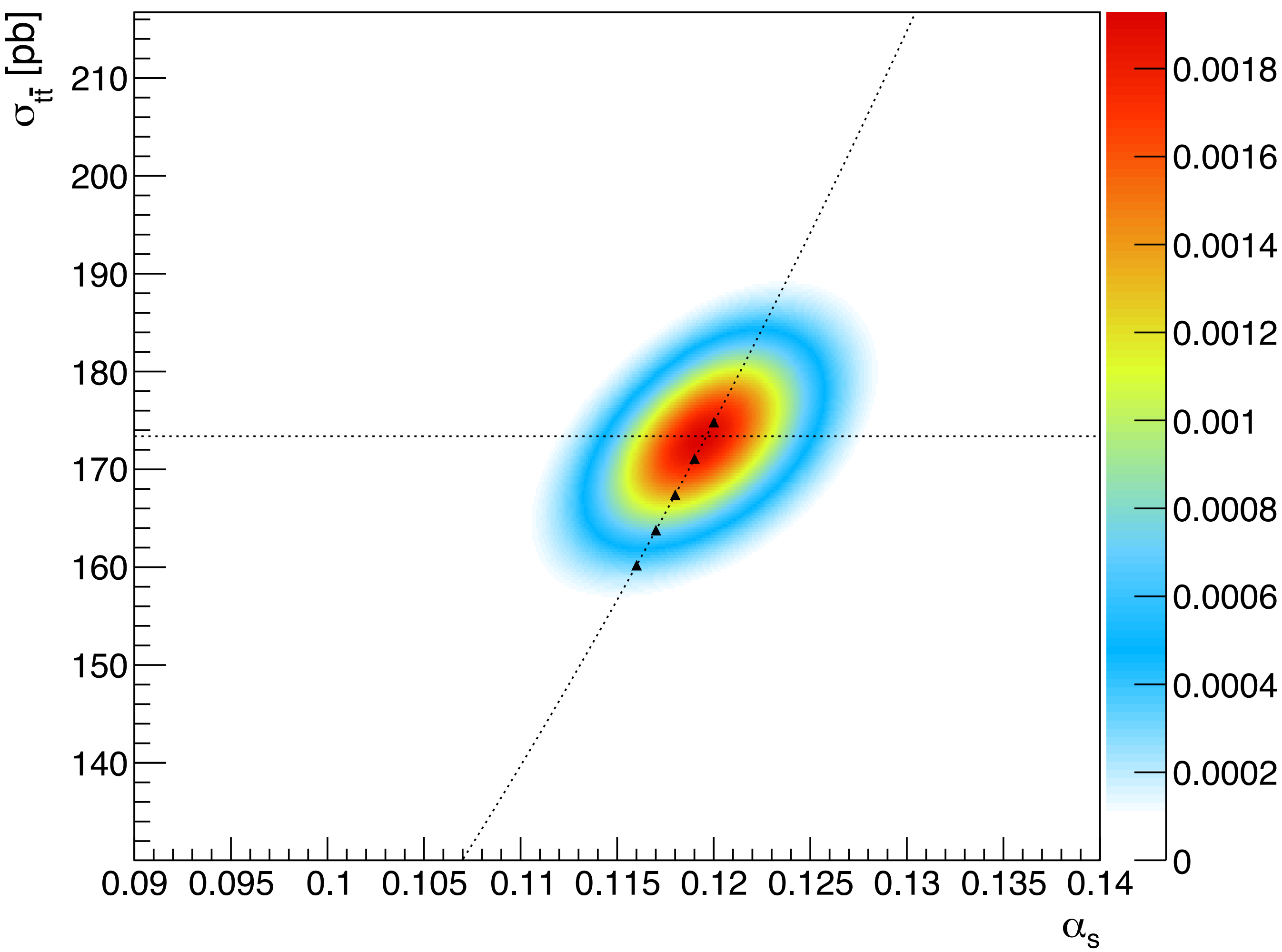
- Independence of α_s is once again assumed

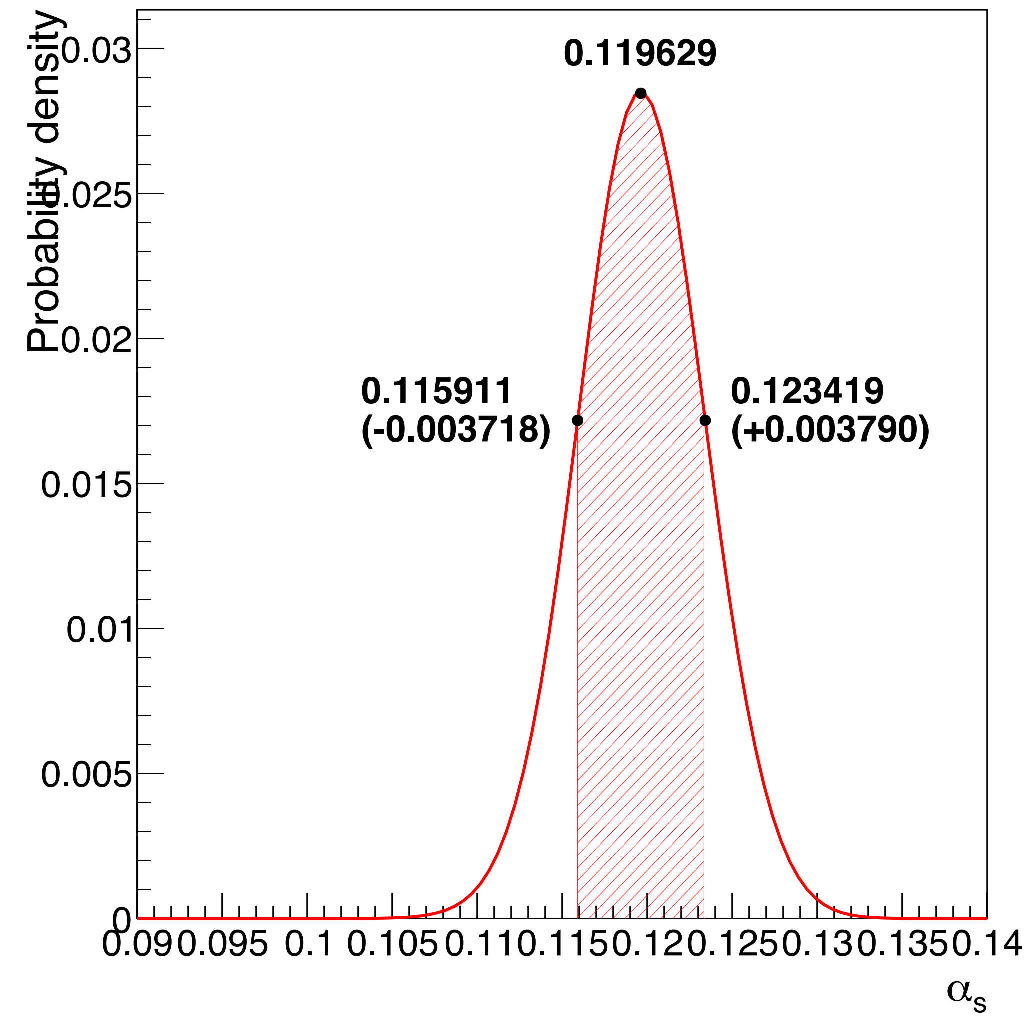
  (dependence of acceptance corrections on α_s is small)



Examples shown here concern the CMS experiment at 7 TeV

Extracting α_s (4) — Getting uncertainties on α_s

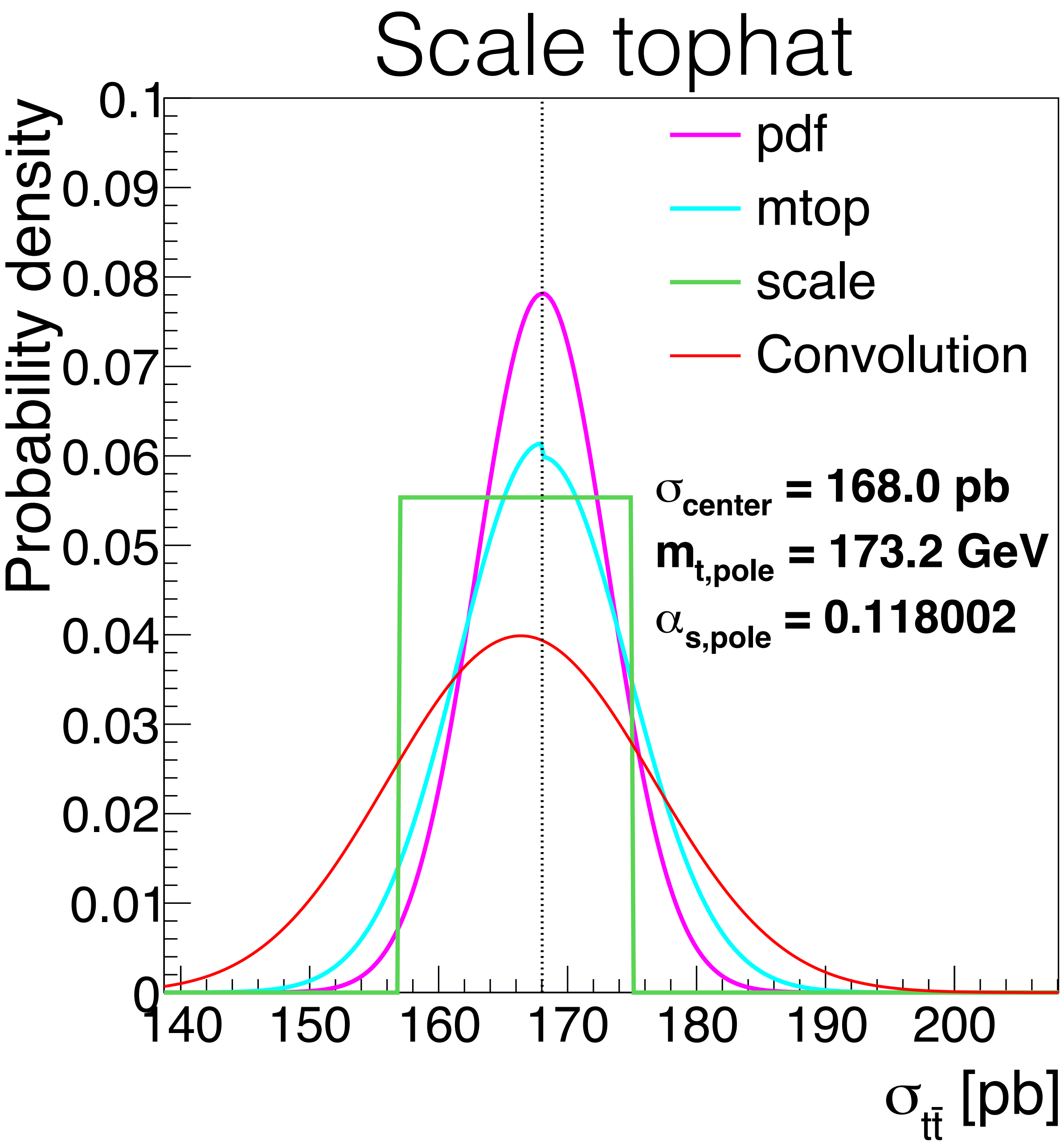




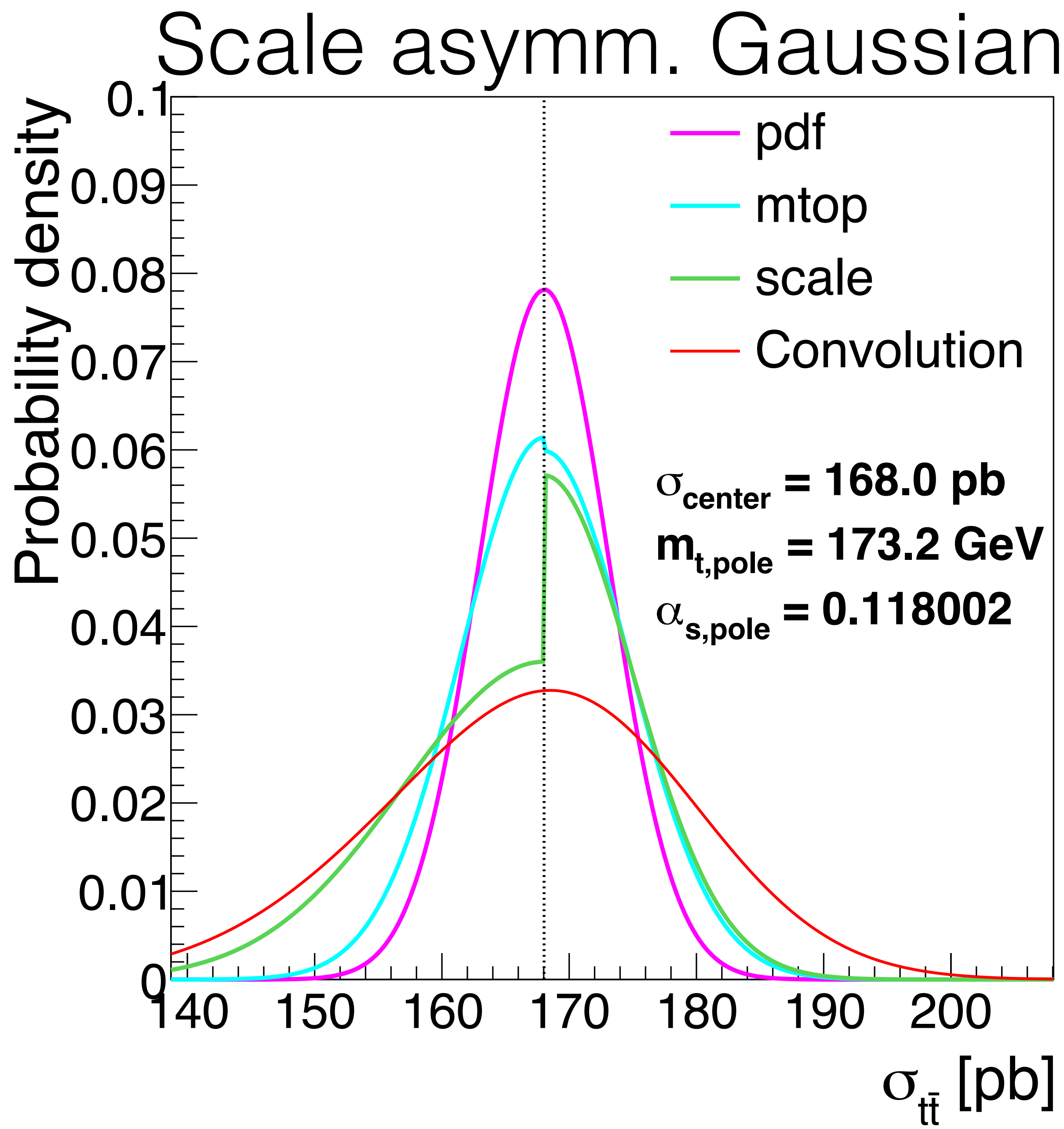
- The described procedure returns 1 center value for α_s and a 'total' uncertainty (which comprises all error sources)
  - To account for correlations in certain error sources during the combination, it is necessary to break down the uncertainty from the extraction into separated error sources
  - Solution: Repeat the extraction, omitting one different error source every time; error on α_s is then:

$$\mathrm{err.}_{\alpha_s,\,\text{one error source}} = \sqrt{\mathrm{err.}^2_{\alpha_s,\,\text{all error sources}} - \mathrm{err.}^2_{\alpha_s,\,\text{all except one error sources}}}$$

Extracting α_s (5) — Scale: Tophat vs. Gaussian



$\alpha_S = 0.1201 \pm 0.0032$



$\alpha_S = 0.1196 \pm 0.0038$

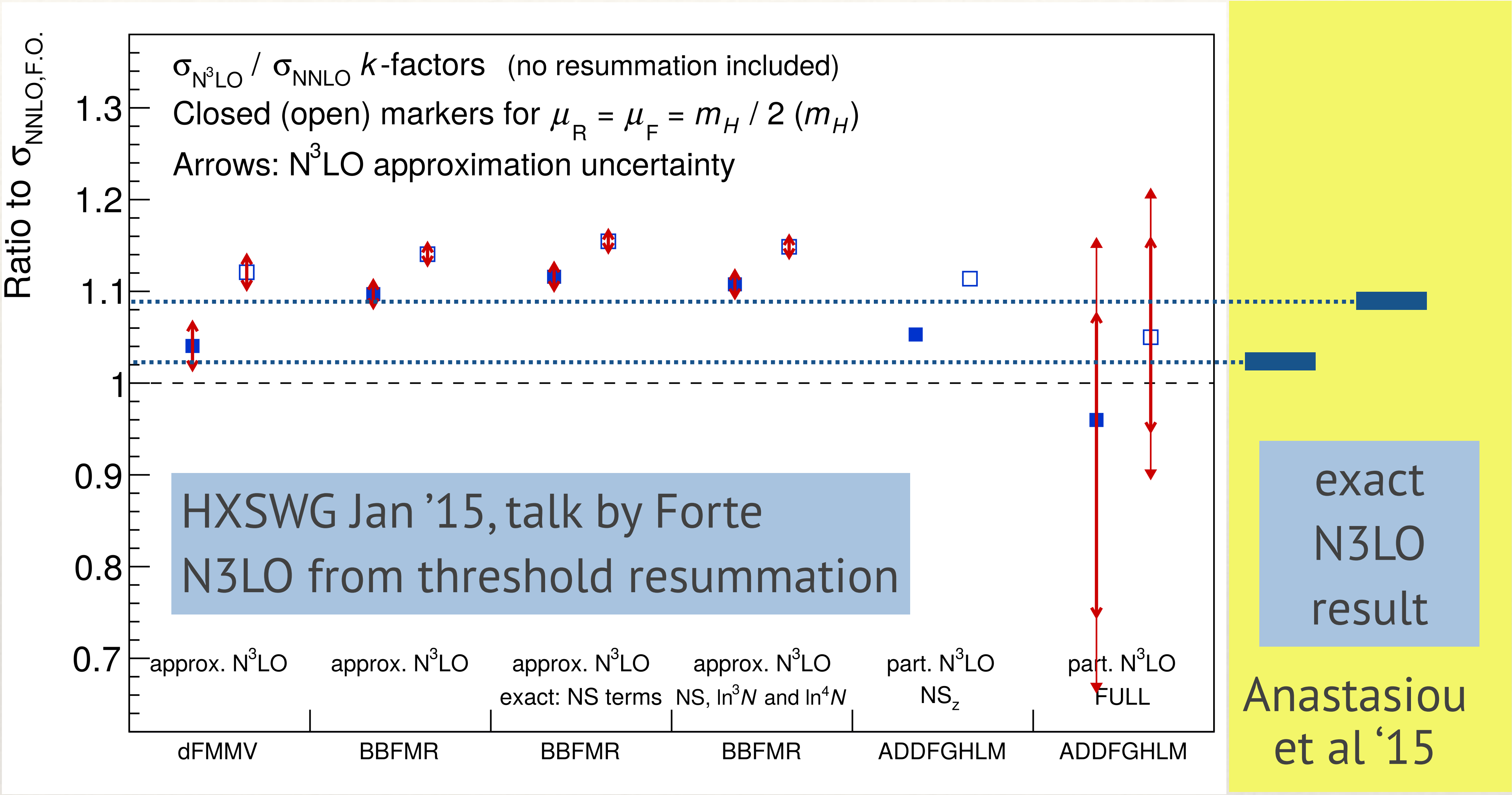
From CMS data at 7 TeV

- Asymm. Gaussian is slightly more conservative

Slide from Gavin Salam (2015)

NNLO v. NNLL+NNLO?

$N^3LO/NNLO$ k-FACTOR in gluon fusion → Higgs



In case of Higgs production (only process known at N3LO), threshold approx.for N3LO was off by 2−10%.

We will consider results with and without NNLL

Combination input (1): Measurements

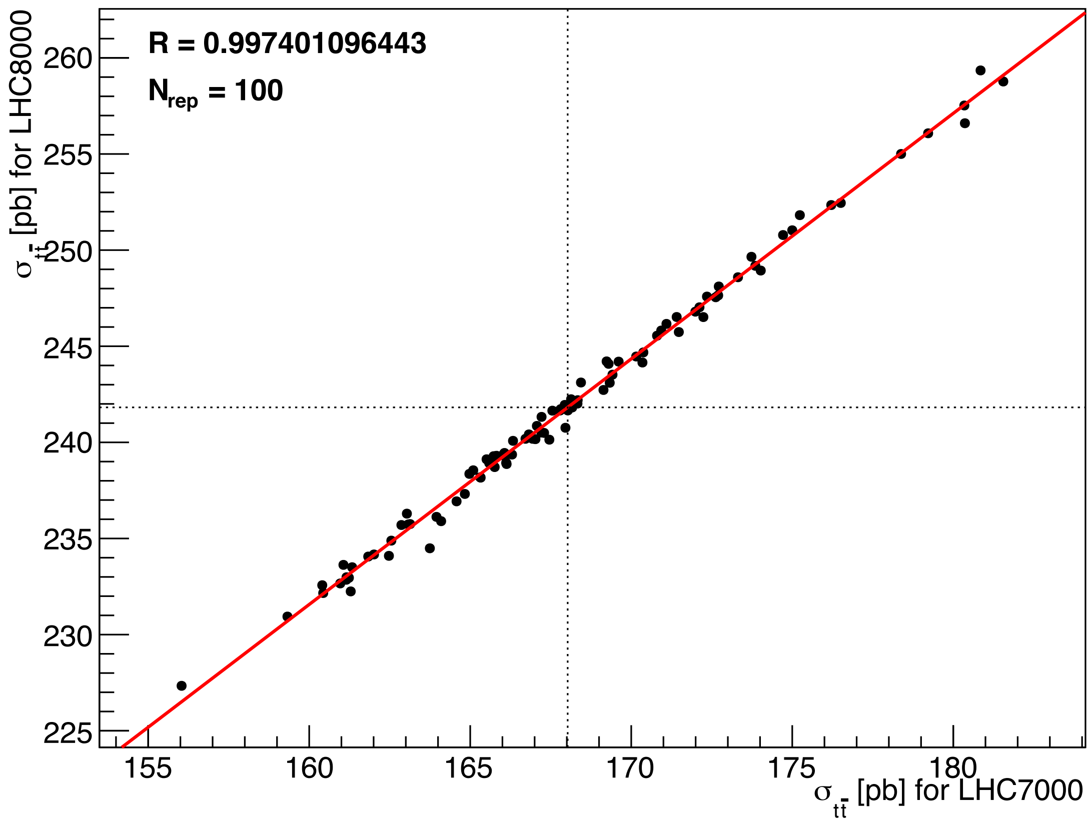
- Currently 5 measurements of σ_{tt} are considered:
  - ATLAS:
    - @ 7 TeV: 182.9 ± 3.1 (stat.) ± 4.2 (syst.) ± 3.6 (lumi.) pb
    - @ 8 TeV: 242.4 ± 1.7 (stat.) ± 5.5 (syst.) ± 7.5 (lumi.) pb
  - CMS:
    - @ 7 TeV: 173.6 ± 2.1 (stat.) ± 4.3 (syst.) ± 3.8 (lumi.) pb
    - @ 8 TeV: 244.9 ± 1.4 (stat.) ± 5.9 (syst.) ± 6.4 (lumi.) pb
  - Tevatron (D0 and CDF combination)
    - @ 1.96 TeV: 7.60 ± 0.20 (stat.) ± 0.29 (syst.) ± 0.21 (lumi.) pb
- Each experiment produces:
    - 1 α_s center value
    - 7 α_s uncertainties:
      - **stat**, **syst**, **lumi**, **Ebeam** (<u>experimental uncertainties</u>)
      - **mtop**, **pdf** and **scale** (<u>theoretical uncertainties</u>)

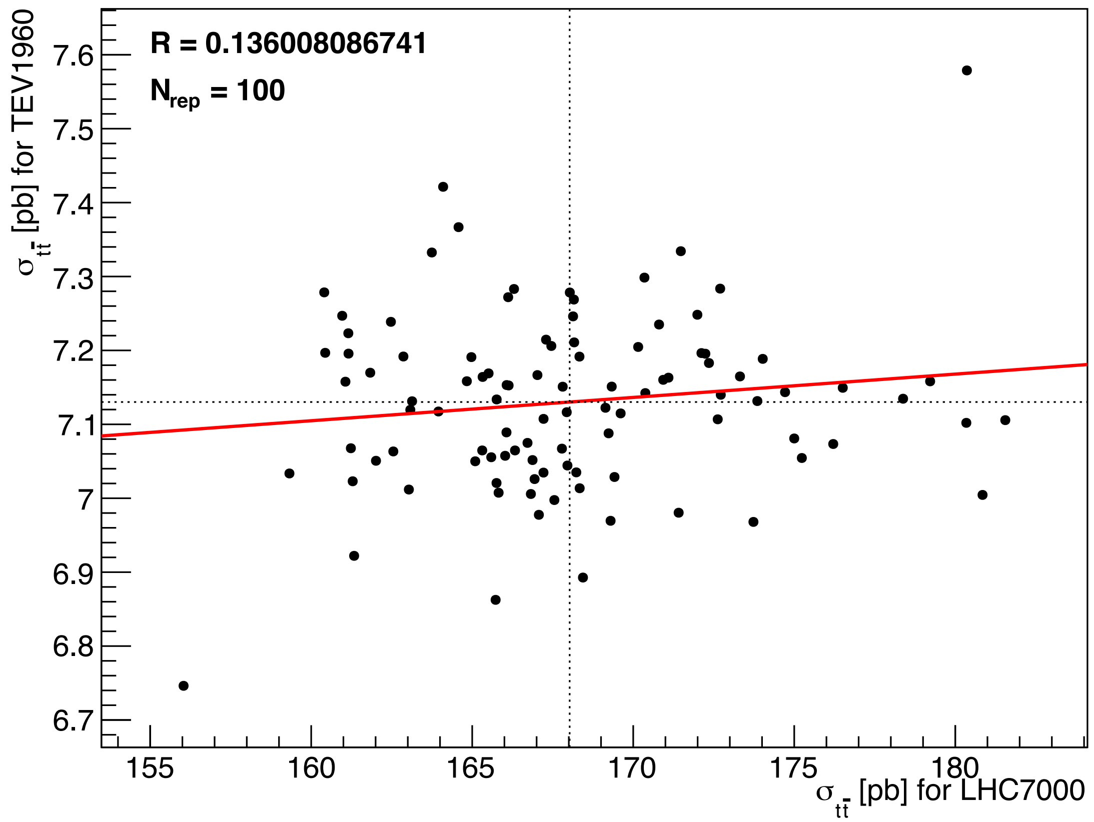
Combination input (2): Correlations

- Breakdown of chosen correlation values:
  - **stat**: ρ = 0.0 between all measurements
  - **syst**: ρ = 1.0 for measurements at the same experiment, 0.0 elsewhere
  - **lumi**: Partly correlated for measurements at the same center of mass energy, 0.0 elsewhere
    - Bunch current uncertainty is the same for CMS and ATLAS (100% correlated)
    - Individual luminosity determinations are considered uncorrelated
  - **Ebeam**: ρ = 1.0 between all LHC experiments, ρ = 0.0 between LHC and Tevatron

Combination input (3): Correlations

- Breakdown of chosen correlation values:
  - **scale**: ρ = 1.0 between all LHC measurements, ρ = 0.5 between LHC and Tevatron measurements
  - **mtop**: ρ = 1.0 for all measurements
  - **pdf**: ρ can be determined by calculating the correlation coefficient of the PDF members





pdf plotted here: NNPDF2.3 (NNLO)

BLUE in more detail

Best **L**inear **U**nbiased **E**stimate:

- Method to combine measurements with correlated error sources
- The center value from the combination is a linear combination of the inputs:

$$y_{BLUE} = \sum_i w_i \, y_i$$

y_i: center value from extraction i ; w_i: weight given to experiment i

- Weights are set so that σ^2_{BLUE} is minimized:

$$\sigma^2_{BLUE} = w^T \, \mathbf{E} \, w$$

Where **E** is the error matrix:

$$\mathbf{E}_{\text{one error source}} = \begin{bmatrix} \sigma_1^2 & \dots & \rho_{1k}\,\sigma_1\sigma_k \\ \vdots & \ddots & \vdots \\ \rho_{k1}\,\sigma_k\sigma_1 & \dots & \sigma_k^2 \end{bmatrix}, \quad \mathbf{E} = \sum_i \mathbf{E}_i$$